ECE 486 (Control Systems) – Homework 1

Due: Sep. 9, midnight

Problem 1. (15 points) Calculate the following Laplace transforms $F_i = \mathcal{L}\{f_i\}$ **by hand**:

(a) $f_1(t) = 3\cos(t) + 2\sin(t)$

Hint: Recall Euler's formula.

(b) $f_2(t) = e^{-3t}$

(c) $f_3(t) = 3\cos(t) + 2\sin(t) + e^{-3t}$

Problem 2. (30 points) Consider the following four transfer functions.

$$H_1(s) = \frac{4}{s+10} \qquad H_2(s) = \frac{4}{s-10} \qquad H_3(s) = \frac{4}{s^2+13s+30} \qquad H_4(s) = \frac{4}{s^2+2s+2}$$

For each part below, answer the questions for $H_i, i = 1, 2, 3, 4$ in turn before proceeding to next part.

(a) What are the poles and zeros? (5 points)
(b) What is the general form of the free response? (5 points)
(c) What is the general form of the forced response? (5 points)
(d) Recall that a step response is the output of the system when all the initial conditions are zero and input is as follows:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Compute the step responses **by hand**. Calculate the steady-state value of each step response. (5 points)
(e) Next, calculate the response with zero initial conditions and the following input (5 points):

$$u(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(f) Finally, calculate the response with zero initial conditions and the following input (5 points):

$$u(t) = \begin{cases} t^2 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Problem 3. (20 points) Consider the following transfer functions:

(a) $H_1(s) = \dfrac{1}{s^2 - s + 3}$

(b) $H_2(s) = \dfrac{s-3}{s^2+5s+4}$

(c) $H_3(s) = \dfrac{3s+7}{s^2+8s+7}$

(d) $H_4(s) = \dfrac{5}{s^2+6s+25}$

Calculate $\lim_{s\to 0} H_i(s)$. Use the MATLAB command `step` to plot their step responses, and attach the plots (the command `ltiview` may also be useful, be sure to select plot pages during upload to Gradescope). Can the Final Value Theorem be invoked? What is the DC gain?

Problem 4. (35 points) Sketch the response $y(t)$ vs. t for the system, initial conditions, and input given below. Label the steady-state value of y and the approximate settling time. Also label the approximate peak value of y. Specify whether the system is over or underdamped.

$$\ddot{y} + 6\dot{y} + 25y = 25u, \tag{1}$$

$$y(0) = 0,\ \dot{y}(0) = 0,\ u(t) = \begin{cases} 0 & t < 0 \text{ sec} \\ 4 & t \geq 0 \text{ sec} \end{cases} \tag{2}$$

Finally, suppose you are given the following time-domain specs: rise time $t_r \leq 0.6$ and settling time $t_s \leq 1.6$. (Here we're considering settling time to within 5% of the steady-state value.) Plot the admissible pole locations in the s-plane corresponding to these two specs. Does this system satisfy these specs?