



Electrical & Computer Engineering

GRAINGER COLLEGE OF ENGINEERING

# Lecture 5 - Nonideal Behavior of Components

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# What are components?

- Resistances, inductances, capacitances (“Real components” and 3 “Ideal Components”) Parasitic effects of passive “Real” components (resistors, capacitors, inductors)
- Interfaces : Wires, cables, tracks, bonding wires, conducting parts in materials (Carbon nanotubes) Ferromagnetic cores (“Ferrites”)
- Electromechanically components: motors, relays, switches  
Semiconductors (IC and packaging)
- ...

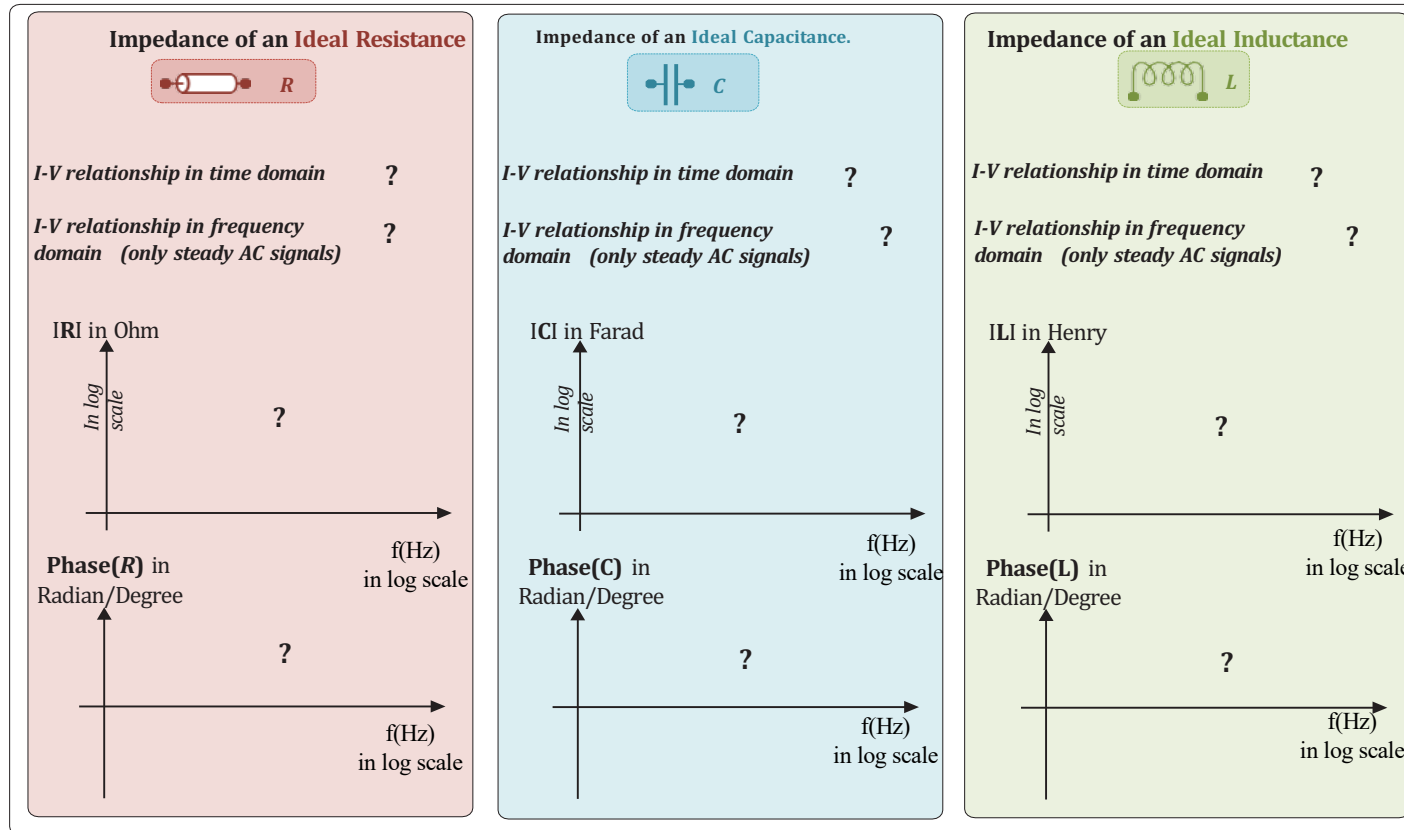


# Different kinds of Components

- 5.1 Ideal Components
  - Resistance
  - Inductance
  - Capacitance
- 5.2 ————
  - Effect of Component Leads
- 5.3 Nonideal Components
  - Nonideal Resistor
  - Nonideal Capacitor
  - Nonideal Inductor



### 5.1 Ideal component – our goal in this first part:



Understands the behavior of the three basic Ideal components: R, L, and C

In other words,

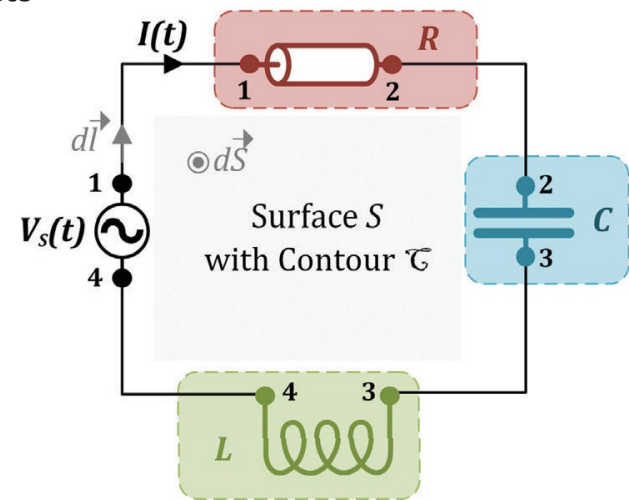
Understand the three “ideal relationships” one can establish between a voltage difference  $V(t)$  (between the terminals of a component) and the current  $I(t)$  crossing it.

## Ideal components - Intro

In this section we will consider a Circuit with Ideal Components

Some assumptions:

- Components are electrically small  
 $\lambda = c / f$ , with  $\lambda$  the wavelength,  $c$  the speed of light in vacuum and  $f$  the frequency
- Circuit is electrically small (short wires, no “interfaces” between components)
- Sinusoidal Steady State



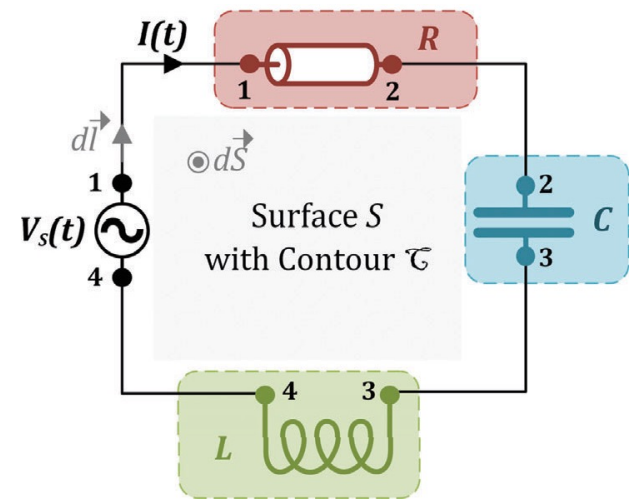
*Circuit with Ideal Components*

# Ideal Components - Intro

In this section we will consider a Circuit with Ideal Components

In the coming slides we will explore the definition of each Ideal components in this circuit:

- The Ideal Resistance,
- The Ideal Capacitance,
- The Ideal Inductance.

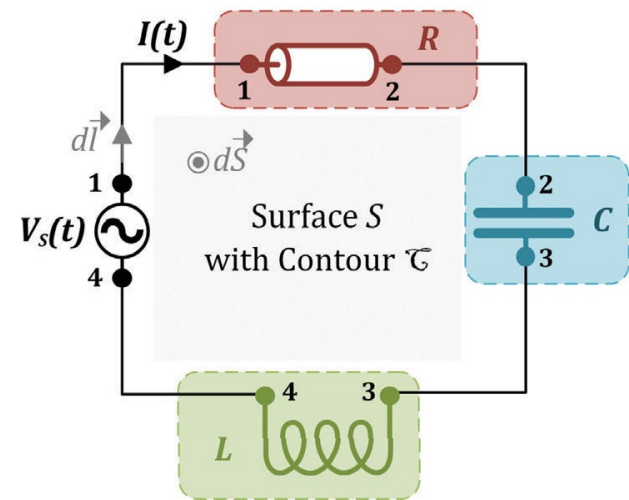


*Circuit with Ideal Components*

## Ideal Components - Intro

Between any two adjacent terminal  $a$  and  $b$ , the Voltage is defined as:

$$V_{ba}(t) = - \int_a^b \vec{E}(t) \cdot d\vec{l}$$



*Circuit with Ideal Components*

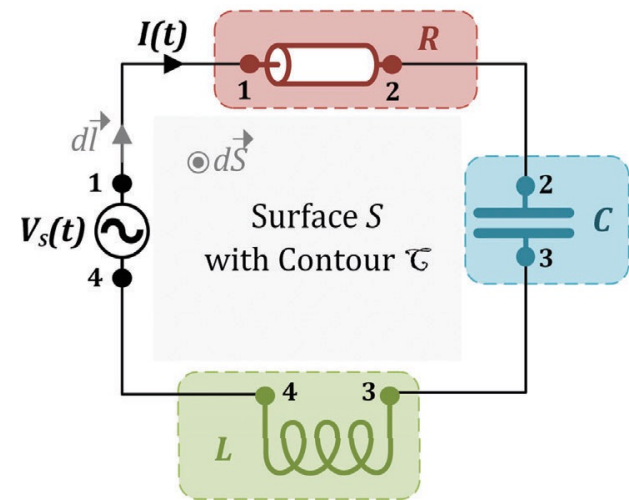
## Ideal Components - Intro

Between any two adjacent terminal  $a$  and  $b$ , the Voltage is defined as:

*“a measure (at one point between points  $b$  and  $a$ ) of how closely the electric field  $\vec{E}(t)$  aligns with the unit vector  $d\vec{l}$ , in terms of the directions they point”*

$$V_{ba}(t) = - \int_a^b \vec{E}(t) \cdot d\vec{l}$$

*“the sum of all contributions along a line between points  $b$  and  $a$ ”*



Circuit with Ideal Components

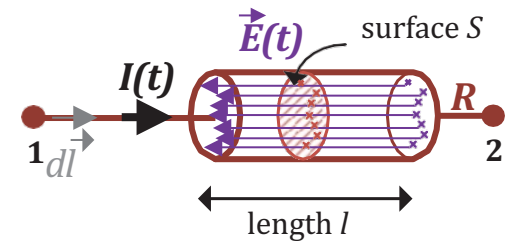
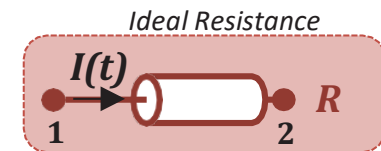
## 5.1.1 The ideal Resistance

An Electric field is pushing electron inside the resistor.

**Resistor are used to reduce current flow.**

Inside a conductive material, density of current:  $\vec{j}(t) = \sigma \vec{E}(t)$   
with  $\sigma$  the electric conductivity of the material and  $A$  the area crossed by the Electric Field.

$$V_{21}(t) = - \int_1^2 \vec{E}(t) \cdot d\vec{l}$$



## 5.1.2 Inductance

Probably the most misunderstood concept in electrical engineering  
– Do not confuse ‘inductance’ with ‘inductors’

Current loop  $\rightarrow$  inductance!

Inductances may be classified as:

- Internal inductance
- External inductance
- Self inductance
- Mutual inductance
- Loop inductance
- Partial inductance



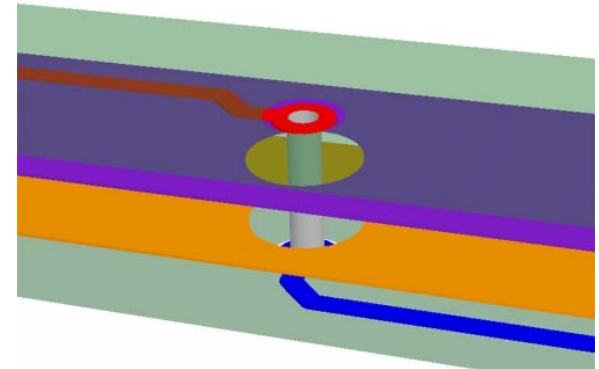
Poll: Do these have inductance?



Ground strap

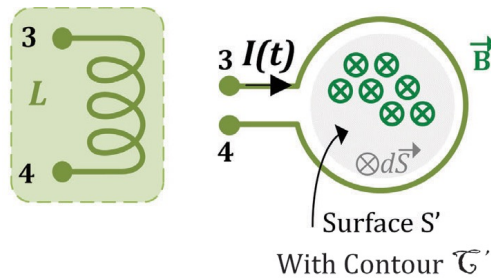


SMT capacitor



PCB via

## The ideal Inductance



Example: Coil  
with one loop

**(self-) Inductance** is the tendency of an electrical conductor to oppose a change in the electric current  $I(t)$  flowing through it.

The flow of electric current creates a magnetic field around the conductor. Any change in magnetic field through a circuit induces an **electromotive force (EMF) (voltage)** in the conductors, a process known as electromagnetic induction.

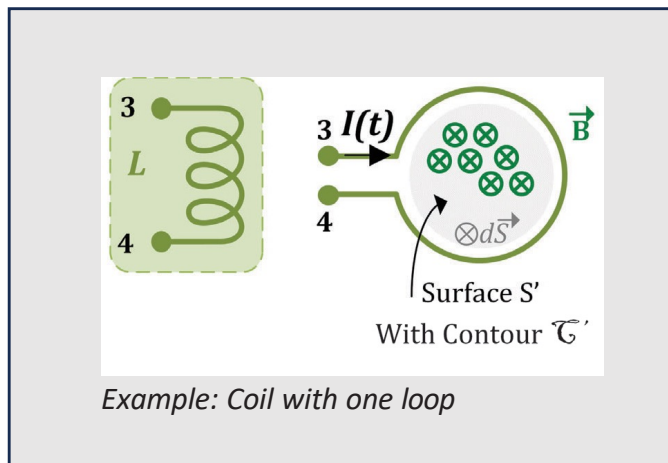
The induced voltage created by the changing current has the effect of opposing the change in current.

The induced  
EMF is

$$V_{34}(t) = -\frac{d}{dt}(\text{Flux of } \vec{B}(t) \text{ through } S') = -L \frac{dI(t)}{dt}$$

The inductance  $L$  is defined as the flux linkage per unit current.

# The ideal Inductance



The inductance  $L$  is defined as the flux linkage per unit current.

$$L = \frac{\int_{C'(S')} \vec{B}(t) \cdot d\vec{S}'}{I(t)} = \frac{B(t)S'}{I(t)}$$

$L$  depends on geometry

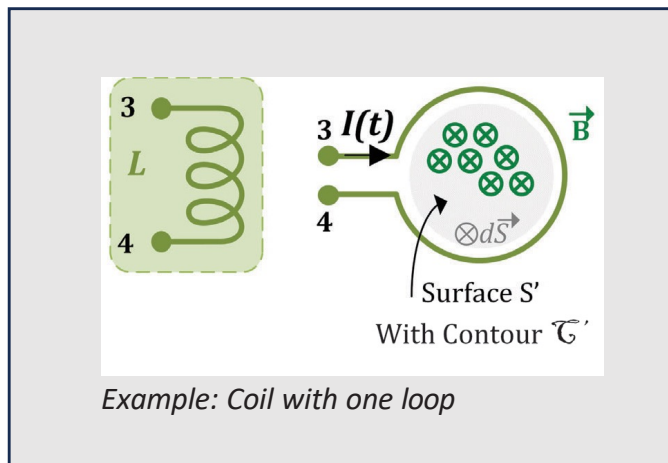
For  $N$  superposed turns in a coil, total  $L_{total} = NL$

“The **Flux** is the integral over the surface  $S'$ . It sums all contributions across the surface  $S'$ ”

$$\int_{C'(S')} \vec{B}(t) \cdot d\vec{S}'$$

“a measure (in the Surface  $S'$ ) of how closely the magnetic field  $\vec{B}(t)$  aligns with  $d\vec{S}'$  (the unit normal vector to the Surface  $S'$ ) in terms of the directions they point”

# The ideal Inductance



The inductance  $L$  is defined as the flux linkage per unit current.

$$L = \frac{\int_{C'(S')} \vec{B}(t) \cdot d\vec{S}'}{I(t)} = \frac{B(t)S'}{I(t)}$$

$L$  depends on geometry

For  $N$  superposed turns in a coil, total  $L_{total} = NL$

$$V_{43}(t) = - \oint_{S'(C')} \vec{E}(t) \cdot d\vec{l} = \frac{d}{dt} \int_{C'(S')} \vec{B}(t) \cdot d\vec{S}' = L \frac{di(t)}{dt}$$

## The ideal Inductance

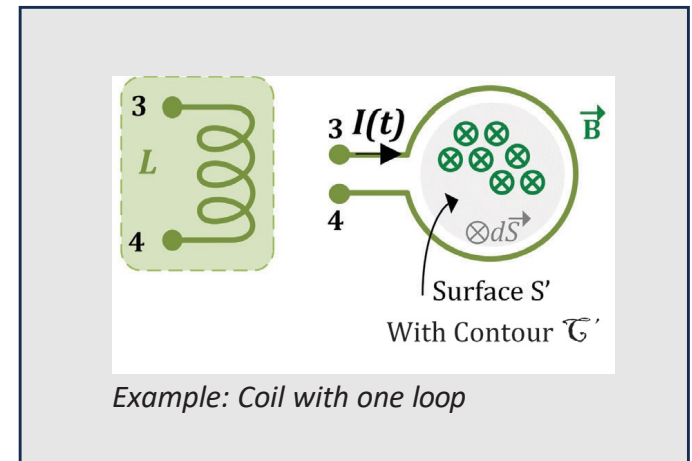
$$V_{43}(t) = - \oint_{S'(C')} \vec{E}(t) \cdot d\vec{l} = \frac{d}{dt} \int_{C'(S')} \vec{B}(t) \cdot d\vec{S}' = L \frac{di(t)}{dt}$$

*Time domain*

$$V_{43}(t) = L \frac{di(t)}{dt}$$

*Frequency domain*

$$V_{43} = j\omega LI$$



# Inductance

- Why inductance is important?
- Internal and external inductance
- Self and Mutual inductances
- Loop and partial inductances
- How to reduce inductance?



