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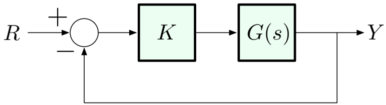
Reading: FPE, Section 6.1

Frequency-Response Design Method: Main Idea



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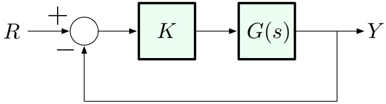

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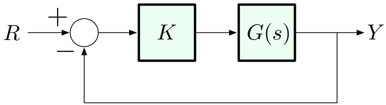

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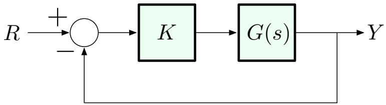

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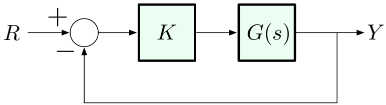

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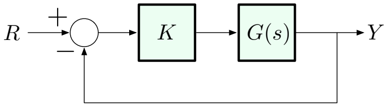

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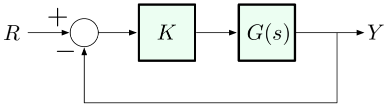

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1. Bode plots: magnitude $|KG(j\omega)|$ and phase $\angle KG(j\omega)$ vs. frequency ω (could have seen it earlier, in ECE 342)
2. Nyquist plots: $\text{Im}\big(KG(j\omega)\big)$ vs. $\text{Re}\big(K(j\omega)\big)$ [Cartesian plot in s-plane] as ω ranges from $-\infty$ to $+\infty$

Scale Convention for Bode Plots

	magnitude	phase
horizontal scale	log	log
vertical scale	log	linear

Advantage of the scale convention: we will learn to do Bode plots by starting from simple factors and then building up to general transfer functions by considering products of these simple factors.

Bode Form of the Transfer Function

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$$\text{rewrite as} \quad \left.\frac{3K\left(\frac{s}{3}+1\right)}{4s\left(\left(\frac{s}{2}\right)^2+\frac{s}{2}+1\right)}\right|_{s=j\omega}$$

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$$= \underbrace{\frac{3K}{4}}_{=K_0}\frac{\frac{j\omega}{3}+1}{j\omega\left(\left(\frac{j\omega}{2}\right)^2+\frac{j\omega}{2}+1\right)}$$

Three Types of Factors

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In our example above,

$$KG(j\omega) = \frac{3K}{4}\frac{\frac{j\omega}{3} + 1}{j\omega\left[\left(\frac{j\omega}{2}\right)^2 + \frac{j\omega}{2} + 1\right]}$$

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Now let's discuss Bode plots for factors of each type.

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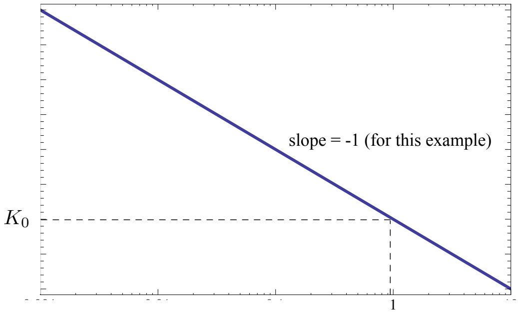

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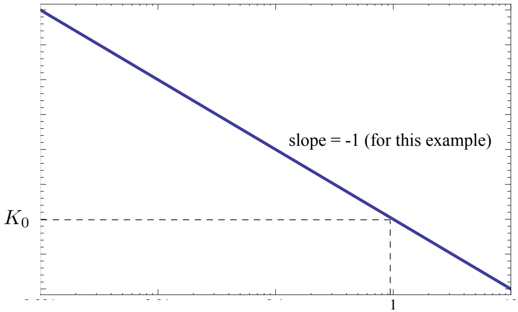

— this is called a low-frequency asymptote (will see why later)

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Phase: $\angle K_0(j\omega)^n = \angle(j\omega)^n = n\angle j\omega = n \cdot 90^\circ$
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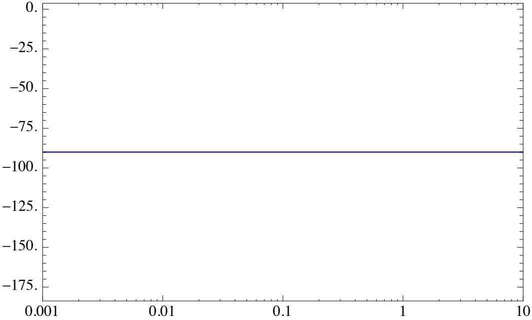

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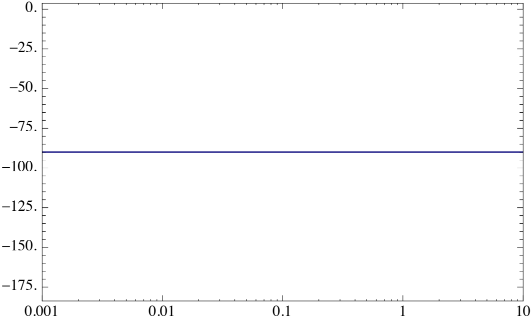

— here, the phase is -90° for all ω.

