**ECE 486** **First Midterm Exam: Sample**

This is a *closed-book closed-notes* test, no calculators allowed. Some useful facts are summarized in the back of the exam. For scratch work you may use backs of pages and the additional sheets provided. You have four problems to solve in 80 minutes.

Good luck!

**Full Name:** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

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|---|---|---|
| Problem 1: | . . . . . | / 15 points |
| Problem 2: | . . . . . | / 15 points |
| Problem 3: | . . . . . | / 10 points |
| Problem 4: | . . . . . | / 10 points |
| TOTAL: | . . . . . | /50 points |

**Problem 1** (15 points) *10+5*

Consider the following linear state-space model:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = x_1$$

with zero intial conditions.

**(a)** Find the transfer function from the input $U$ to the output $Y$.

(Your answer should be a ratio of two polynomials in $s$ with real coefficients.)

**(b)** Is this transfer function stable? Justify your answer.

**Problem 2** (15 points) *5+10*

Consider the prototype second-order transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

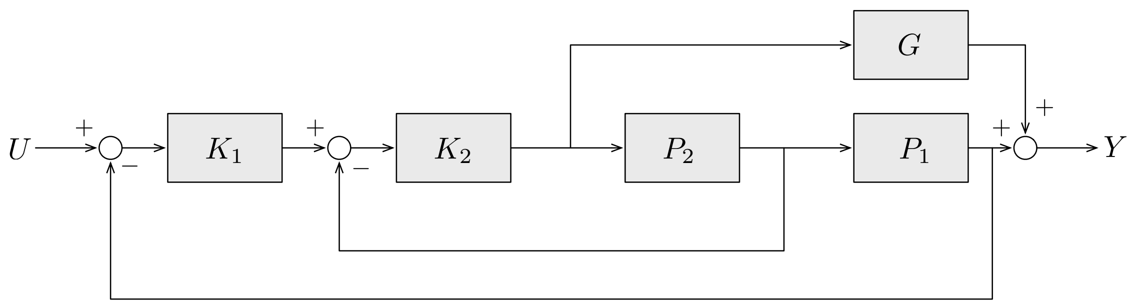
with $\omega_n > 0$ and $0 < \zeta < 1$.

**(a)** Is it possible for two such systems to have the same 5% settling time, but different values for the rise time and for the overshoot? Justify your answer!

**(b)** Write down a state-space model to realize this transfer function.

**Problem 3** (10 points)

Consider the system given by the block diagram below:

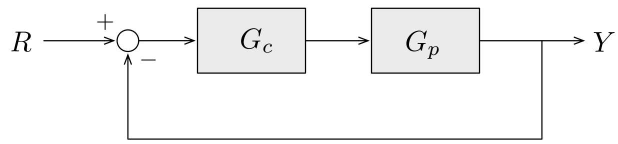



Compute the transfer function from the input $U$ to the output $Y$.

(Your answer should be an expression involving the transfer functions $K_1, K_2, P_1, P_2, G$.)

**Problem 4** (10 points)

A linear time invariant system with transfer function $G_p$ is to be controlled using unity feedback as in the diagram below:




The transfer function of the plant is $G_p(s) = \dfrac{1}{(s-2)^2}$.

A controller with transfer function $G_c(s) = K\dfrac{s+z}{s+1}$ is applied to this system.

Are there any settings for the controller gain $K$ and controller zero $z$ that would result in a stable closed-loop system? Justify your answer!

# Useful Facts

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**Unilateral Laplace transforms:**

$$f(t),\ t \geq 0 \quad \xrightarrow{\mathscr{L}} \quad F(s) = \int_0^\infty f(t) e^{-st} \mathrm{d}t,\ s \in \mathbb{C}$$

$$\mathscr{L}\left[f'(t)\right] = sF(s) - f(0)$$

$$\mathscr{L}\left[f''(t)\right] = s^2 F(s) - sf(0) - f'(0)$$

**Second-order system:**

$$\begin{aligned} H(s) &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\ &= \frac{\omega_n^2}{(s+\sigma)^2 + \omega_d^2} \qquad \omega_n > 0, 0 < \zeta < 1 \end{aligned}$$

Rise time: $t_r \approx \dfrac{1.8}{\omega_n}$

Peak time: $t_p = \dfrac{\pi}{\omega_n \sqrt{1-\zeta^2}}$

Overshoot: $M_p = \exp\left(-\dfrac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$

Settling time: $t_s^{5\%} \approx \dfrac{3}{\zeta\omega_n}$

**Stability criteria for polynomials:**

- a monic polynomial $p(s) = s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n$ is *stable* if all of its roots are in the open LHP
- 2nd-order polynomial
  $$p(s) = s^2 + a_1 s + a_2$$
  is stable if and only if $a_1, a_2 > 0$
- 3rd-order polynomial
  $$p(s) = s^3 + a_1 s^2 + a_2 s + a_3$$
  is stable if and only if $a_1, a_2, a_3 > 0$ and $a_1 a_2 > a_3$