## Why do we care about inductance?

- Why On-chip inductance effects have become increasingly significant?
  - Some global signal and clock wires are routed with large widths and thickness to minimize delays. This decreases the resistance of the wires, making their inductive impedance comparable to the resistive part.

$$Z = \downarrow R + j\omega L \uparrow$$

- Copper wires (with low resistivity) has less RC delays compared with aluminum wires (with higher resistivity). This could make the wire inductive reactance larger than the resistance.
- High-frequency components in electrical signals makes the inductance effects more significant.
- Many wires are long and run in parallel, which increases the inductive crosstalk and delay.



## Adverse Effects of the Inductance

- Signal quality
  - Inductive discontinuity → Overshoot/undershoot, ringing
- $V = L \frac{dI}{dt}$  voltage drop
  - Voltage drop on traces and ground/power planes
- Crosstalk
  - Distributed mutual inductance in uniform transmission lines and lumped mutual inductance
- Electromagnetic interference (EMI)
  - Generation of common mode noise currents



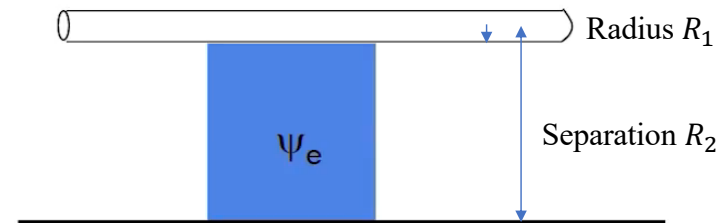
## Internal and external inductance

### Internal inductance



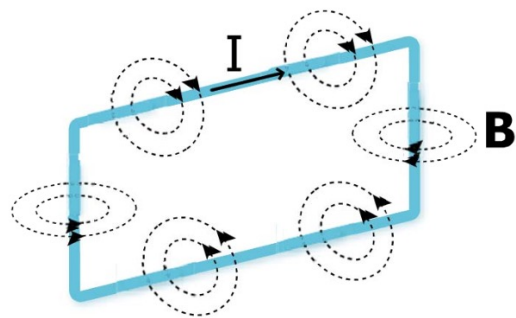
Reference

### External inductance



Reference

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{s} \text{ Webers}$$

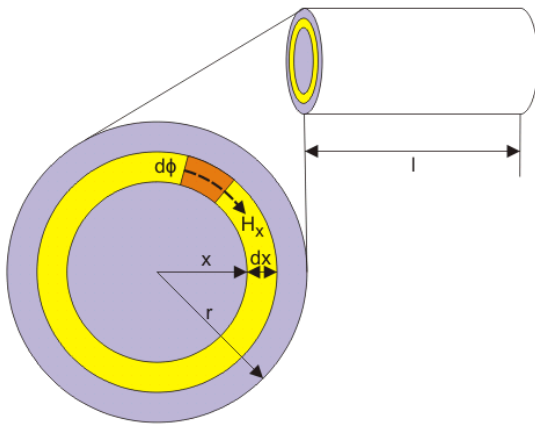


$$L = \frac{\Psi}{I}$$

$$I = \frac{\psi_e}{I} = \frac{2\pi}{\mu_o \ln\left(\frac{R_2}{R_1}\right)} \text{ H/m}$$

## Internal Inductance

**Internal Inductance:** due to magnetic flux internal to the wire



☞ Per-unit-length internal inductance at low freq.

$$l_{lf} = l_{i,DC} = \frac{\mu}{8\pi} \text{ H/m } (r \ll \delta)$$

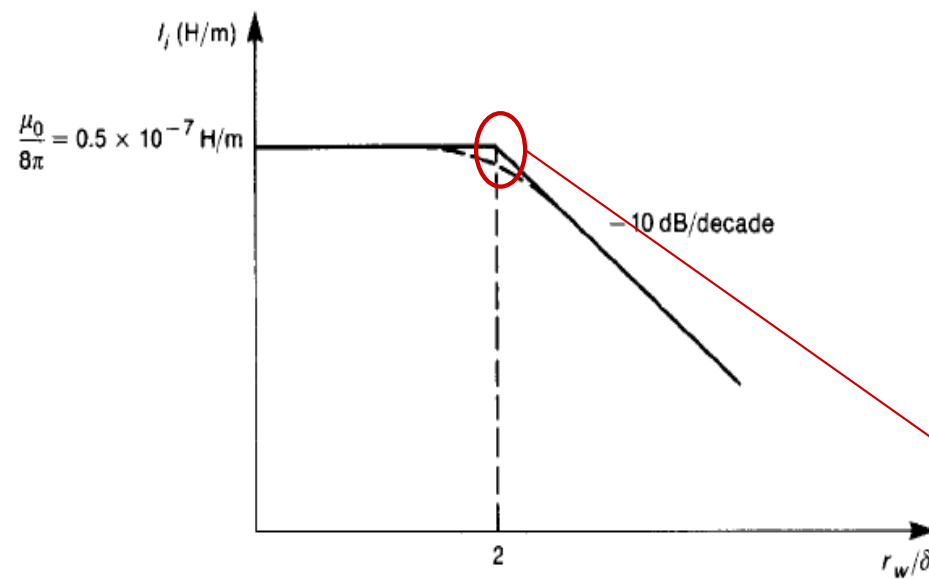
For DC and for low frequencies, internal inductance is a constant

If  $\mu_r = 1$ , internal inductance  $l_{lf} = 50 \text{ nH/m}$

☞ Per-unit-length internal inductance at high freq.

$$l_{hf} = \frac{\mu\delta}{4\pi r} = \frac{1}{4\pi r} \sqrt{\frac{\mu}{\pi\sigma}} \frac{1}{\sqrt{f}} \text{ H/m } (r \gg \delta)$$

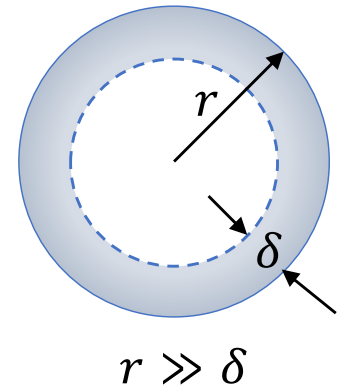
## Internal inductance (Low Freq vs. High Freq.)



1. HF inductance decreases at a rate of **10 dB/decade**.

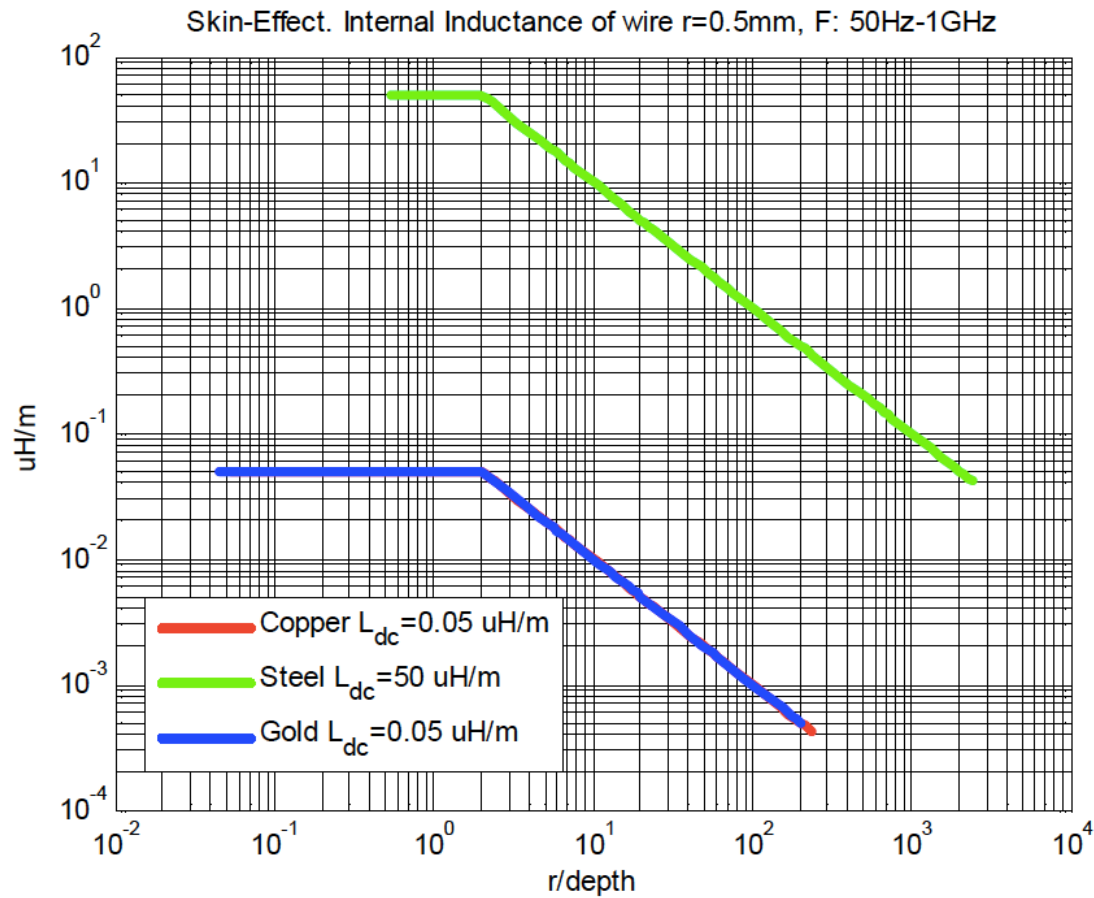
2. The two asymptotes meet at point:

$$r = 2\delta$$



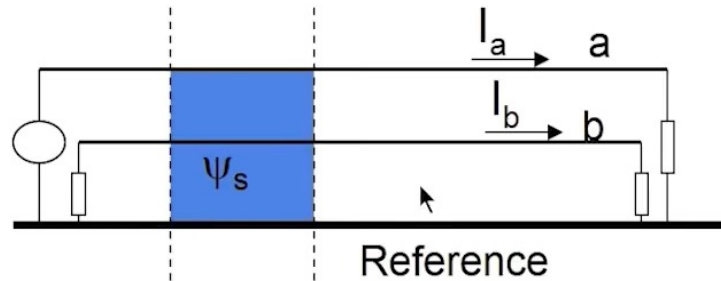
**Dependence of the wire per-unit-length internal inductance on frequency**

### Illustration of the dependence of the per-unit-length Internal Inductance of a wire on frequency (skin effect)



## Self and Mutual inductances

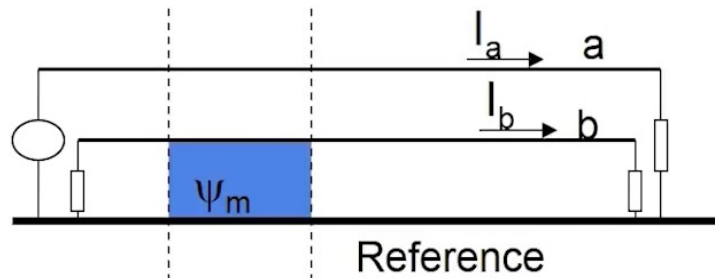
### Self inductance



$$L_s = \frac{\psi_s}{I_a} \text{ H/m}$$

Self inductance = internal inductance  
+ external inductance

### Mutual inductance

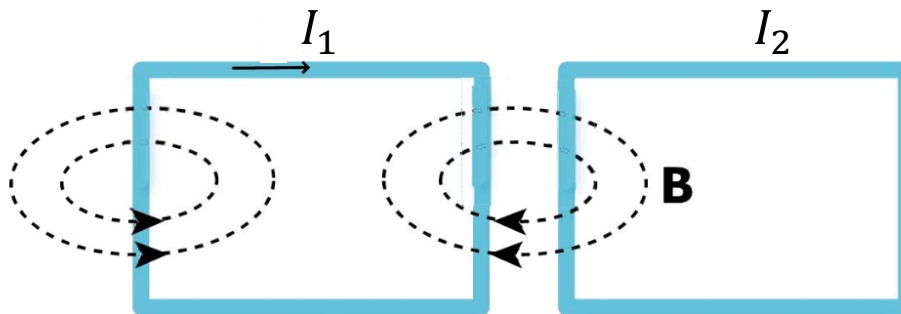


$$L_m = \frac{\psi_m}{I_a} \text{ H/m}$$

How much magnetic flux is induced in the inner loop (loop b and reference) from current  $I_a$ ?

The magnetic field and mutual inductance drop off rapidly with increasing the spacing between the two wires.

## Mutual Inductance

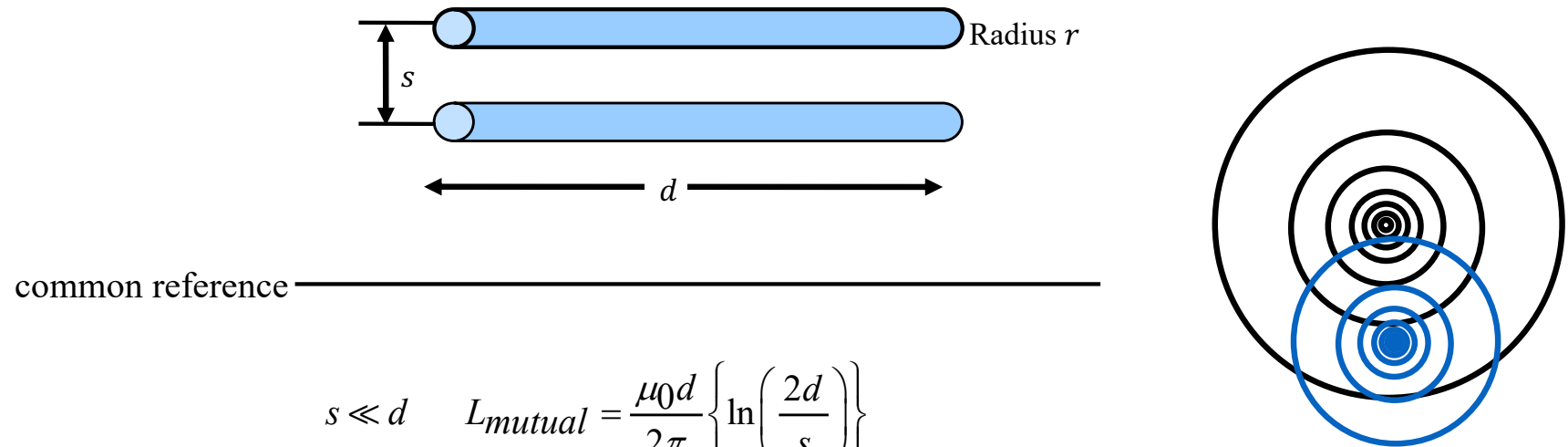


Two current loops with no common reference

$$L_{21} = \frac{\psi_{21}}{I_1} \text{ H}$$

$\psi_{21}$ : flux inside loop 2 due to current  $I_1$

## First-order Equation for Mutual Inductance



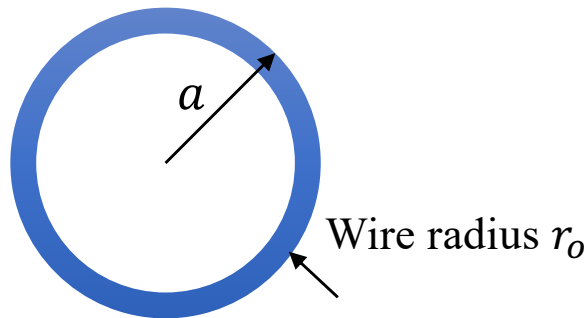
$$s \ll d \quad L_{mutual} = \frac{\mu_0 d}{2\pi} \left\{ \ln \left( \frac{2d}{s} \right) \right\}$$

$$s < d \quad L_{mutual} = \frac{\mu_0 d}{2\pi} \left\{ \ln \left( \frac{2d}{s} \right) - 1 + \frac{s}{d} - \left( \frac{s}{2d} \right)^2 \right\}$$

- Notice the change in mutual inductance with rod separation

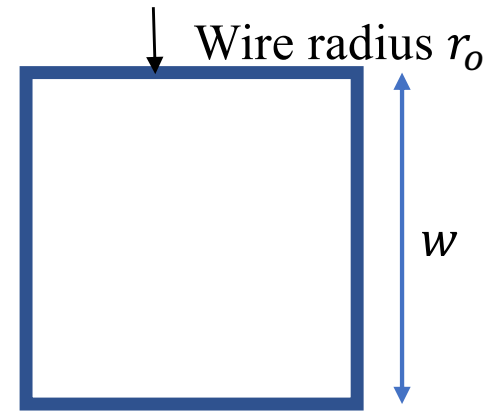
## Loop and partial inductances

### inductance of a loop



$$L_{loop} \approx a\mu_o\mu_r \left[ \ln \left( \frac{8a}{r_o} \right) - 2.0 \right] \text{ H}$$

**Simple Round Loop** ( $r_o \ll r$ )



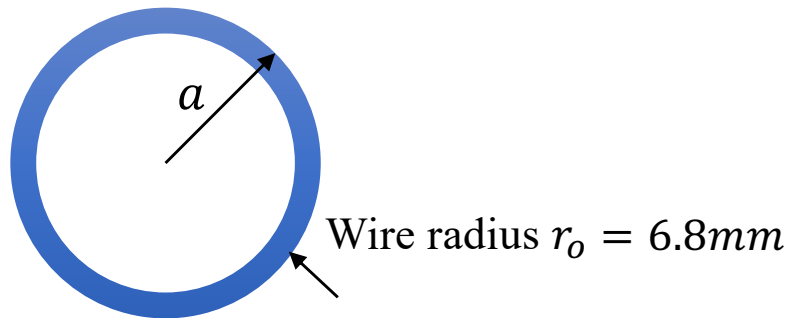
$$L = \frac{2\mu_o w}{\pi} \left( \ln \frac{w}{r_o} - 0.774 \right) \text{ H} \quad (r_o \ll w^2)$$

**Simple Square Loop**

Loop inductance is directly influenced by loop AREA and less influenced by conductor size!