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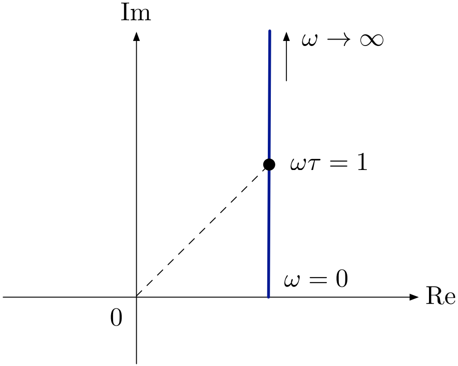

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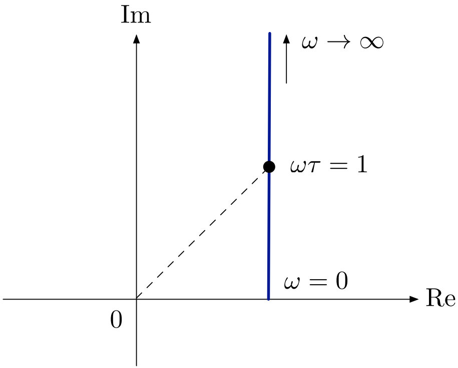

For $\omega\tau \ll 1,\ j\omega\tau + 1 \approx 1$

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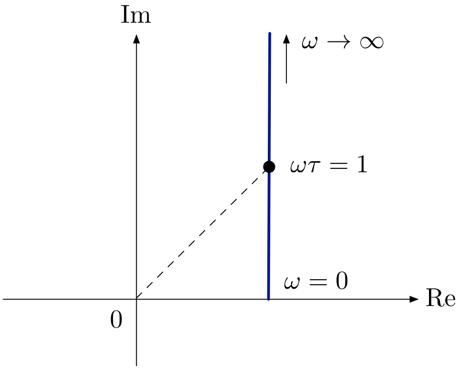

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Transition:

$$\omega\tau = 1 \iff \omega = 1/\tau$$

— this is the breakpoint

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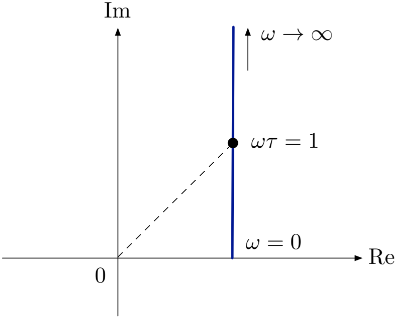
Magnitude:



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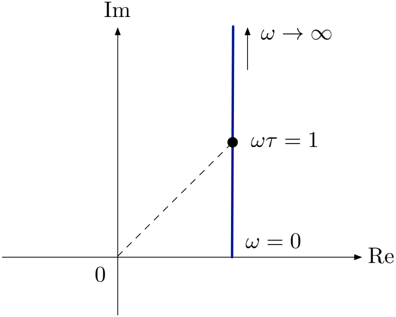

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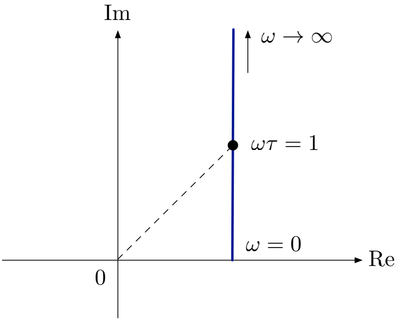

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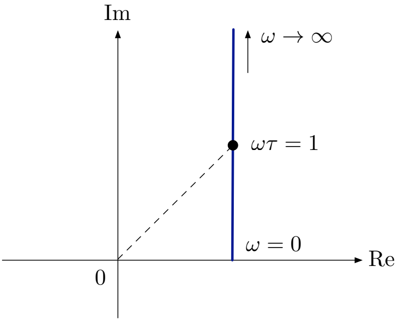

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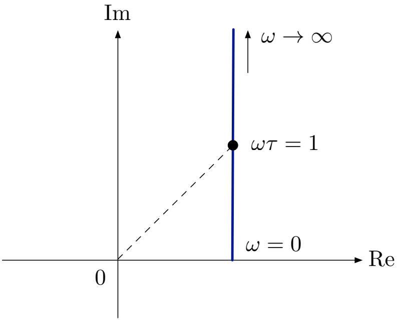

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  – a line of slope 1 passing through the point $(1/\tau, 1)$ (log-log scale)
- Careful: these are just *asymptotes* (the actual value of M at $\omega = 1/\tau$ is $\sqrt{2}$)

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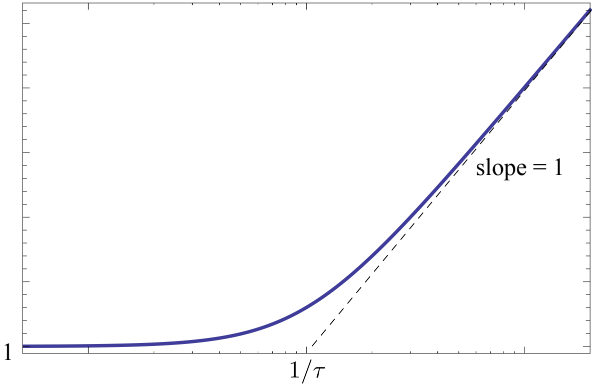
Magnitude plot:



For a stable real zero, the magnitude slope "steps up by 1" at the break-point.

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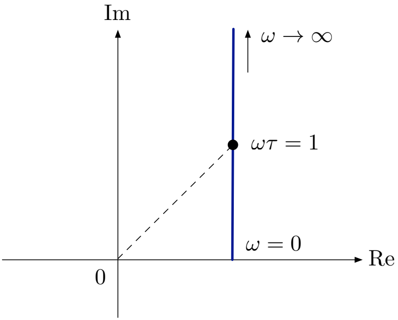
Phase:



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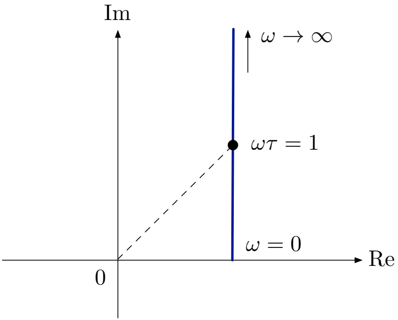