## Loop and partial inductances

### Poll: inductance of a loop



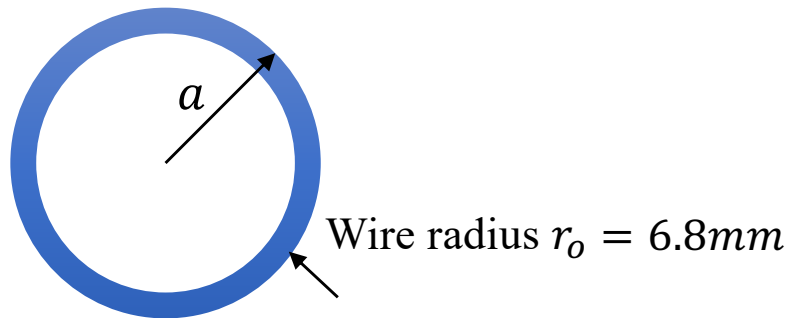
$$L_{loop} \approx r \mu_o \mu_r \left[ \ln \left( \frac{8a}{r_o} \right) - 2.0 \right] \text{ H}$$

The inductance of a 5-cm radius loop is

- a) about 130 nH
- b) about 13 nH
- c) undefined

## Loop and partial inductances

### Poll: inductance of a loop



The inductance of a 5-cm radius loop is

- a) about 130 nH
- b) about 13 nH
- c) undefined

$$L_{loop} \approx r \mu_o \mu_r \left[ \ln \left( \frac{8a}{r_o} \right) - 2.0 \right] \text{ H}$$

$$L_{loop} \approx 0.05 \times 4\pi \times 10^{-7} \left[ \ln \left( \frac{8 \times 0.05}{0.0068} \right) - 2.0 \right] \approx 130 \text{ nH}$$

## Low/high inductance decoupling capacitor connection

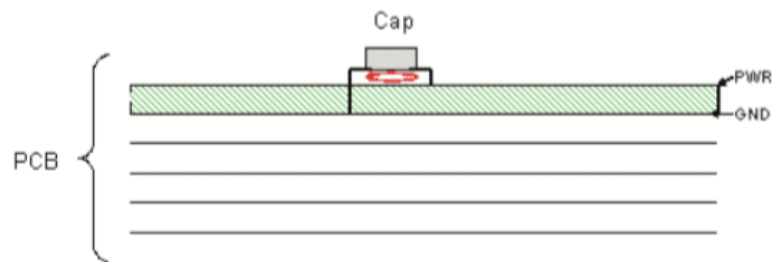


Fig 1. Low inductance connection with capacitor mounted on top of board.

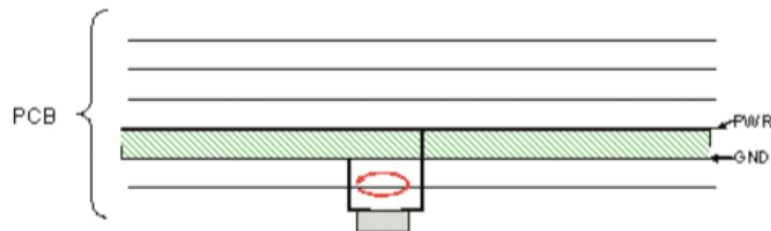


Fig 3. Low inductance connection with capacitor mounted on bottom of board.

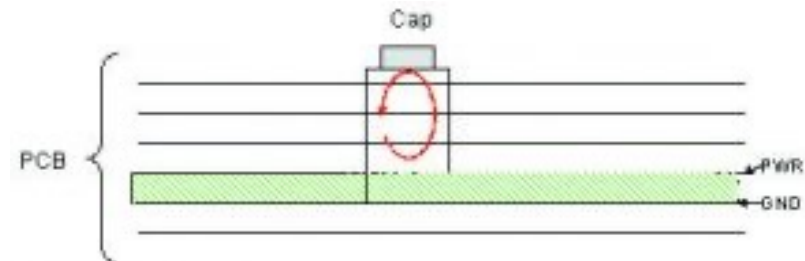


Fig 2. High inductance connection with capacitor mounted on top of board.

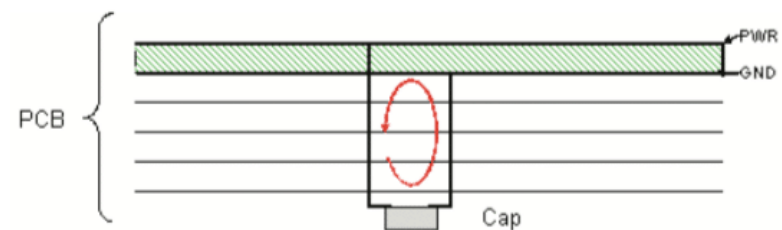
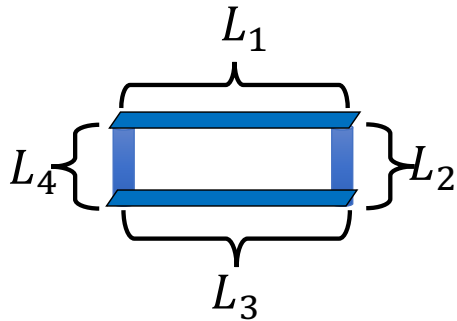


Fig 4. High inductance connection with capacitor mounted on top of board.

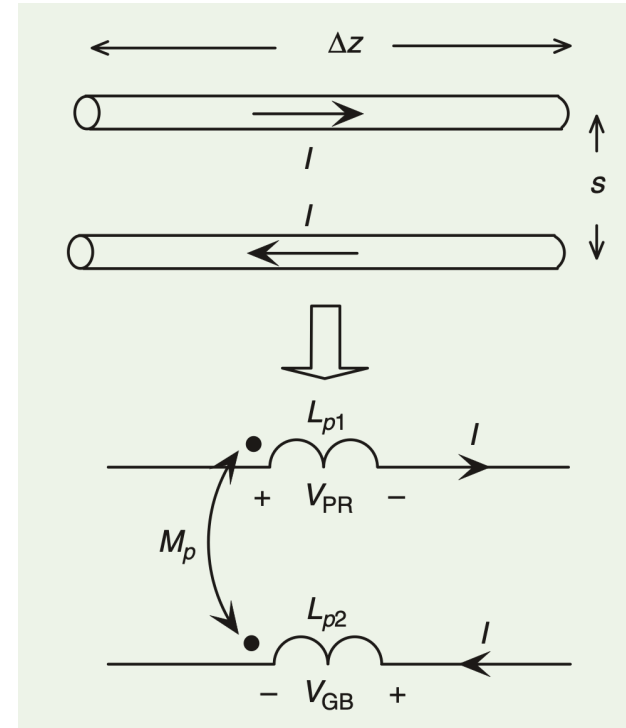
## Partial inductances

Use partial inductance to find the total inductance



In analogy to

$$L_{total} = L_{11} + L_{22} + L_{33} + L_{44} - 2L_{13} - 2L_{24}$$



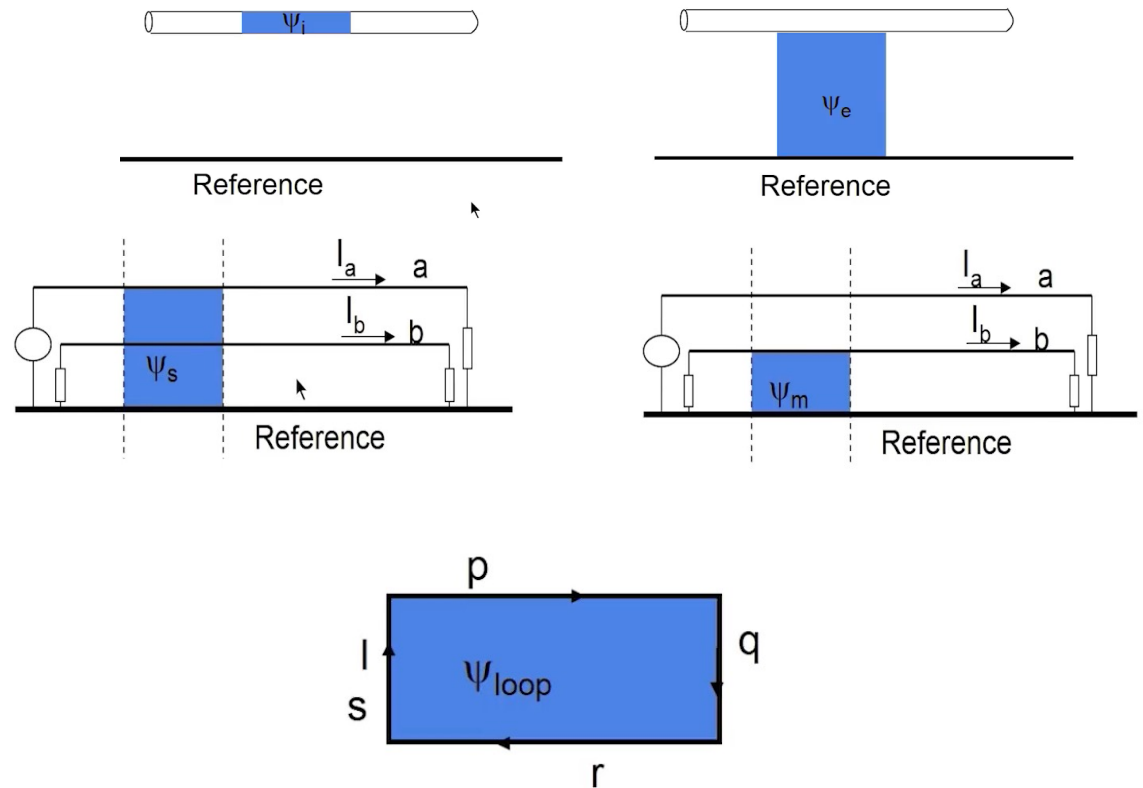
$$V_{PR} = L_{p1} \frac{dI}{dt} - M_p \frac{dI}{dt} = (L_{p1} - M_p) \frac{dI}{dt}$$

$$V_{GB} = L_{p2} \frac{dI}{dt} - M_p \frac{dI}{dt} = (L_{p2} - M_p) \frac{dI}{dt}$$

## Inductance

- Internal inductance
- External inductance
- Self inductance
- Mutual inductance
- Loop inductance
- Partial inductance

Current loop  $\rightarrow$  inductance!



## How to reduce inductance?

- Reducing loop area is most importance
  - Run the wire close to ground plane or return wire
- Reducing the length of wire
- Use large surface area of planes

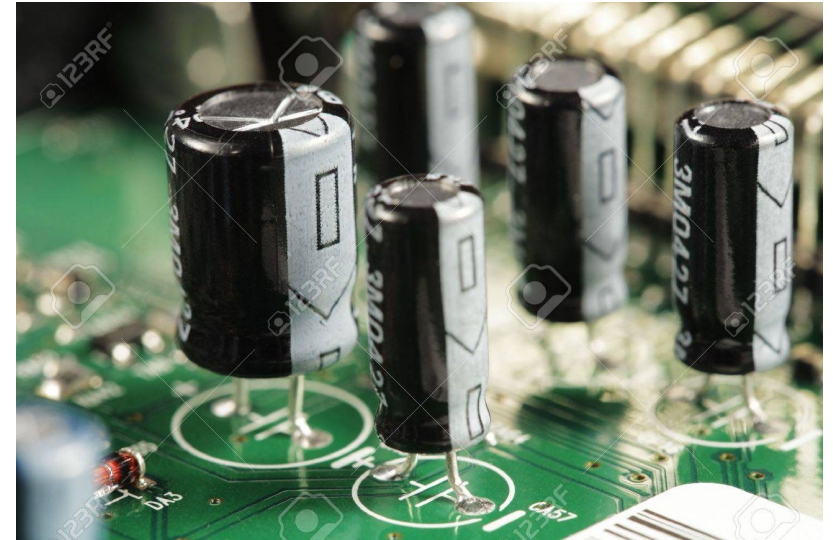
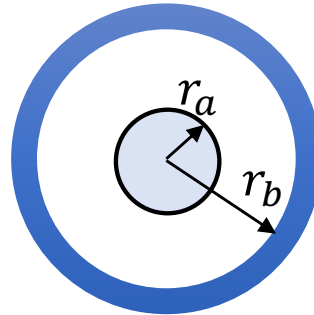
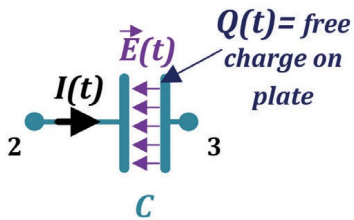
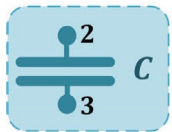
Eg. for round conductors, diameter increase from 1mm to 10mm, but L decreases from  $1.5 \mu H/m$  to  $1.0 \mu H/m$ .

**Rule of thumb:** Inductance of a conductor approximately  $1 \mu H/m \sim 1.5 \mu H/m$   
( $1 nH/mm \sim 1.5 nH/mm$ )

1 meter earth wire has about  $126 \Omega$  impedance at 20 MHz compared with less than 1 ohm at 60 Hz.

Inductance is everywhere!!!

Capacitance  $C = \frac{Q}{V}$  farads

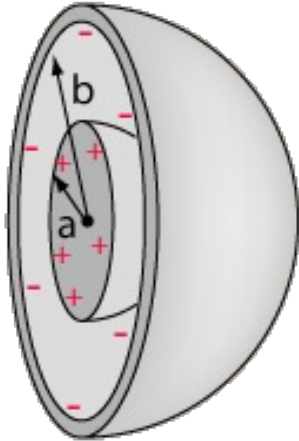


Capacitance is the ability of a structure to hold charge (electrons) for a given voltage

Consider a capacitor as a *bucket* holding lots of electrons!

For coaxial cables, per unit length  $C = \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)}$

### 3 Capacitance



**Spherical Capacitor**

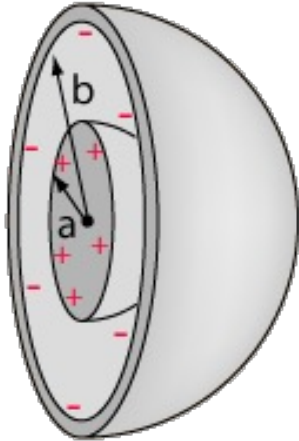
$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad a < r < b$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{a} - \frac{1}{b} \right]$$

$$C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0}{\left[ \frac{1}{a} - \frac{1}{b} \right]}$$

Q: Does an isolated charged sphere have capacitance?

### 3 Capacitance



**Spherical Capacitor**

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad a < r < b$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{a} - \frac{1}{b} \right]$$

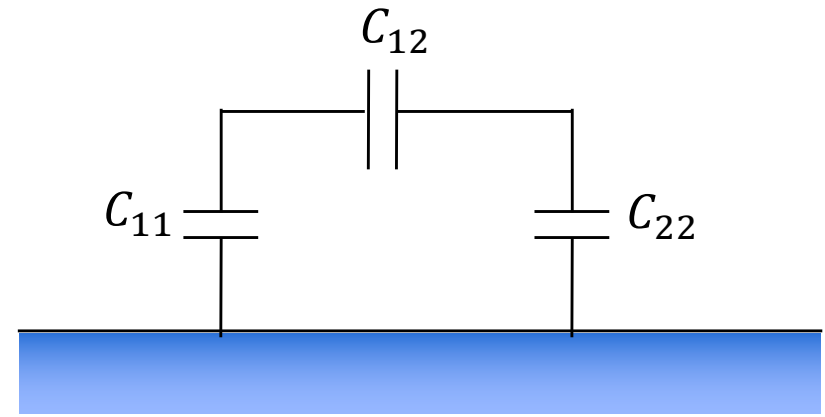
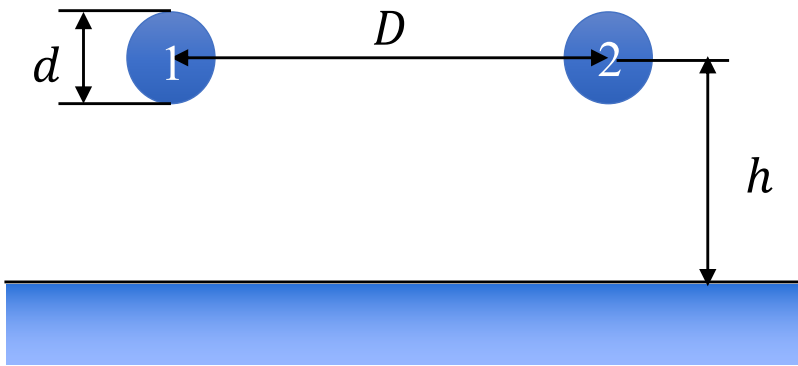
$$C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0}{\left[ \frac{1}{a} - \frac{1}{b} \right]}$$

Q: Does an isolated charged sphere have capacitance?