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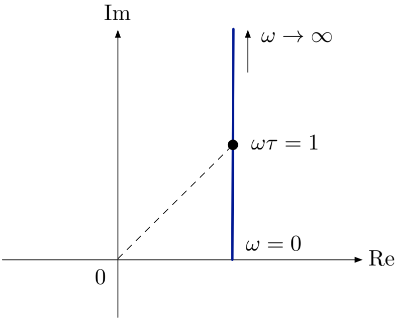

- For small ω (below break-point), $\phi \approx 0°$
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$$\begin{aligned}\phi &\approx \angle(j\omega\tau) \\ &= 90°\end{aligned}$$

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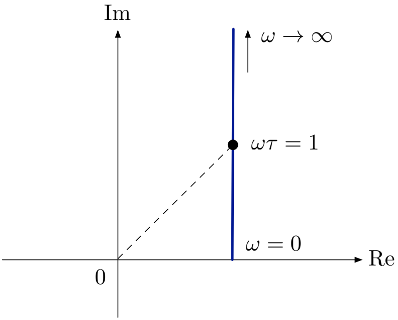

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- At break-point ($\omega\tau = 1$),

$$\begin{aligned}\phi &= \angle(j + 1) \\ &= 45°\end{aligned}$$

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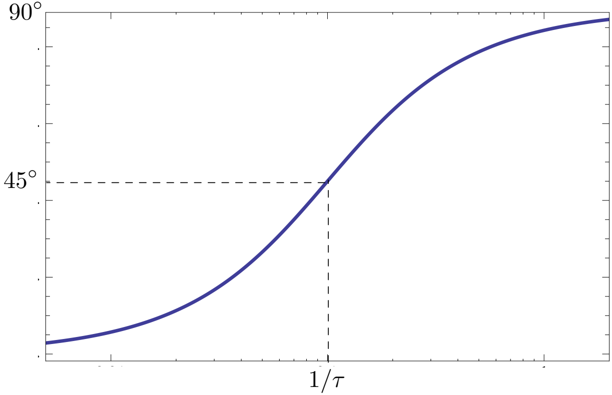
Phase plot:



For a stable real zero, the phase "steps up by 90°" as we go past the break-point.

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Example: Type 1 and Type 2 Factors

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$$KG(j\omega) = \frac{2000 \cdot 0.5 \cdot \left(\frac{j\omega}{0.5} + 1\right)}{10 \cdot 50 \cdot j\omega \left(\frac{j\omega}{10} + 1\right)\left(\frac{j\omega}{50} + 1\right)}$$

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$$= \frac{2}{j\omega} \cdot \left(\frac{j\omega}{0.5}+1\right) \cdot \frac{1}{\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{50}+1\right)}$$

Example 1: Magnitude

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Type 1 term:

- $K_0 = 2, n = -1$ — it contributes a line of slope -1 passing through the point $(\omega = 1, M = 2)$.

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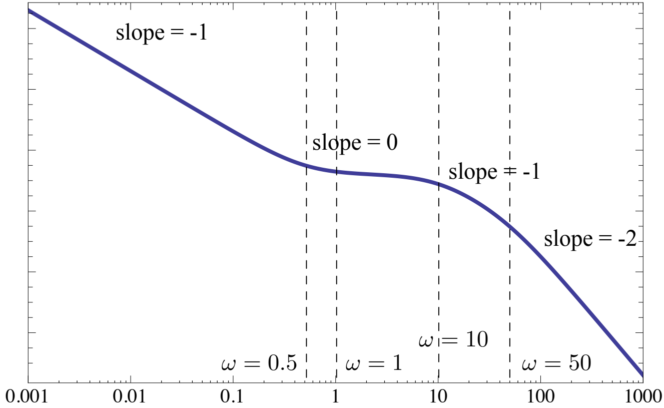

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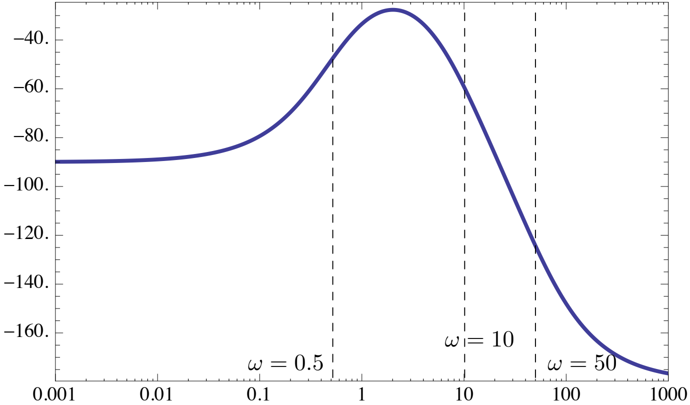

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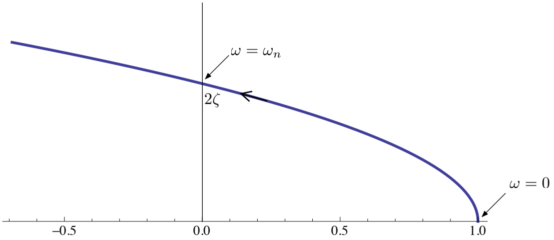
And here is the Nyquist plot, for $0 < \omega < \infty$:



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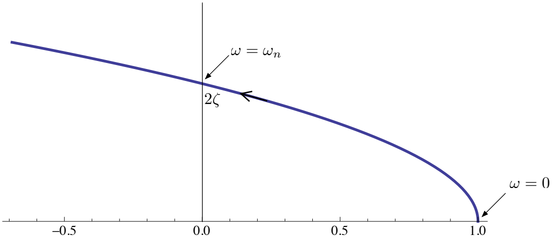

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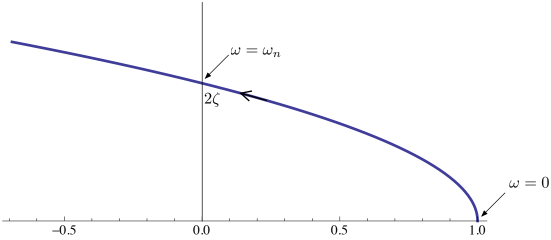

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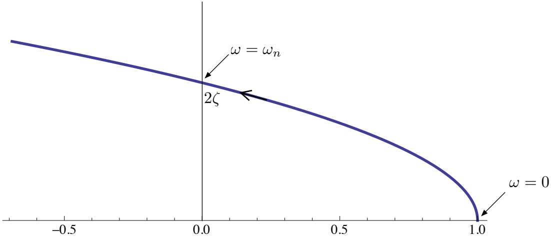

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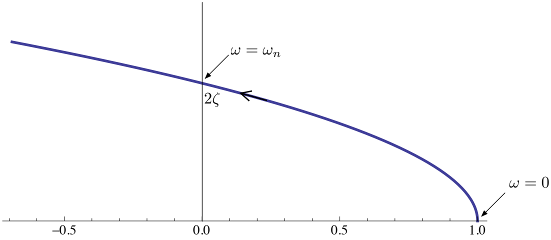

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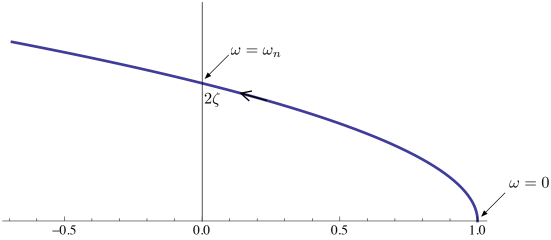

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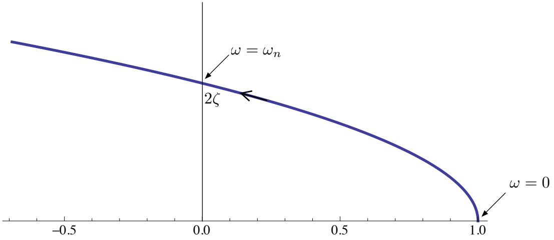

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What happens as $\omega \to \infty$?

- real part $\approx -(\omega/\omega_n)^2 \to -\infty$, quadratic in ω
- imaginary part $= 2\zeta(\omega/\omega_n) \to \infty$, linear in ω

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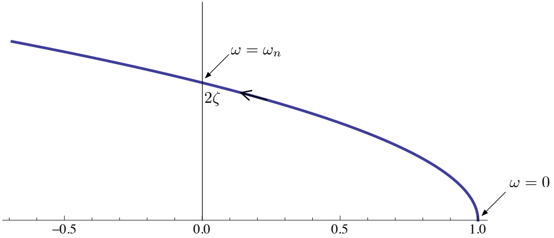

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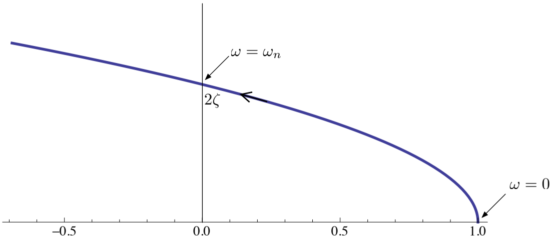

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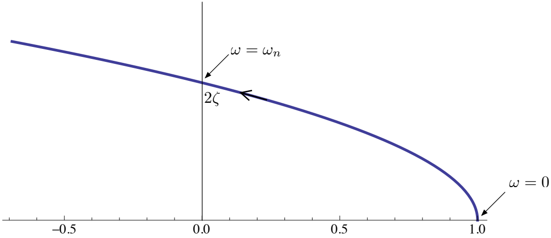

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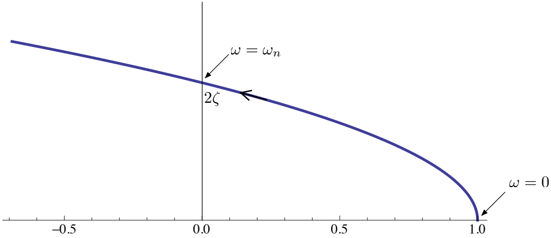

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- for $\omega \gg \omega_n$, $M \approx \left(\frac{\omega}{\omega_n}\right)^2 \Rightarrow \log M \approx 2\log\omega - 2\log\omega_n$

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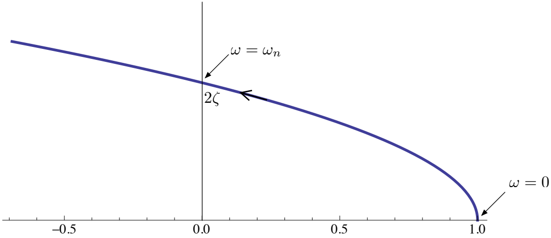

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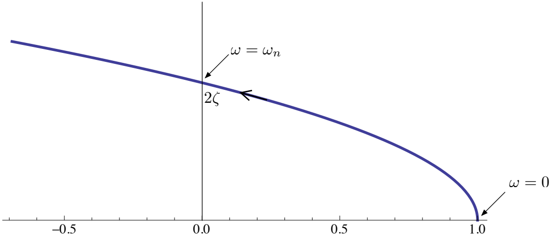
- for $\omega \gg \omega_n$, $M \approx \left(\frac{\omega}{\omega_n}\right)^2 \Rightarrow \log M \approx 2\log\omega - 2\log\omega_n$
  The asymptote is a line of slope 2 passing through the point $(\omega = \omega_n, M = 1)$

Type 3: $\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1$, Magnitude



Nyquist plot
$(0 < \omega < \infty)$

$$(R(\omega), I(\omega)) = \left(1 - \left(\frac{\omega}{\omega_n}\right)^2, 2\zeta\frac{\omega}{\omega_n}\right)$$

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For a stable complex zero, the magnitude slope steps up by 2 as we go through the breakpoint.