Self capacitance

$$a \rightarrow R \quad \text{and} \quad b \rightarrow \infty \quad C = 4\pi\epsilon_0 R$$

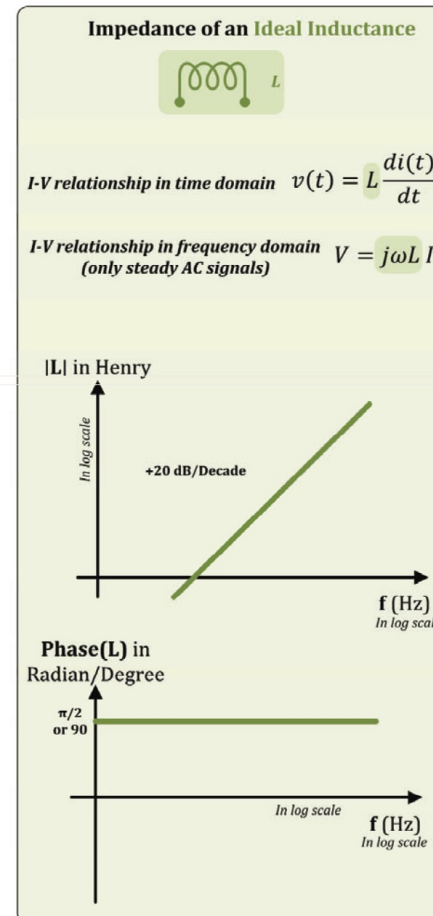
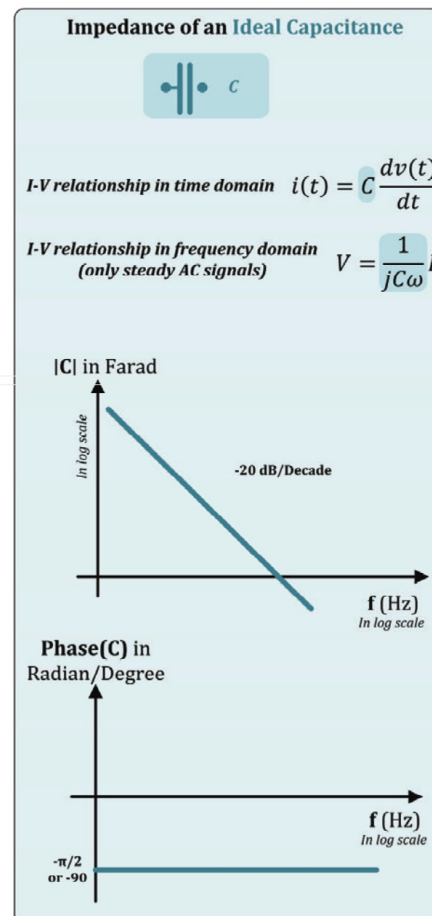
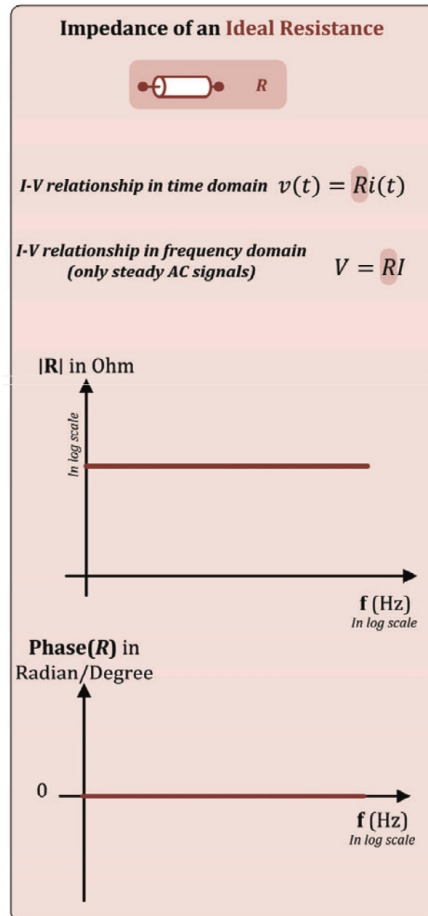
## Self and mutual capacitance



| d=1 mm, h=infinity |                    | d=1 mm, h=2 mm |                   |
|--------------------|--------------------|----------------|-------------------|
| D=5 mm             | $C_{12}=12$ pF/m   | D=5 mm         | $C_{12}=5$ pF/m   |
| D=10 mm            | $C_{12}=10.3$ pF/m | D=10 mm        | $C_{12}=2$ pF/m   |
| D=20 mm            | $C_{12}=7.5$ pF/m  | D=20 mm        | $C_{12}=0.5$ pF/m |

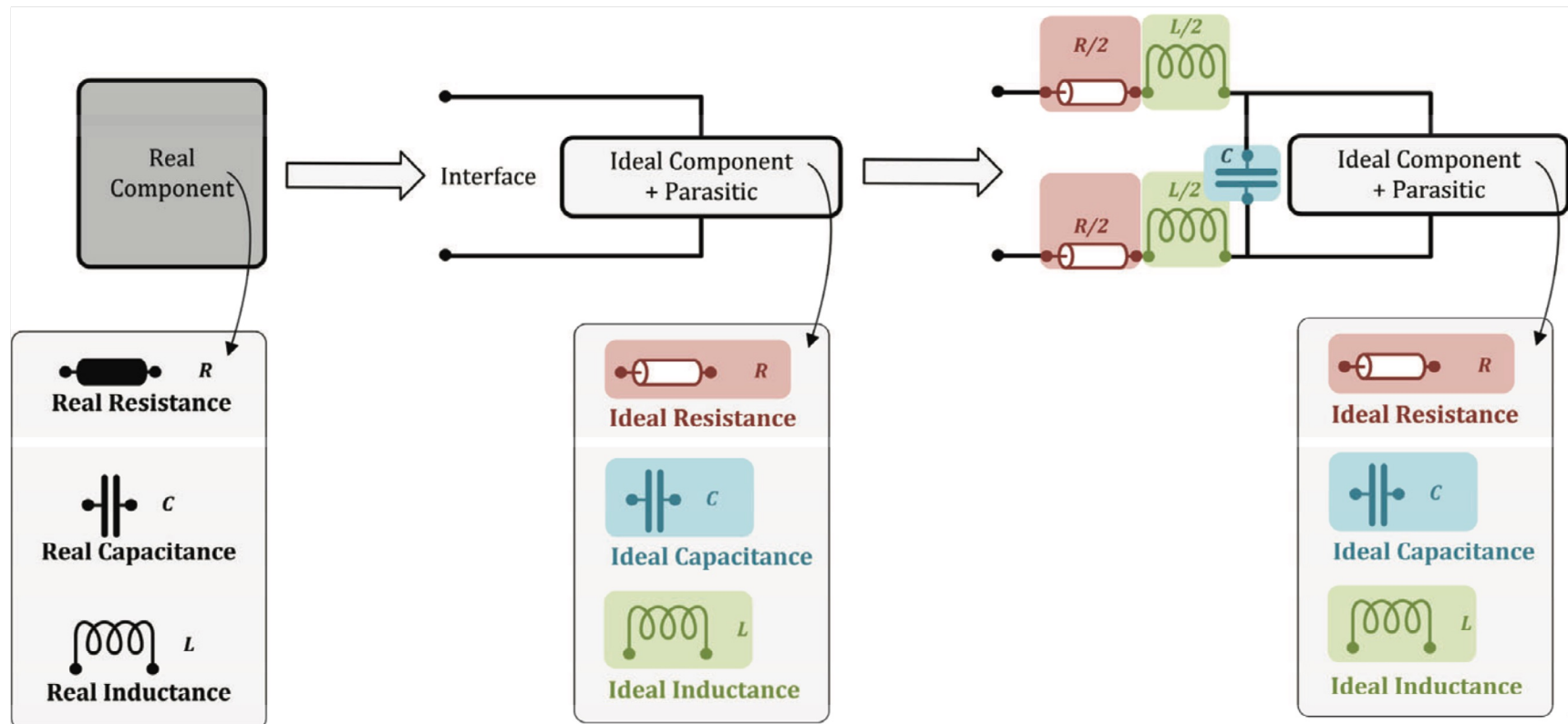
Coupling capacitance  $C_{12}$  is reduced with the proximity of reference plane

# Ideal Components recap

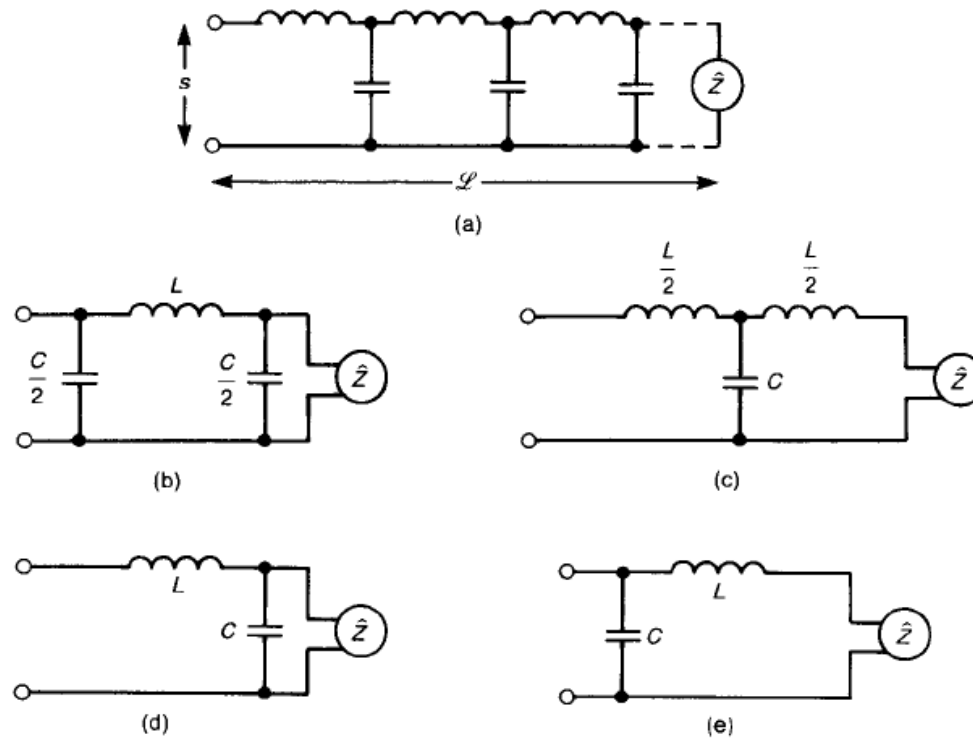


- We have three circuit elements: R, L and C.
- These are IDEAL components: they are mathematical “perfect things”
- We will use them as building blocks to describe REAL components

## 5.2 Effect of Component Leads



## Effect of Component Leads

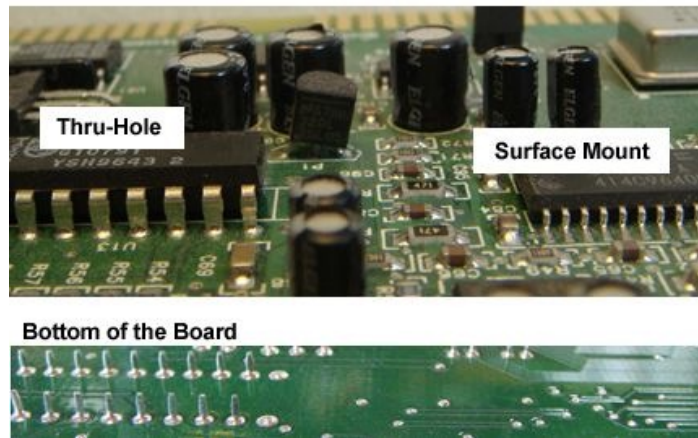


Equivalent circuits of component leads

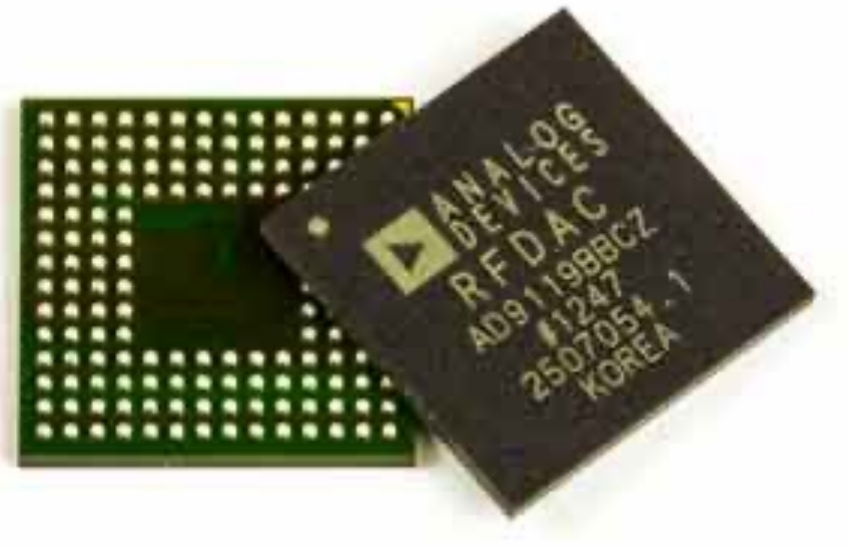
## Effect of Component Leads

**SMT:** surface-mount technology

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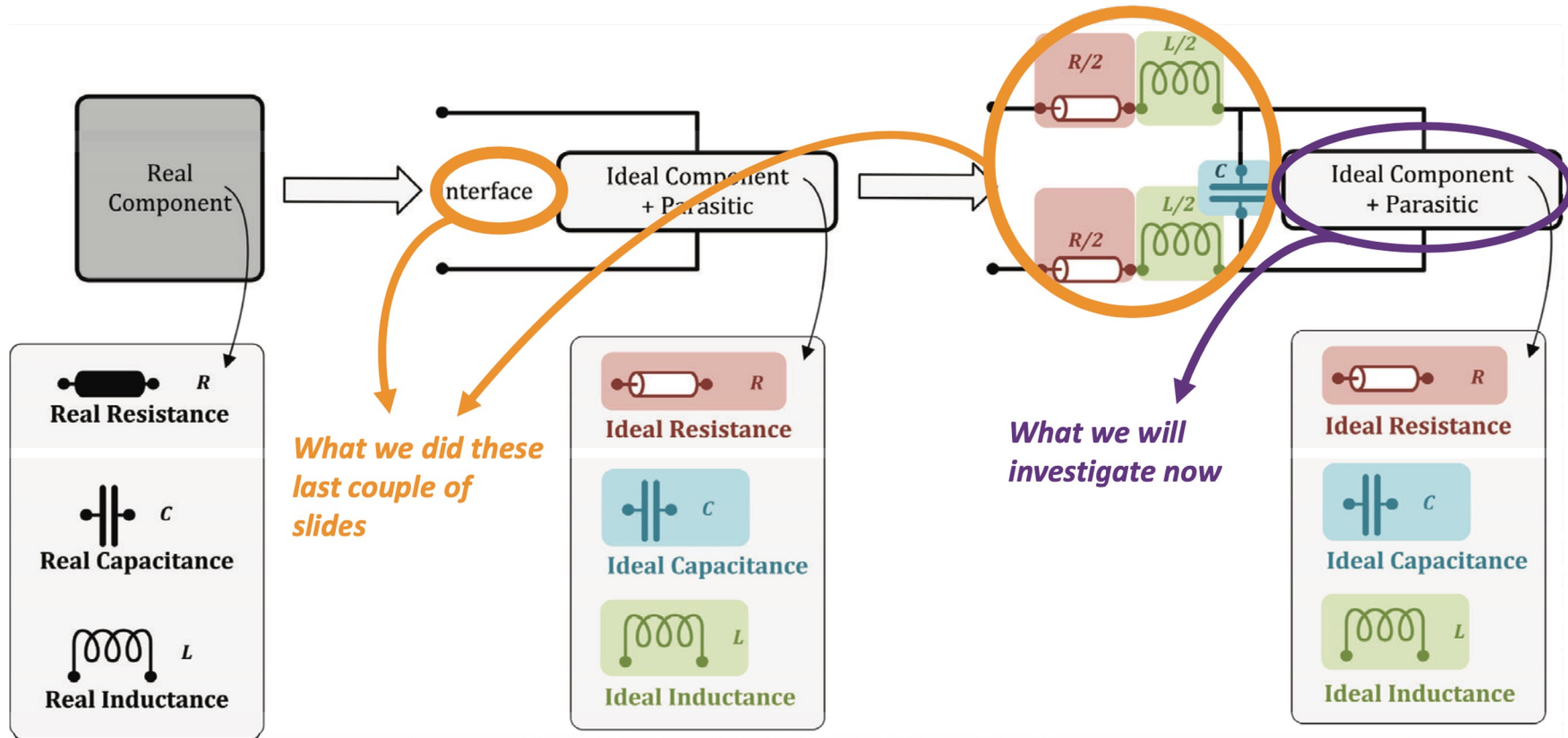


Surface mount vs. Thru-Hole

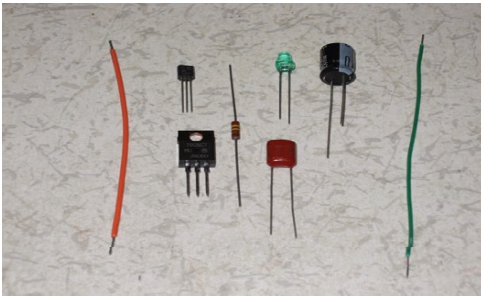


Top and bottom view of a BGA (Ball grid array)

## 5.3 Nonideal R, L, and C Components

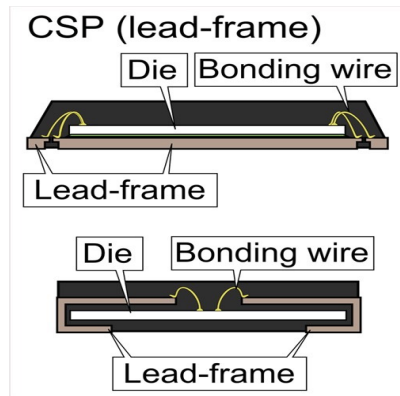


## 5.3.1 Nonideal Resistors



### Several kinds of lead wires

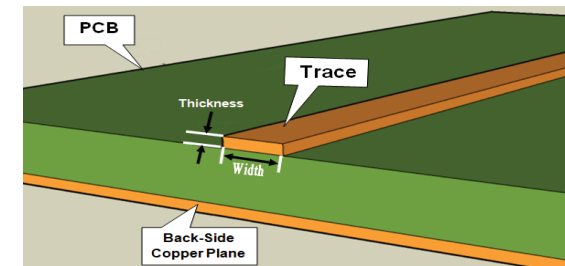
- A lead wire is a metal wire connected from the electric pole of an electronics part or an electronic component.
- The lead wire is a coated copper wire, a tinned copper wire or another electrically conductive wire used to connect two locations electrically.