Type 3: $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1\right]^{-1}$

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Magnitude:

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Phase:

$$\phi = \angle\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1} = -\angle\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1\right]$$

Type 3: Magnitude, Complex Pole Case

How does the magnitude plot look?

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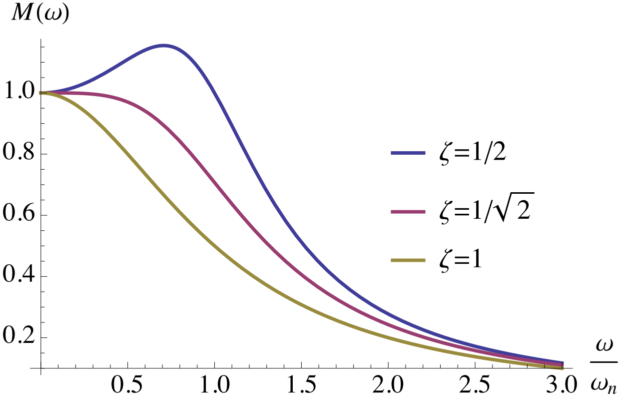
How does the magnitude plot look? Depends on the value of ζ:



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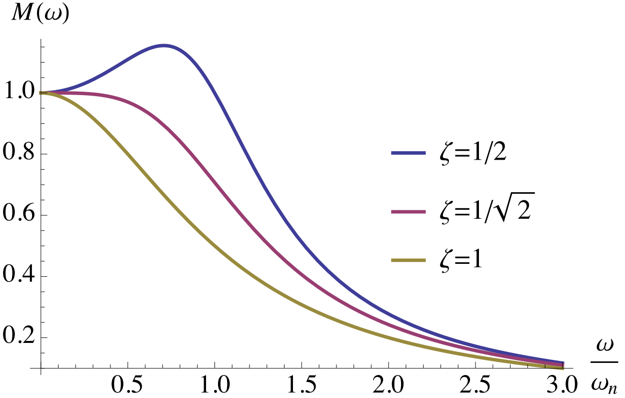

The magnitude hits its peak value (for $\zeta < 1/\sqrt{2} \approx 0.707$) occurs when $\omega = \omega_r$, where

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2} < \omega_n$$

Type 3: Magnitude

For small enough ζ (below $1/\sqrt{2}$), the magnitude of

$$\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1}$$

has a resonant peak at the resonant frequency

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2}.$$

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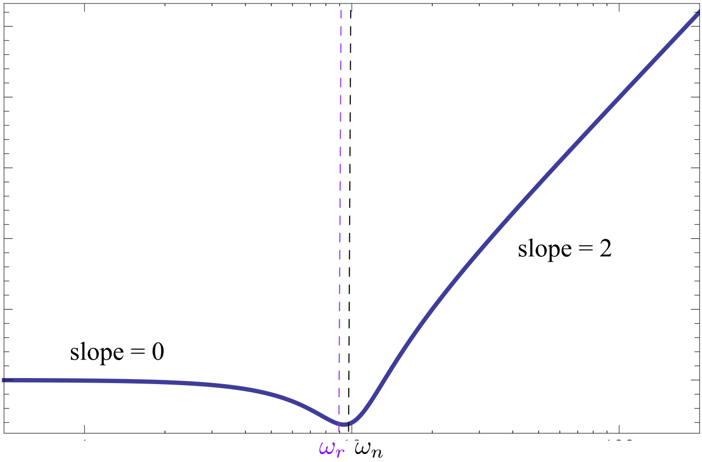
$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2}.$$

Likewise, the magnitude of

$$\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1$$

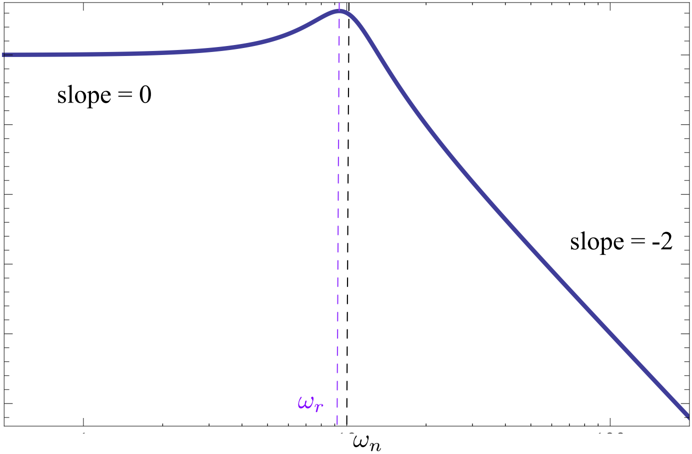
has a resonant dip at ω_r.

Type 3 Zero: Magnitude



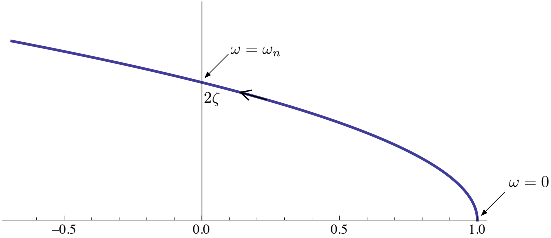
For a stable real zero, the magnitude slope “steps up by 2” at the break-point.

Type 3 Pole: Magnitude



For a stable real pole, the magnitude slope "steps down by 2" at the break-point.

Type 3: $\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1$, Phase



Nyquist plot
$(0 < \omega < \infty)$

$$(R(\omega), I(\omega))$$
$$= \left(1 - \left(\frac{\omega}{\omega_n}\right)^2, 2\zeta\frac{\omega}{\omega_n}\right)$$

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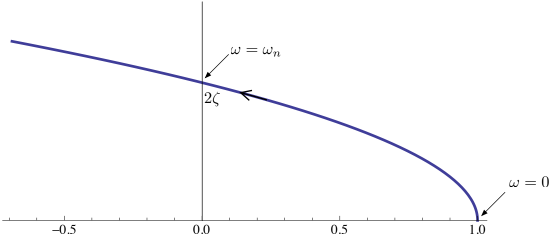

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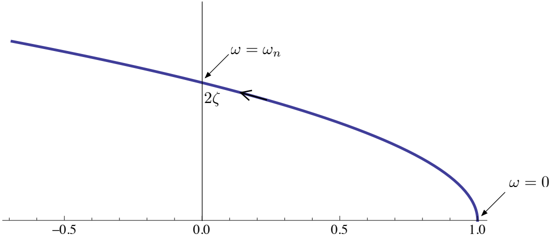

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Phase:

- for $\omega \ll \omega_n$, $\phi \approx 0^\circ$ (real and positive)

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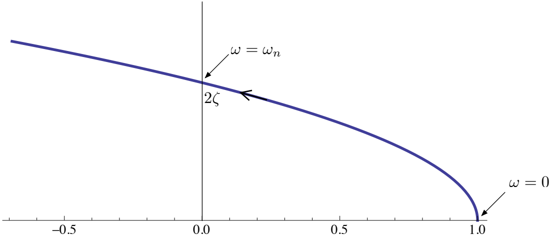

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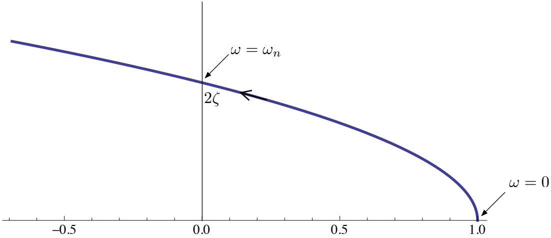

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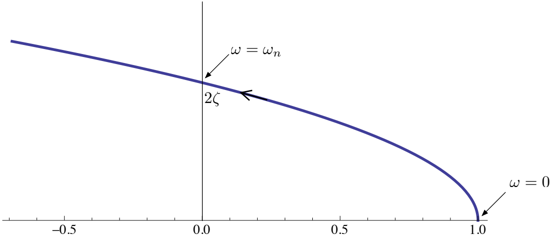

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For a stable complex zero, the phase steps up by 180° as we go through the breakpoint; as $\zeta \to 0$, the transition through the break-point gets sharper, almost step-like.

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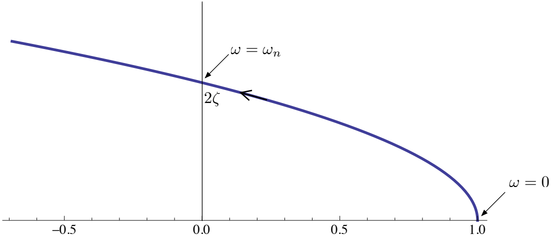

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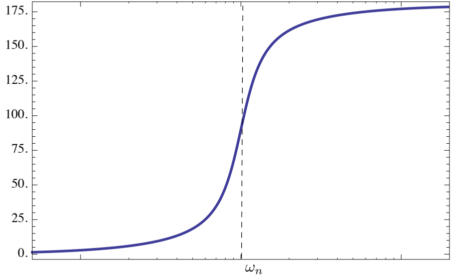
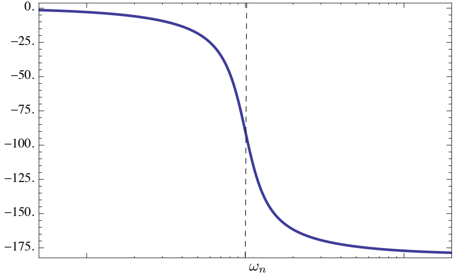
For a pole, the phase is multiplied by -1.

Type 3: Phase



(stable complex zero — phase steps up by 180°)



(stable complex pole — phase steps down by 180°)

Example 2

$$KG(s) = \frac{0.01\left(s^2 + 0.01s + 1\right)}{s^2\left(\dfrac{s^2}{4} + 0.02\dfrac{s}{2} + 1\right)}$$

— already in Bode form

Example 2

$$KG(s) = \frac{0.01\left(s^2 + 0.01s + 1\right)}{s^2\left(\frac{s^2}{4} + 0.02\frac{s}{2} + 1\right)} \qquad \text{— already in Bode form}$$

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- low-frequency term $\dfrac{0.01}{(j\omega)^2}$ with $K_0 = 0.01$, $n = -2$

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- complex zero with break-point at $\omega_n = 1$ and $\zeta = 0.005$

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- complex pole with break-point at $\omega_n = 2$ and $\zeta = 0.01$

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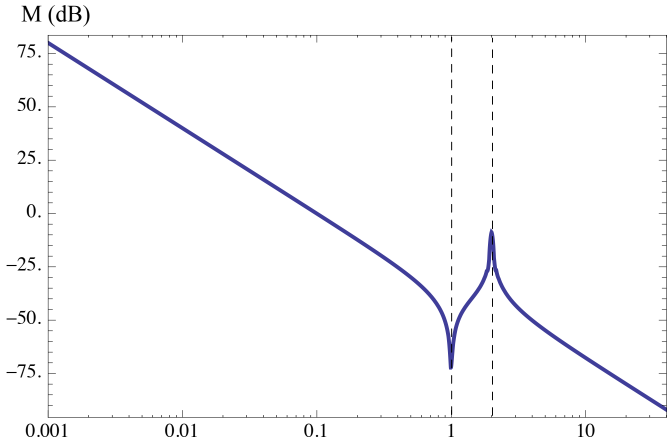
- complex pole with break-point at $\omega_n = 2$ and $\zeta = 0.01$ — slope down by 2; large resonant peak

Example 2: Magnitude Plot



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What can we tell about phase?