Lead frame of a surface-mount technology IC package



Cables and multi strand wires

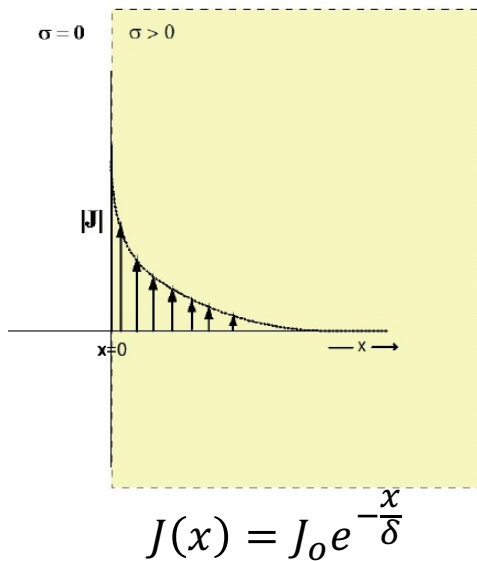


PCB trace

\*Ref: pictures: Wikipedia article "Lead (Electronic)" June 2022  
[https://en.wikipedia.org/wiki/Lead\\_%28electronic%29](https://en.wikipedia.org/wiki/Lead_%28electronic%29)  
\*\* Ref: PCB design world  
<https://pcbdesignworld.com/article/pcb-track-width-and-track-resistance-importance-calculation-and-design-tips>

## Skin Depth

High-frequency electric fields and currents decay exponentially with distance from the surface of a good conductor.



$$I = \int_0^d J(x) dx = \int_0^d J_0 e^{-\frac{x}{\delta}} dx = J_0 \delta (1 - e^{-\frac{d}{\delta}})$$

### Skin depth $\delta$

The skin depth  $\delta$  is the distance over which the amplitude,  $E_0 e^{-\alpha x}$  of the wave decreases by  $1/e$  ( $\sim 37\%$ ).

$$\delta = 1/\alpha$$

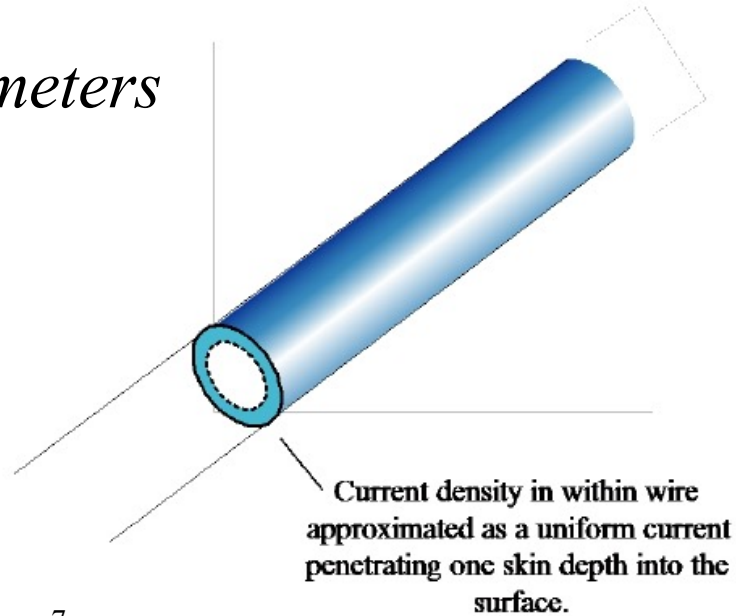
$\alpha$  is the attenuation constant.

For good conductors ( $\frac{\sigma}{\omega\epsilon} \gg 1$ ),

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

## Skin Depth

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \text{ meters}$$



copper  $\sigma = 5.8 \times 10^7 S / m$

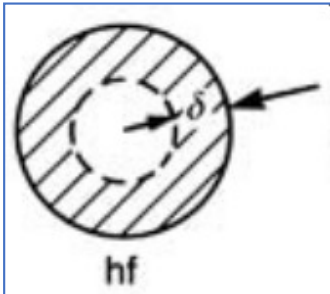
| $f$    | $\delta$ | $f$    | $\delta$    |
|--------|----------|--------|-------------|
| 50Hz   | 9.3mm    | 1MHz   | 2.6mils     |
| 1kHz   | 2.09 mm  | 10MHz  | 0.82 mils   |
| 10kHz  | 0.66 mm  | 100MHz | 0.26 mils   |
| 100kHz | 0.21 mm  | 1GHz   | 0.0823 mils |

## Wire resistance

Per-unit-length resistance at low freq.

$$r_{lf} = r_{DC} = \frac{1}{\pi \sigma r_w^2} \quad (r_w \ll \delta)$$

Per-unit-length resistance at high freq.



Influence of skin effect

$$r_{hf} = \frac{1}{\sigma [\pi r_w^2 - \pi (r_w - \delta)^2]}$$

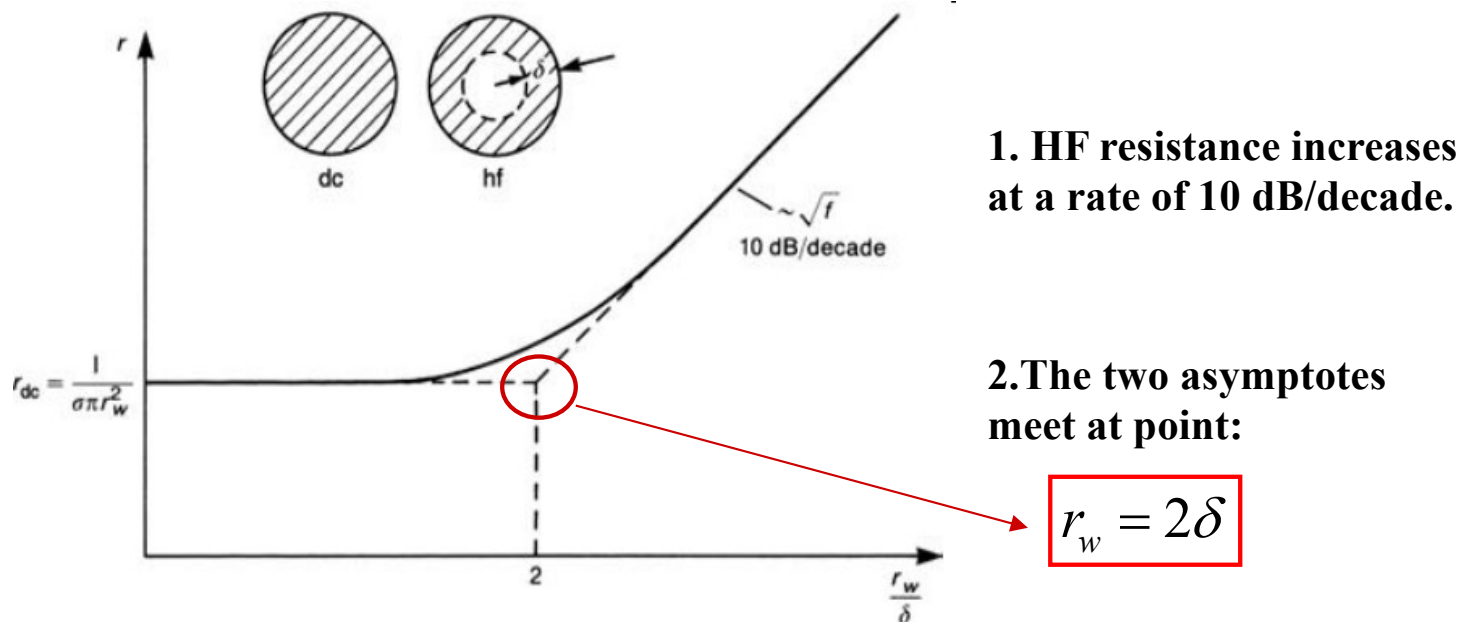
Reduced cross-section area

$$= \frac{1}{\sigma (2\pi r_w \delta - \pi \delta^2)}$$

$$\approx \frac{1}{\sigma 2\pi r_w \delta} = \frac{r_w}{2\delta} r_{DC} = \frac{1}{2r_w} \sqrt{\frac{\mu_0}{\pi \sigma}} \sqrt{f} \quad (r_w \gg \delta)$$

Increased resistance at high freq.

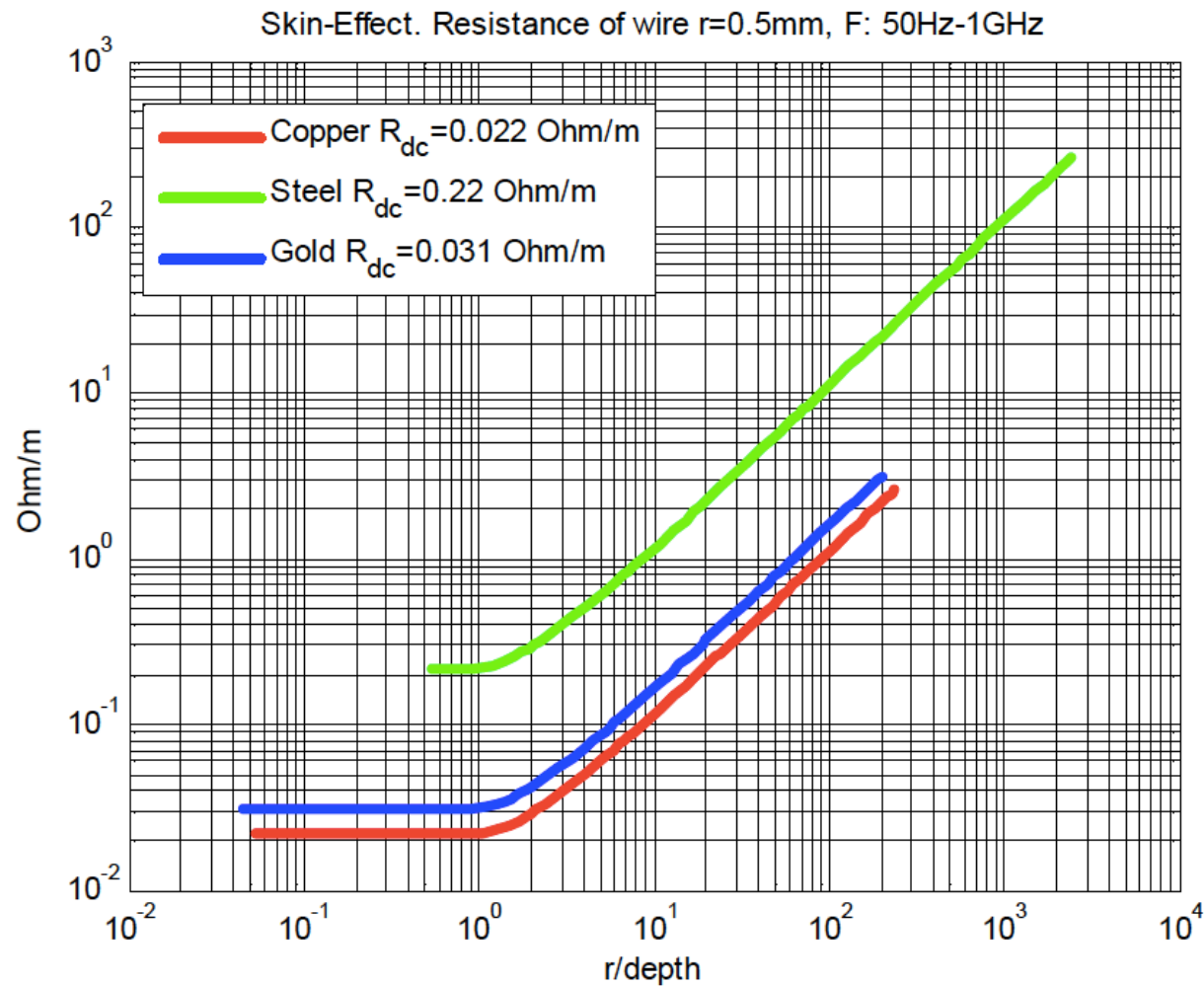
## Wire resistance asymptotic dB plot



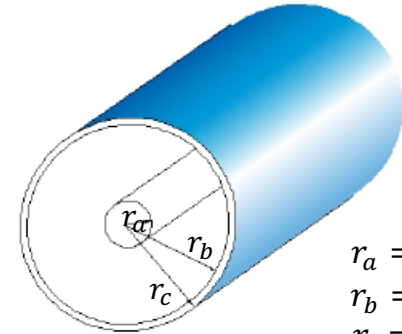
Dependence of the wire per-unit-length resistance on frequency

### 5.3 Nonideal Components — Nonideal Resistors

Illustration of the dependence of the per-unit-length resistance of a wire on frequency (skin effect)



## Resistance per unit length of a coaxial cable

**At 60 Hz**

$$\begin{aligned} r_a &= 0.5 \text{ mm} \\ r_b &= 4.9 \text{ mm} \\ r_c &= 5.0 \text{ mm} \end{aligned}$$

$$\delta_{60\text{Hz}} = \frac{1}{\sqrt{\pi f \mu_o \sigma}} = \frac{1}{\sqrt{\pi(60)(4\pi \times 10^{-7})(5.8 \times 10^7)}} = 8.53 \text{ mm}$$

$$R_{\text{inner conductor}} = \frac{1}{\sigma A} = \frac{1}{(5.8 \times 10^7) \pi (0.5 \times 10^{-3})^2} = 22.0 \text{ m}\Omega/\text{m}$$

$$R_{\text{outer conductor}} \approx \frac{1}{\sigma A} = \frac{1}{(5.8 \times 10^7) 2\pi (5 \times 10^{-3})(0.1 \times 10^{-3})} = 5.5 \text{ m}\Omega/\text{m}$$

$$R_{\text{total}} = 22.0 + 5.5 = 27.5 \text{ m}\Omega/\text{m}$$

## Resistance per unit length of a cable

**At 1 GHz**

$$\delta_{1GHz} = \frac{1}{\sqrt{\pi f \mu_o \sigma}} = \frac{1}{\sqrt{\pi(10^9)(4\pi \times 10^{-7})(5.8 \times 10^7)}} = 0.0021 \text{ mm}$$

$$R_{inner \text{ conductor}} = \frac{1}{\sigma A} = \frac{1}{(5.8 \times 10^7)2\pi(0.5 \times 10^{-3})(2.1 \times 10^{-6})} \approx 2.61 \text{ } \Omega/m$$

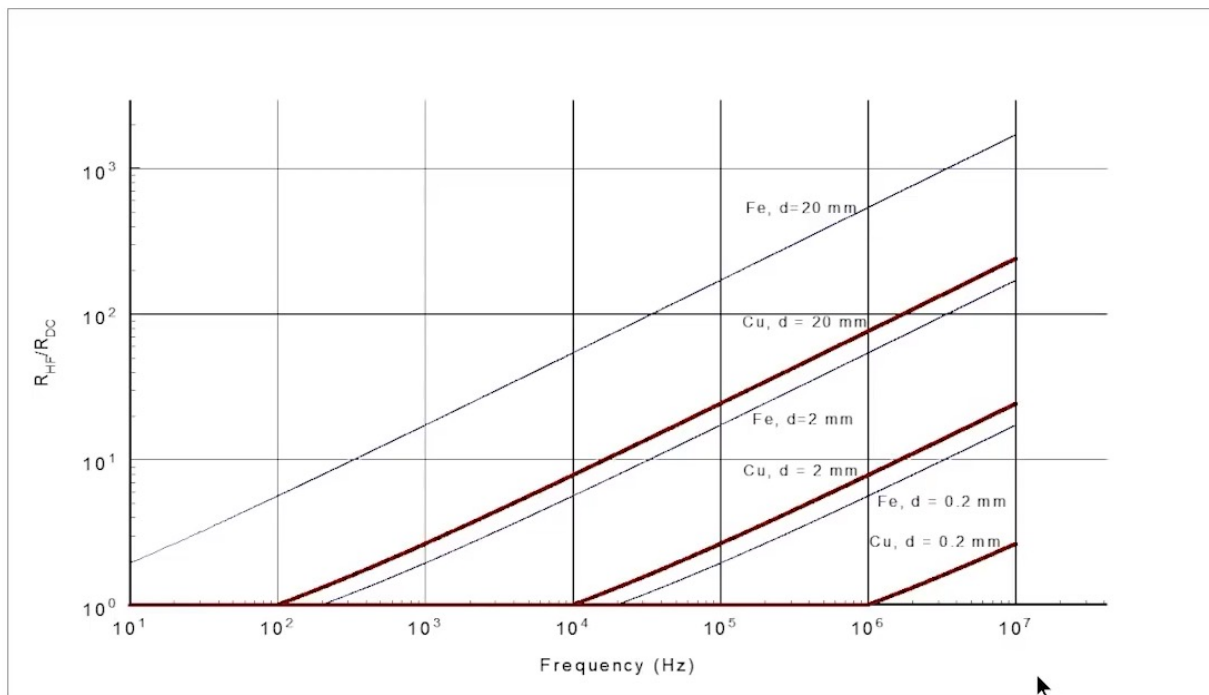
$$R_{outer \text{ conductor}} \approx \frac{1}{\sigma A} = \frac{1}{(5.8 \times 10^7)2\pi(5 \times 10^{-3})(2.1 \times 10^{-6})} \approx 0.261 \Omega/m$$