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  — phase starts at $n \times 90^\circ = -180^\circ$

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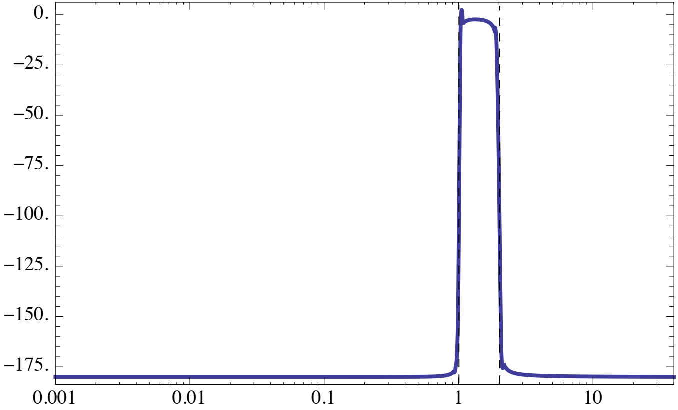
- complex zero with break-point at $\omega_n = 1$ — phase up by 180°
- complex pole with break-point at $\omega_n = 2$ — phase down by 180°
- since ζ is small for both pole and zero, the transitions are very sharp

Example 2: Phase Plot



Unstable Zeros/Poles?

So far, we've only looked at transfer functions with stable poles and zeros (except perhaps at the origin). What about RHP?

Example: consider two transfer functions,

$$G_1(s) = \frac{s+1}{s+5} \qquad \text{and} \qquad G_2(s) = \frac{s-1}{s+5}$$

Note:

- G_1 has stable poles and zeros; G_2 has a RHP zero.
- Magnitude plots of G_1 and G_2 are the same —

$$|G_1(j\omega)| = \left|\frac{j\omega+1}{j\omega+5}\right| = \sqrt{\frac{\omega^2+1}{\omega^2+5}}$$

$$|G_2(j\omega)| = \left|\frac{j\omega-1}{j\omega+5}\right| = \sqrt{\frac{\omega^2+1}{\omega^2+5}}$$

- All the difference is in the phase plots!

Phase Plot for G_1

$$G_1(j\omega) = \frac{j\omega + 1}{j\omega + 5} = \frac{1}{5}\frac{j\omega + 1}{\frac{j\omega}{5} + 1}$$

Phase Plot for G_1

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- Low-frequency term: $\frac{1}{5}(j\omega)^0$

Phase Plot for G_1

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- Break-points at $\omega_n = 1$

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- Break-points at $\omega_n = 1$ (phase goes up by $90°$)

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$$G_1(j\omega) = \frac{j\omega + 1}{j\omega + 5} = \frac{1}{5}\frac{j\omega + 1}{\dfrac{j\omega}{5} + 1}$$

- Low-frequency term: $\dfrac{1}{5}(j\omega)^0$— $n = 0$, so phase starts at $0°$
- Break-points at $\omega_n = 1$ (phase goes up by $90°$) and at $\omega_n = 5$

Phase Plot for G_1

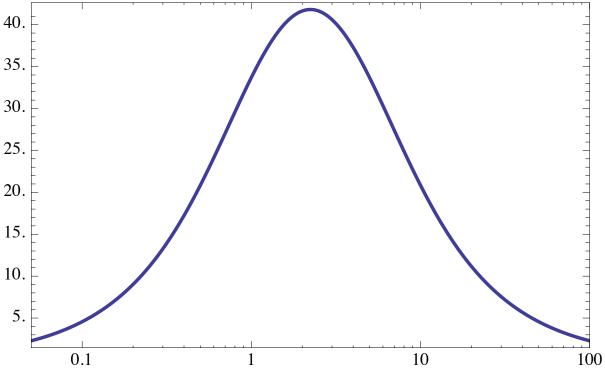
$$G_1(j\omega) = \frac{j\omega + 1}{j\omega + 5} = \frac{1}{5} \frac{j\omega + 1}{\frac{j\omega}{5} + 1}$$

- Low-frequency term: $\frac{1}{5}(j\omega)^0$— $n = 0$, so phase starts at $0°$
- Break-points at $\omega_n = 1$ (phase goes up by $90°$) and at $\omega_n = 5$ (phase goes down by $90°$)

Phase Plot for G_1

$$G_1(j\omega) = \frac{j\omega + 1}{j\omega + 5} = \frac{1}{5} \frac{j\omega + 1}{\frac{j\omega}{5} + 1}$$

- Low-frequency term: $\frac{1}{5}(j\omega)^0$— $n = 0$, so phase starts at 0°
- Break-points at $\omega_n = 1$ (phase goes up by 90°) and at $\omega_n = 5$ (phase goes down by 90°)



Phase Plot for G_2

$$G_2(j\omega) = \frac{j\omega - 1}{j\omega + 5} = \frac{1}{5}\frac{j\omega - 1}{\frac{j\omega}{5} + 1}$$

Phase Plot for G_2

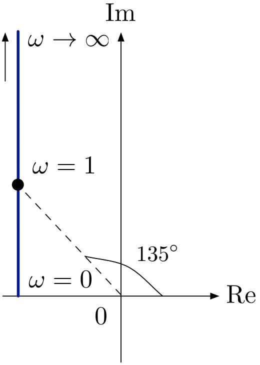
$$G_2(j\omega) = \frac{j\omega - 1}{j\omega + 5} = \frac{1}{5}\,\frac{j\omega - 1}{\frac{j\omega}{5} + 1}$$

Let's do a Nyqiust plot for $j\omega - 1$:

Phase Plot for G_2

$$G_2(j\omega) = \frac{j\omega - 1}{j\omega + 5} = \frac{1}{5}\frac{j\omega - 1}{\frac{j\omega}{5} + 1}$$

Let's do a Nyqiust plot for $j\omega - 1$:



Phase Plot for G_2

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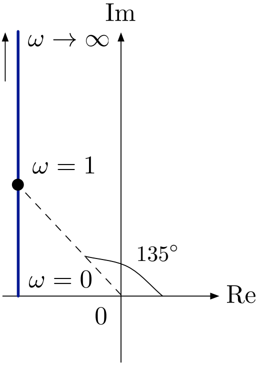

New type of behavior —

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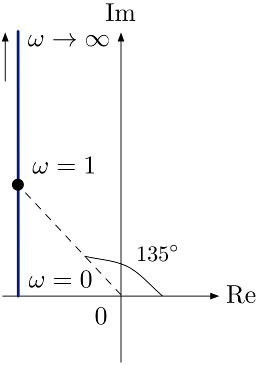

New type of behavior —

- $\omega \approx 0$: $\phi \approx 180°$ (real and negative)

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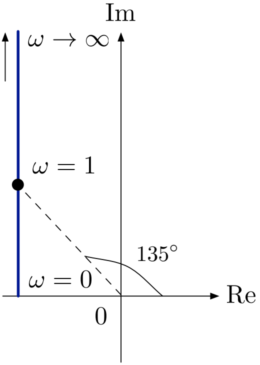


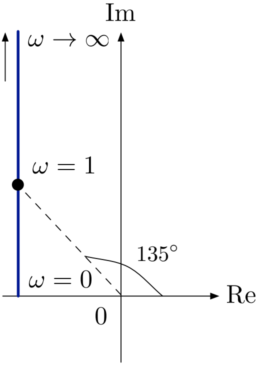
New type of behavior —

- $\omega \approx 0$: $\phi \approx 180°$ (real and negative)
- $\omega \gg 1$: $\phi \approx 90°$ ($\mathrm{Re} = -1$, $\mathrm{Im} = \omega \gg 1$)

Phase Plot for G_2

$$G_2(j\omega) = \frac{j\omega - 1}{j\omega + 5} = \frac{1}{5}\frac{j\omega - 1}{\frac{j\omega}{5} + 1}$$

Let's do a Nyqiust plot for $j\omega - 1$:



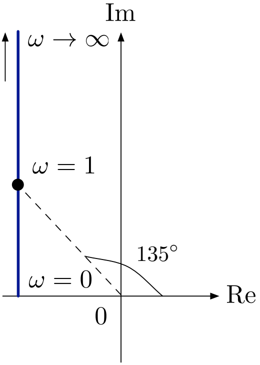
New type of behavior —

- $\omega \approx 0$: $\phi \approx 180°$ (real and negative)
- $\omega \gg 1$: $\phi \approx 90°$ (Re $= -1$, Im $= \omega \gg 1$)
- $\omega \approx 1$: $\phi \approx 135°$

Phase Plot for G_2

$$G_2(j\omega) = \frac{j\omega - 1}{j\omega + 5} = \frac{1}{5}\frac{j\omega - 1}{\frac{j\omega}{5} + 1}$$

Let's do a Nyqiust plot for $j\omega - 1$:

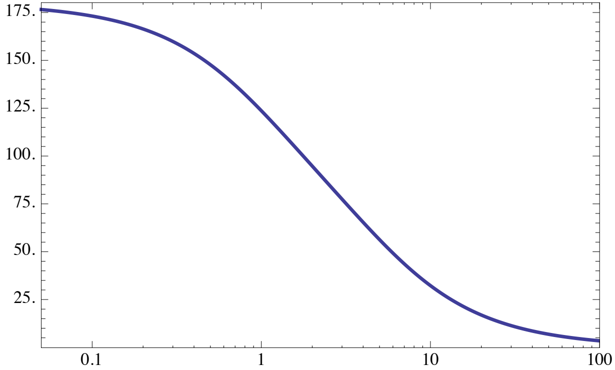


New type of behavior —

- $\omega \approx 0$: $\phi \approx 180^\circ$ (real and negative)
- $\omega \gg 1$: $\phi \approx 90^\circ$ (Re $= -1$, Im $= \omega \gg 1$)
- $\omega \approx 1$: $\phi \approx 135^\circ$

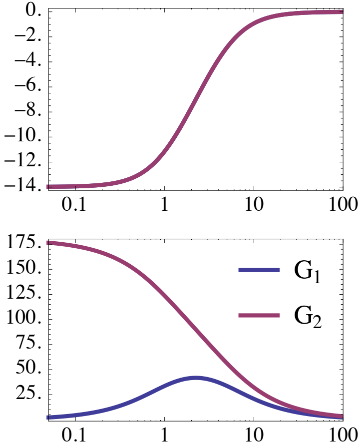
For a RHP zero, the phase starts out at 180° and goes down by 90° through the break-point (135° at break-point).

Phase Plot for G_2



For a RHP zero, the phase plot is similar to what we had for a LHP pole: goes down by 90° ... However, it starts at 180°, and not at 0°.

Minimum-Phase and Nonminimum-Phase Zeros



Among all transfer functions with the same magnitude plot, the one with only LHP zeros has the minimal net phase change as ω goes from 0 to ∞ — hence the term *minimum-phase* for LHP zeros.