$$R_{total} = 2.61 + 0.261 = 2.871 \text{ } \Omega/m$$



## Wire resistance

$r_{hf}/r_{DC}$  for copper and iron wires

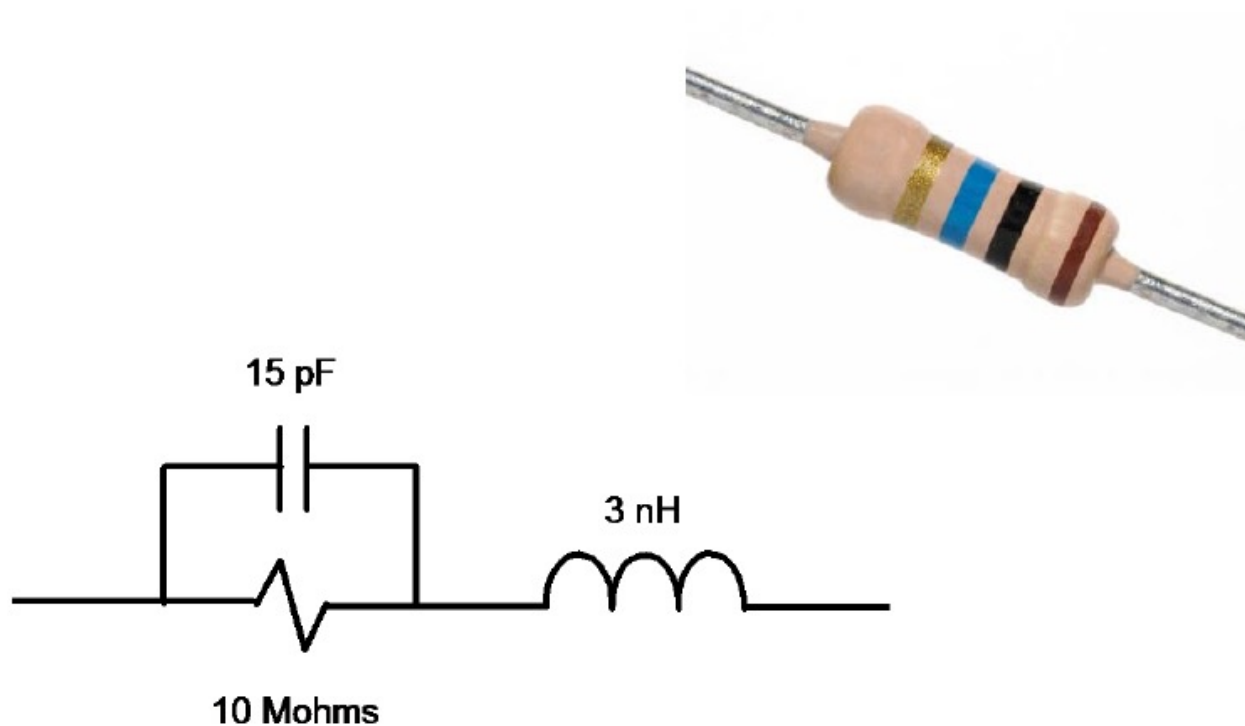


Stranded wire

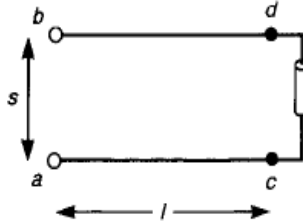


Solid wire

## Do Resistors Have Capacitance and Inductance?

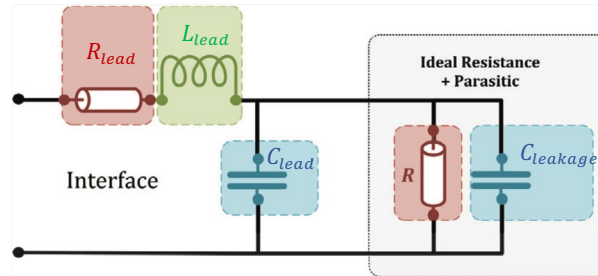


# The nonideal resistor including the effects of the leads

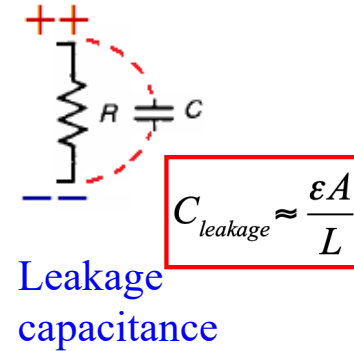


Typically

$$R_{lead} \ll R$$

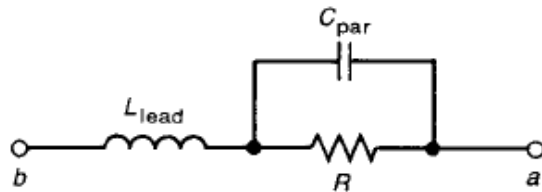
$$C_{lead} \ll C_{leakage}$$


Lumped backward  $\Gamma$   
model of leads



Parasitic capacitance

$$C_{par} = C_{lead} + C_{leakage}$$



Equivalent circuit

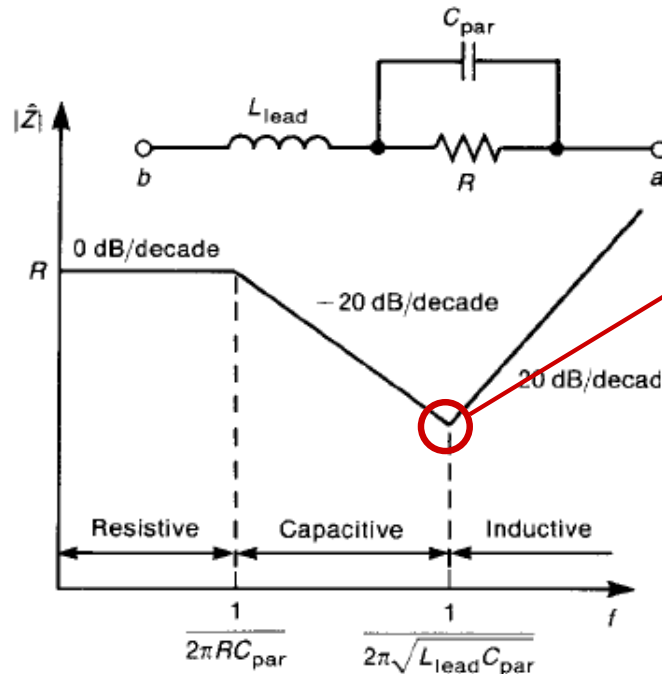
Total Impedance:

$$\hat{Z}(j\omega) = L_{lead} \frac{1/L_{lead} C_{par} - \omega^2 + j\omega/RC_{par}}{j\omega + 1/RC_{par}}$$

Typical values are  $C_{par} = 1 \sim 2$  pF.  $L_{lead} = 14$  nH

lead lengths of 0.5 in., lead separation of 0.25 in., and lead wires (20-gauge with  $r_w = 16$  mils)

# The nonideal resistor including the effects of the leads

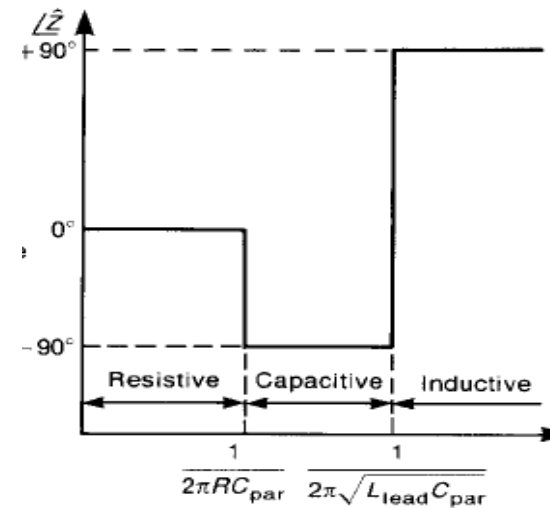


Resonate freq.

$$f = \frac{1}{2\pi \sqrt{L_{lead} C_{par}}}$$

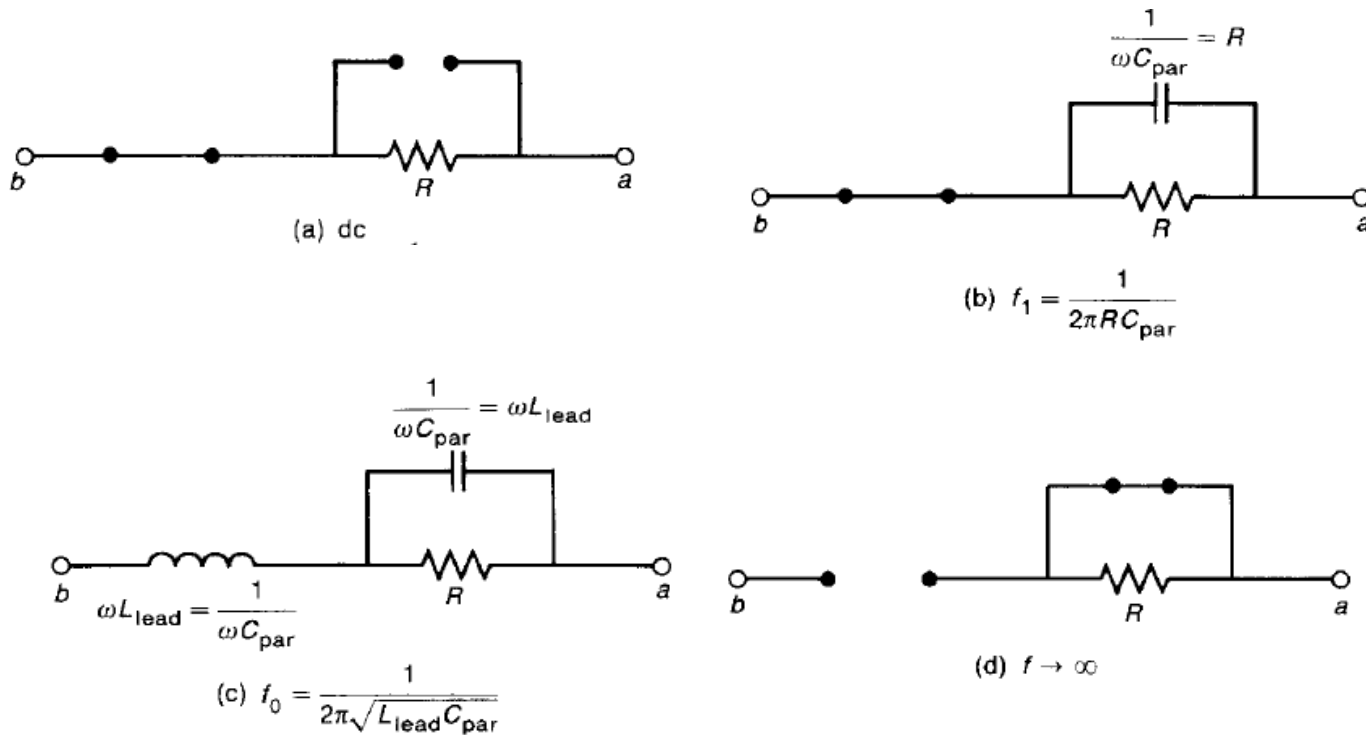
Magnitude bode plot of the impedance variation with frequency

$$\hat{Z}(j\omega) = L_{lead} \frac{1/L_{lead} C_{par} - \omega^2 + j\omega/RC_{par}}{j\omega + 1/RC_{par}}$$



Phase bode plot of the impedance variation with frequency

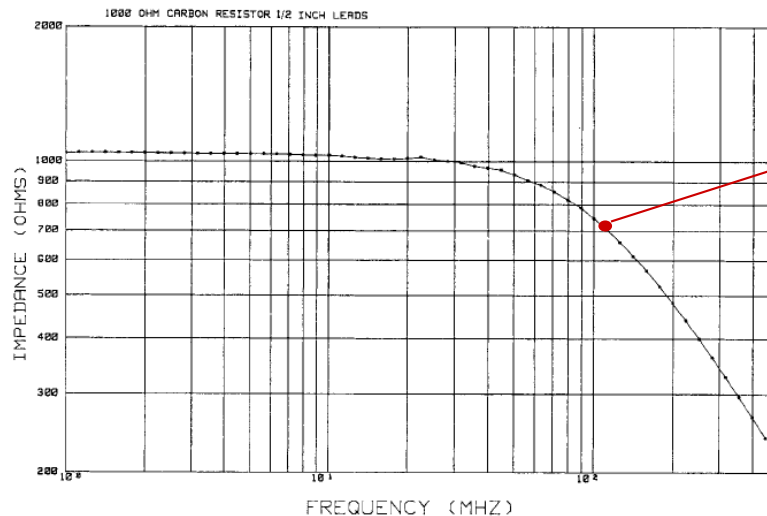
## Simplification of the equivalent circuit of a resistor



At various frequencies

## Nonideal resistor: Example

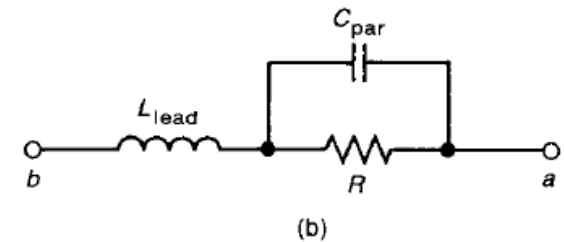
**Example:** Measure the impedance of a 1k $\Omega$ , 1/8W carbon resistor having 0.5in (1 in = 2.54cm) lead lengths, and 0.25in lead separation over a frequency range of 1–500 MHz.



$$f_1 = \frac{1}{2\pi RC_{par}} \approx 120 \text{ MHz}$$

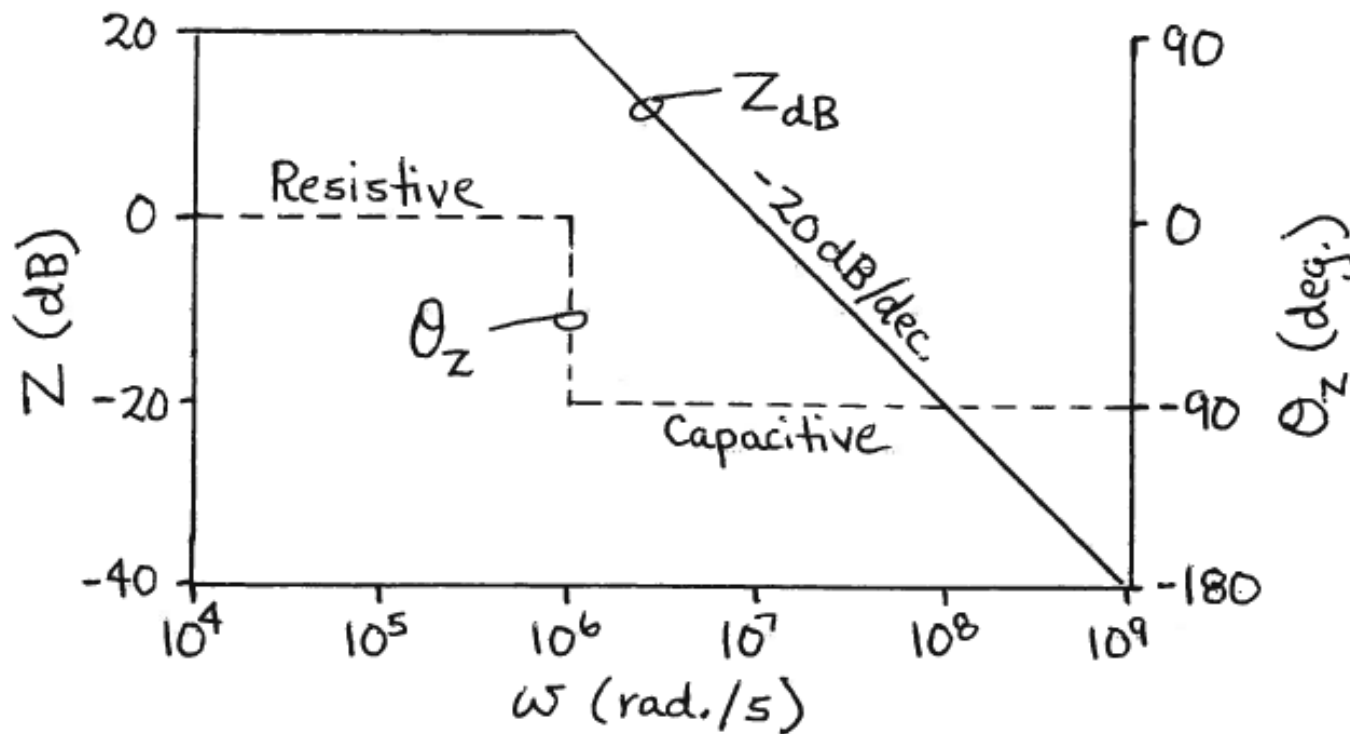
$R = 1.05 \text{ k}\Omega$ ,  $C_{par} = 1.2 \text{ pF}$ , and  $L_{lead} = 14 \text{ nH}$ .

$$f_2 = \frac{1}{2\pi\sqrt{L_{lead}C_{par}}} \approx 1.23 \text{ GHz}$$



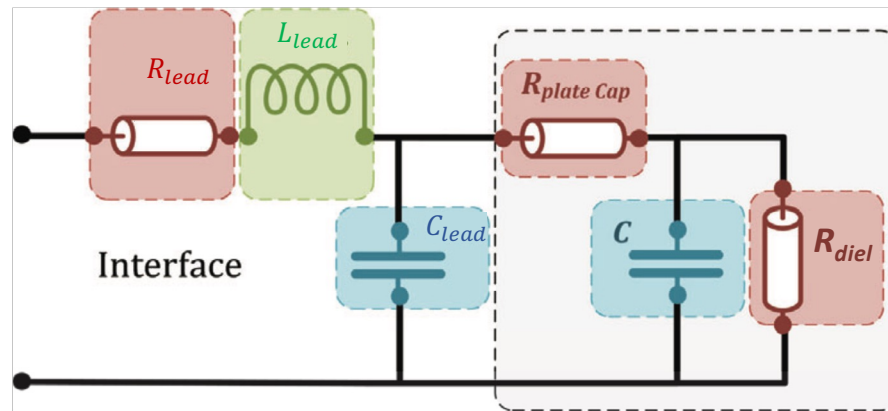
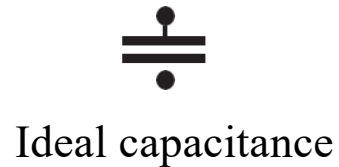
## Quiz: RC Bode Plot

Find the values of R and C corresponding to the following asymptotic  $Z(\omega)$



## 5.3.2 Nonideal Capacitors

Do Capacitors Have Resistance and Inductance?



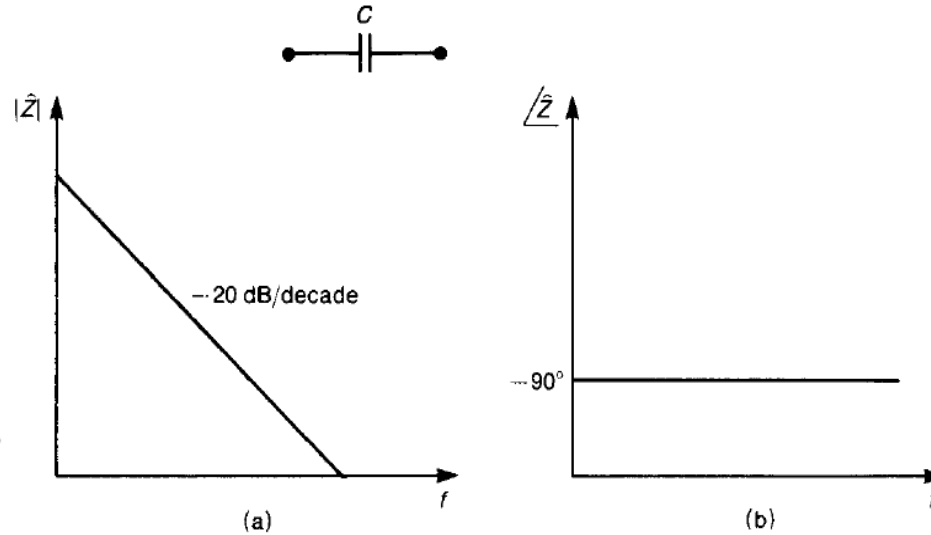
Interface with Real Capacitance

## Nonideal Capacitors

Ideal capacitor C

$$\hat{Z}(j\omega) = \frac{1}{j\omega C}$$

$$= \frac{1}{\omega C} \angle -90^\circ$$



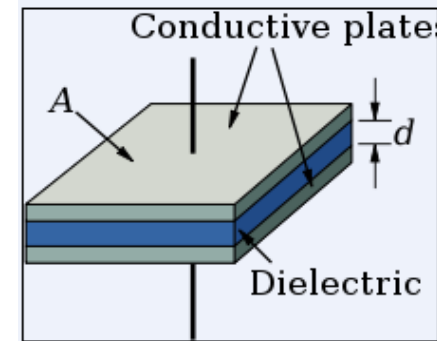
### Nonideal capacitor parameters

$R_{\text{diel}}$ : loss in the dielectric, large value

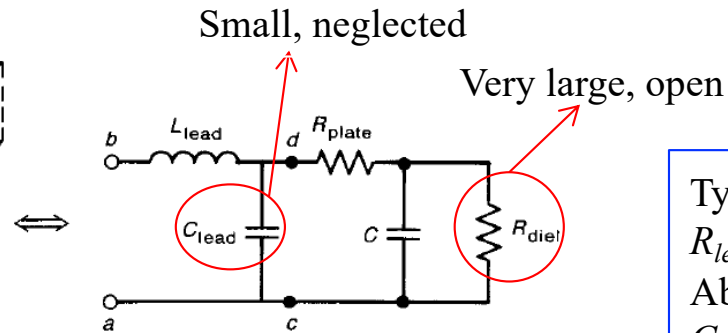
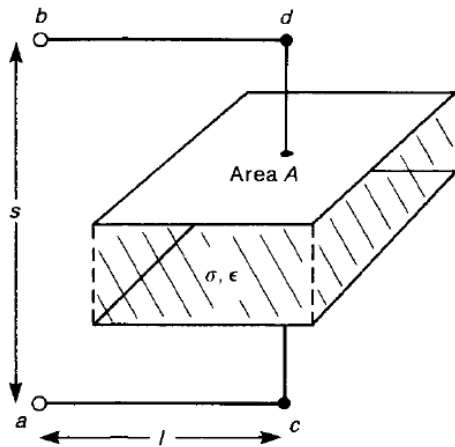
$R_{\text{plate}}$ : resistance of the plates, small value

$L_{\text{lead}}$ : inductance of the leads attached to the capacitor

$C_{\text{lead}}$ : capacitance of the leads attached to the capacitor



## Equivalent model of nonideal capacitors



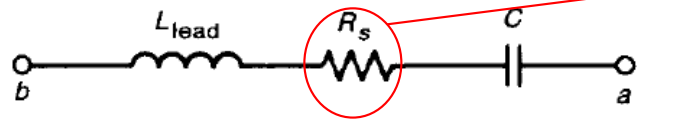
Typically

$$R_{lead} \ll R_{plate}$$

$$\text{Absolute value}(R_{diel}) \gg \text{Absolute value}(Z_C)$$

$$C_{lead} \ll C$$

Equivalent circuit

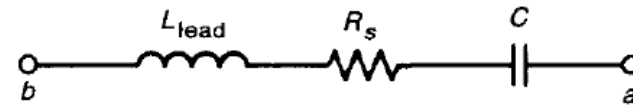


$R_{plate}$  referred to as the equivalent series resistance  $R_s$ .

$$\hat{Z}(j\omega) = L_{lead} \frac{1/L_{lead}C - \omega^2 + j\omega R_s/L_{lead}}{j\omega}$$

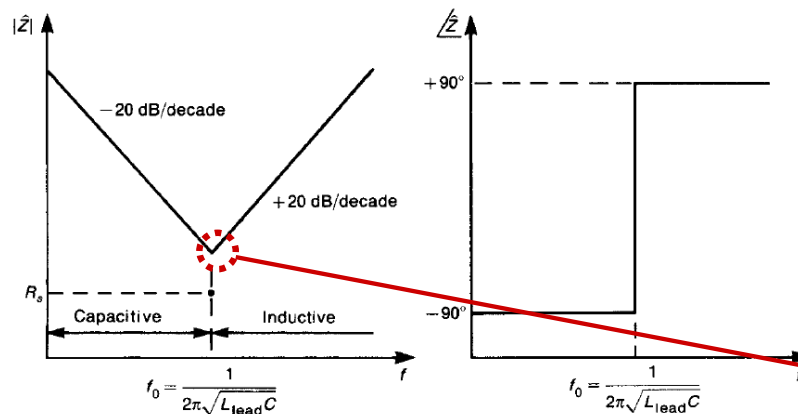
## Equivalent model of nonideal capacitors

$$\hat{Z}(j\omega) = L_{\text{lead}} \frac{1/L_{\text{lead}}C - \omega^2 + j\omega R_S/L_{\text{lead}}}{j\omega}$$



1. At DC  $\rightarrow$  open circuit

2. As frequency increases, the impedance of the capacitor dominates and decreases linearly with frequency at a rate of -20 dB/decade.



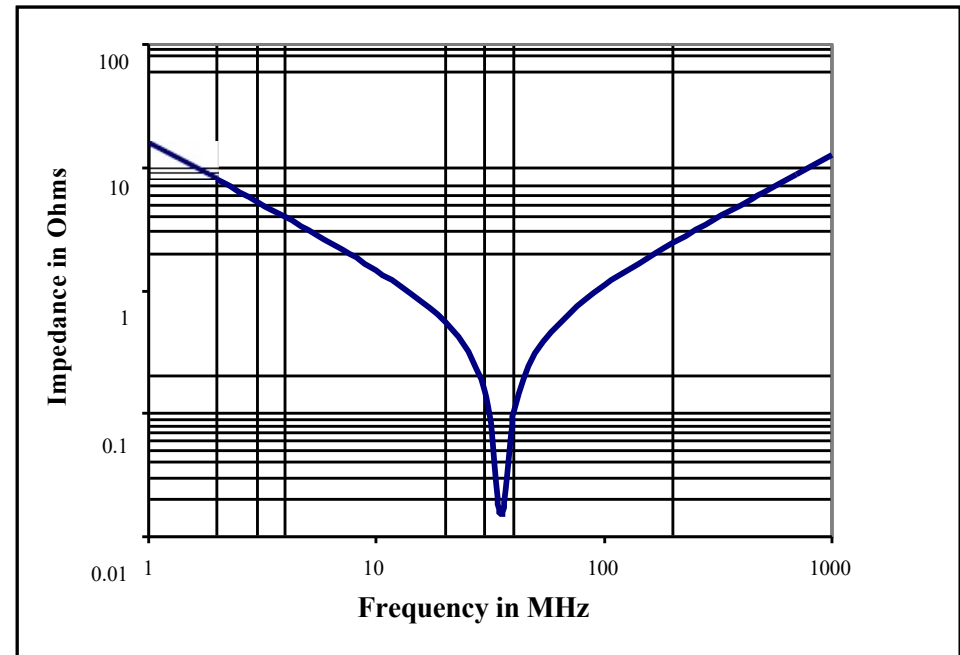
3. self-resonant frequency of the capacitor

$$f_0 = \frac{1}{2\pi\sqrt{L_{\text{lead}}C}}$$

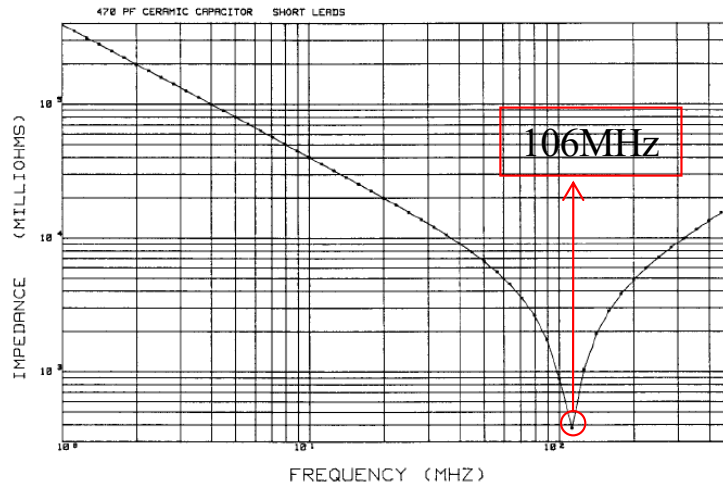
4. For higher frequencies the magnitude of the impedance of the inductor dominates and the impedance increases at a rate of +20 dB/decade

For a fixed lead length and spacing, the larger the capacitance value the lower the self-resonant frequency

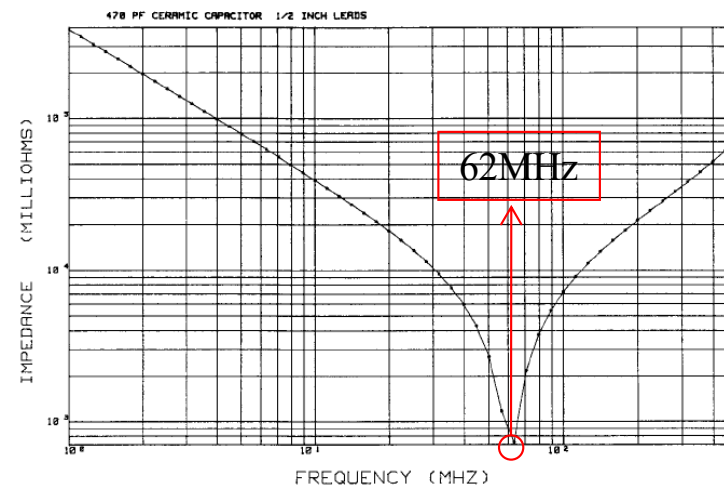
## Impedance of a 0.01- $\mu$ F Capacitor



## Lead lengths



Impedance of a 470-pF ceramic capacitor with very short lead lengths

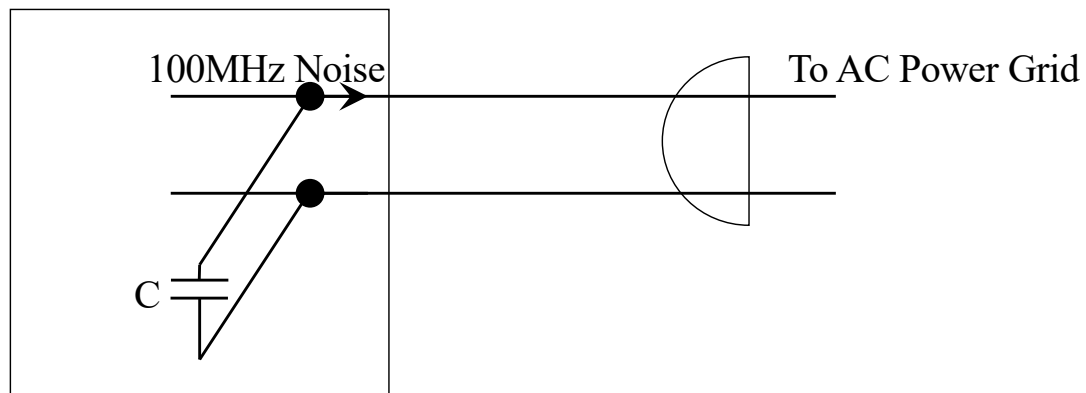


Impedance of a 470-pF ceramic capacitor with 1/2 in. lead lengths

Q: If one is interested in providing a low impedance to shunt a 200MHz signal, which one do you choose?

A: Neither. The 470-pF capacitor with either lead length will give an impedance larger than expected.

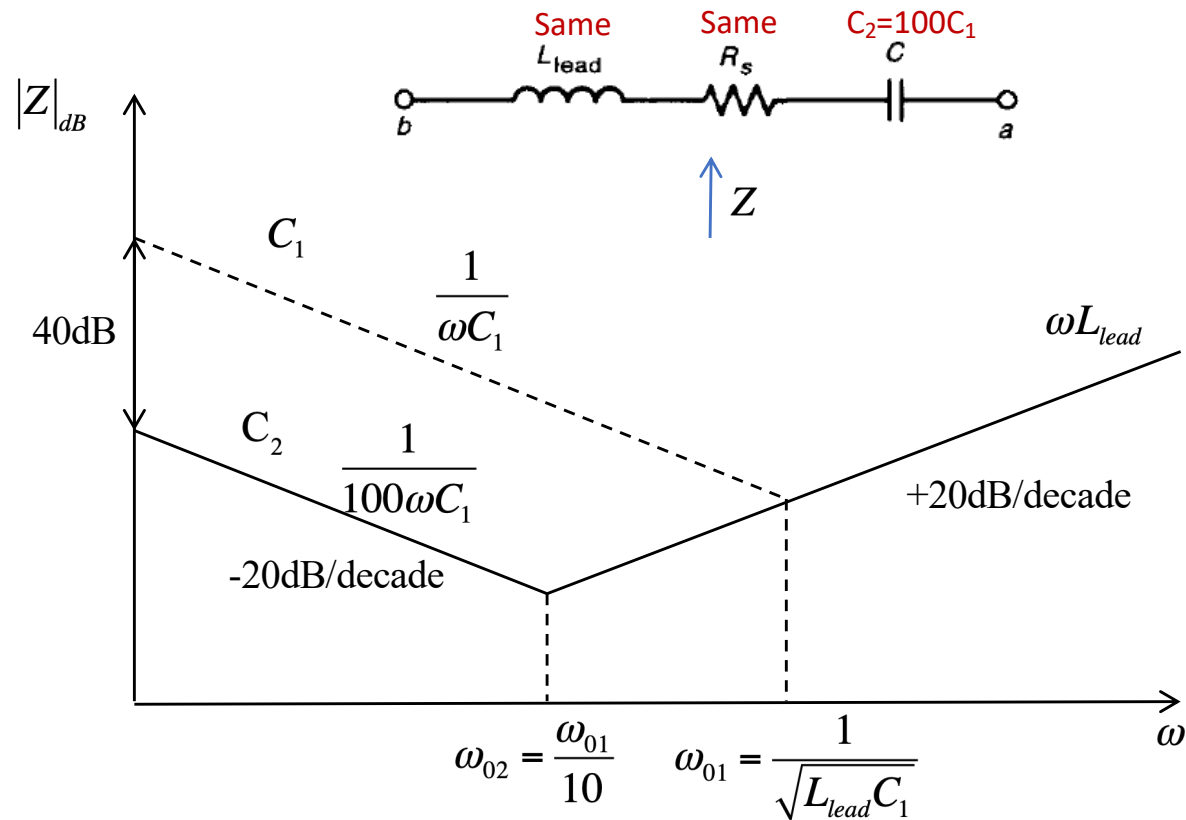
## Noise suppression via shunt capacitance



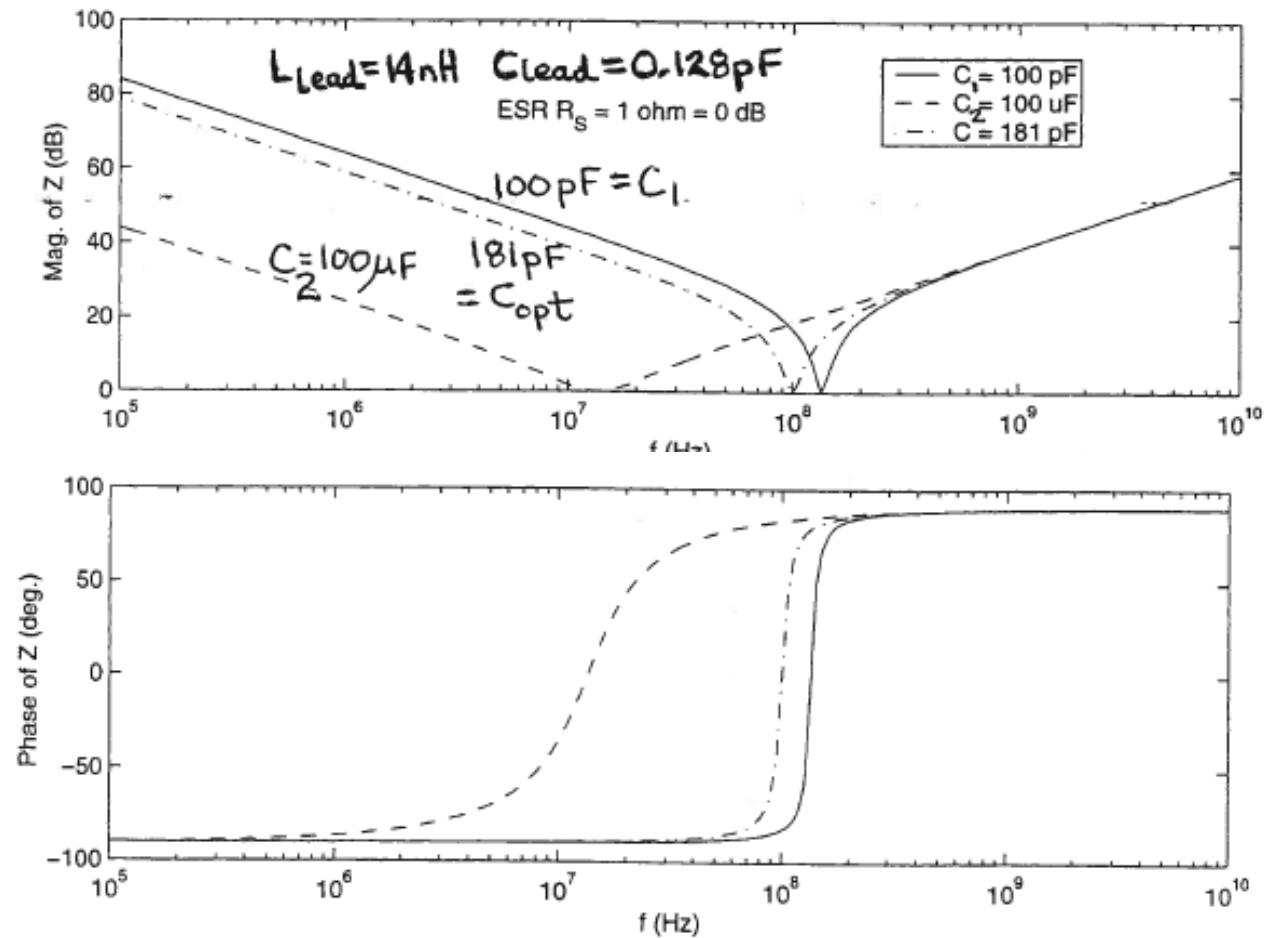
$$C_1 = 100\text{pF}$$

$$C_2 = 0.01\mu\text{F}$$

## Bold plot of $C_1$ and $C_2$ capacitor with same leads



## Optimum shunt capacitor value $C$ @ 10MHz

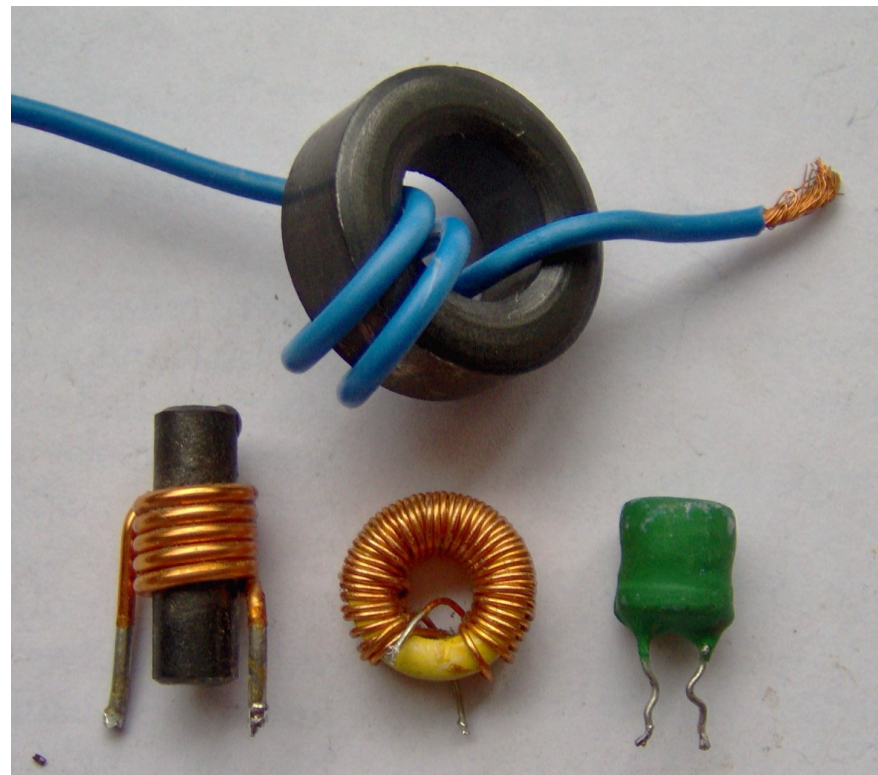


## 5.3.3 Nonideal Inductors

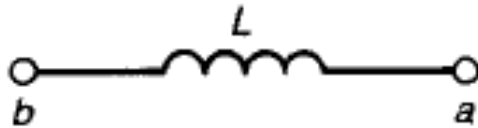
### Do Inductors Have Resistance and Capacitance?

**Inductors** are an electrical/electronic component that opposes a change in current due to its self-inductance.

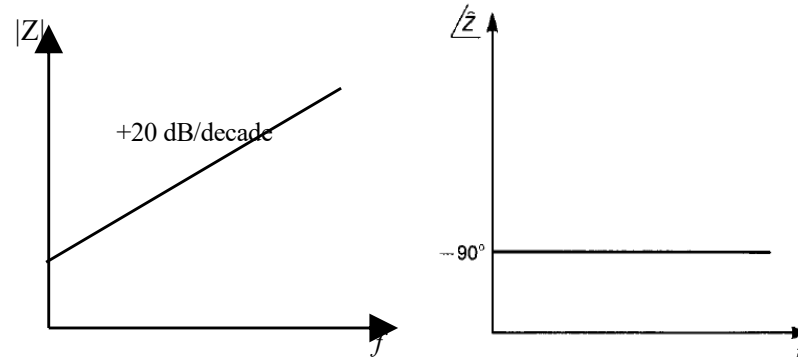
It consists of a number of turns of fine wire of copper wound on a core of some material ( air, iron, powdered iron, or ferrite) in spiral form.



## Nonideal Inductors



Ideal conductor:  $L$



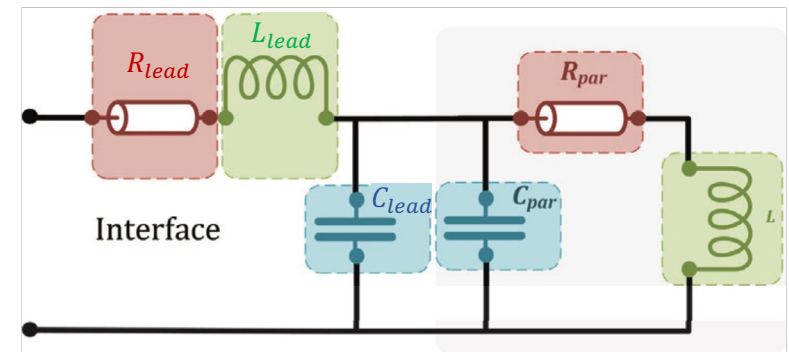
### Nonideal inductor parameters

$R_{\text{par}}$ : Series self-resistance of the inductor's windings

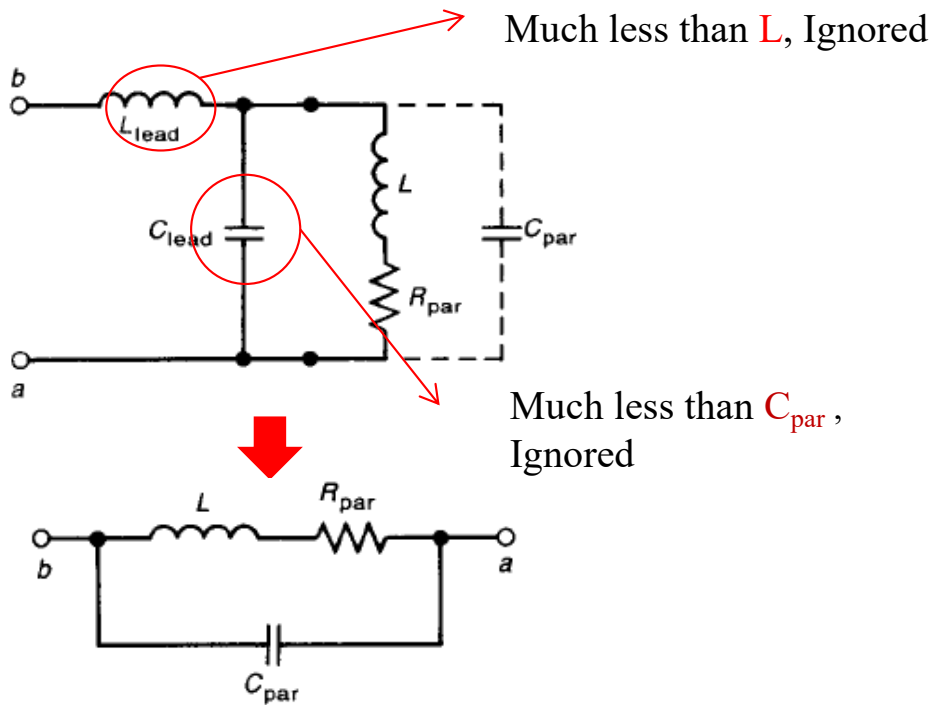
$C_{\text{par}}$ : Parallel parasitic capacitance

$L_{\text{lead}}$ : Lumped inductance of the leads, usually much less than  $L$ , neglected

$C_{\text{lead}}$ : Lumped capacitance of the leads, usually much less than  $C_{\text{par}}$ , neglected



## Equivalent model of nonideal inductors



Typically

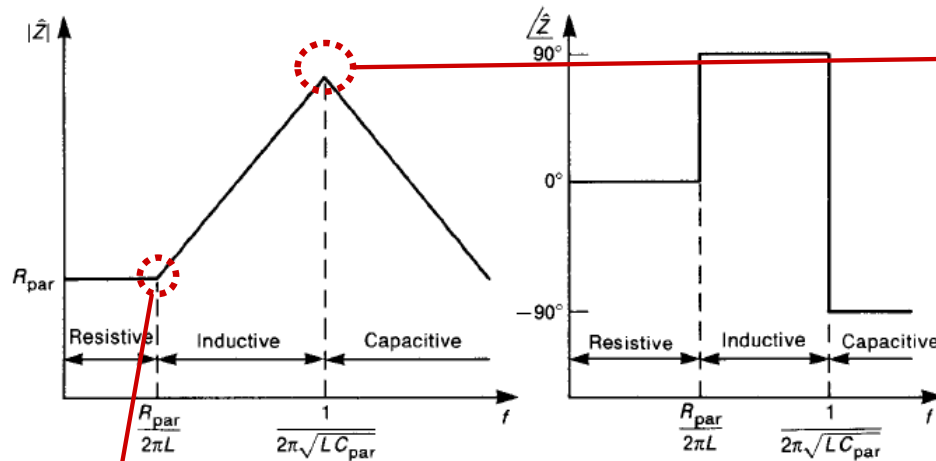
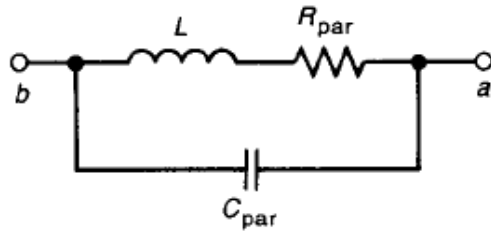
$$R_{\text{lead}} \ll R_{\text{par}}$$

$$L_{\text{lead}} \ll L$$

$$C_{\text{lead}} \ll C$$

$$\hat{Z}_L(j\omega) = R_{\text{par}} \frac{1 + j\omega L/R_{\text{par}}}{1 - \omega^2 LC_{\text{par}} + j\omega R_{\text{par}} C_{\text{par}}}$$

# Frequency response of the impedance of a nonideal inductor



$$f_1 = \frac{R_{\text{par}}}{2\pi L}$$

2. As frequency increases, when  $f > f_1$ , the inductor impedance dominates, and increases at a rate of 20 dB/decade.,

1. DC:  $Z = R_{\text{par}}$

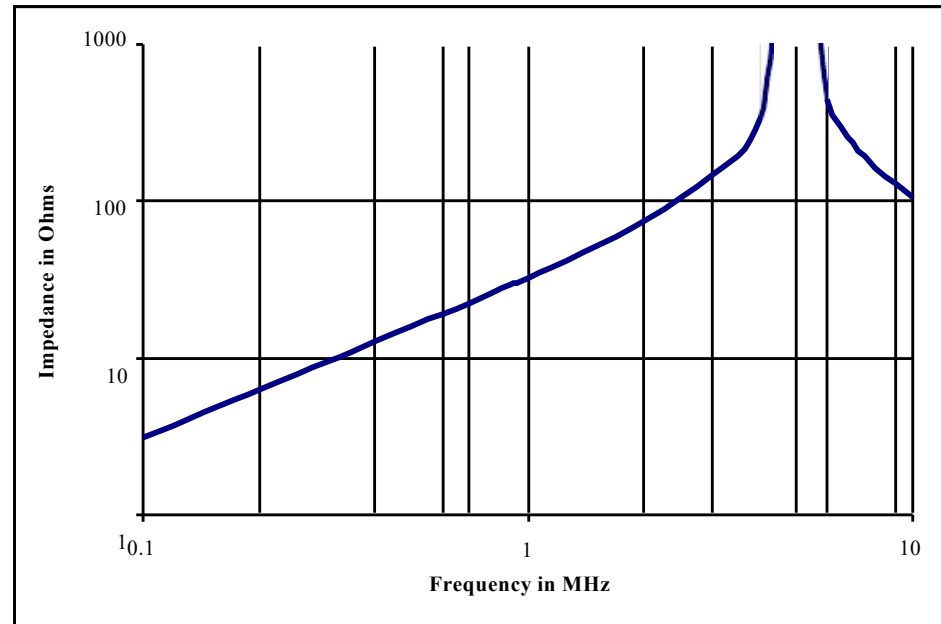
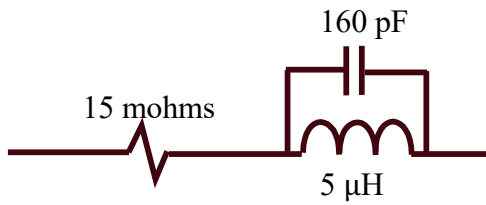
3. Self-resonant frequency:

$$f_0 = \frac{1}{2\pi\sqrt{LC_{\text{par}}}}$$

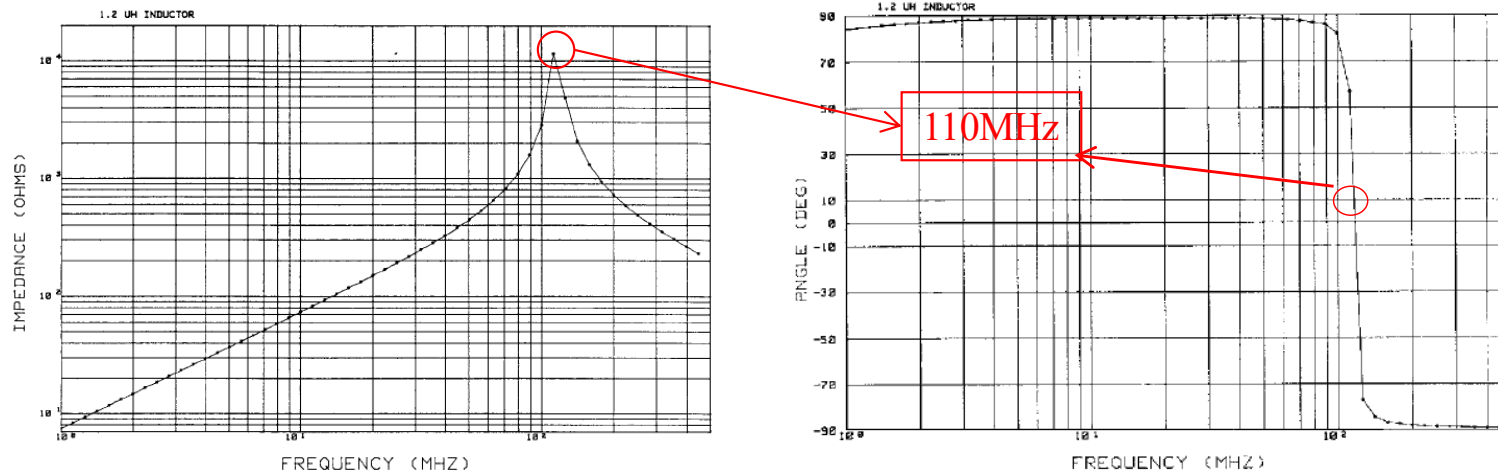
For a fixed lead length and spacing, the larger the inductance value the lower the self-resonant frequency

4. As frequency increases, the impedance of the capacitor dominates, and decreases linearly with frequency at a rate of -20 dB/decade.

## Impedance of a 5- $\mu$ H Inductor



## Example: Influence of the Parasitic capacitance



The measured impedance of a 1.2uH inductor from 1MHz to 500MHz.  
The self-resonant frequency of this inductor is about 110 MHz.

**Quiz:** Find the parallel capacitance for the L=1.2uH inductor

$$(C_{\text{par}} = 1.7 \text{ pF})$$

## Example: Conducted Emission: Line Filter Design

Real impedance behavior of the C and L

L: series insertion

C: parallel insertion

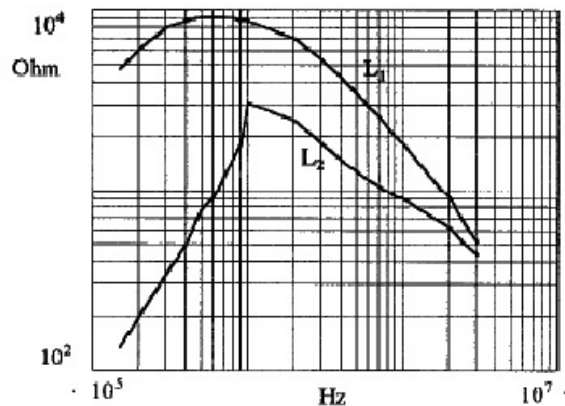


Figure 2: measure of the high frequency impedance of inductors  $L_1$  and  $L_2$

$L_1=3\text{mH}$ ,  $L_2=400\mu\text{H}$

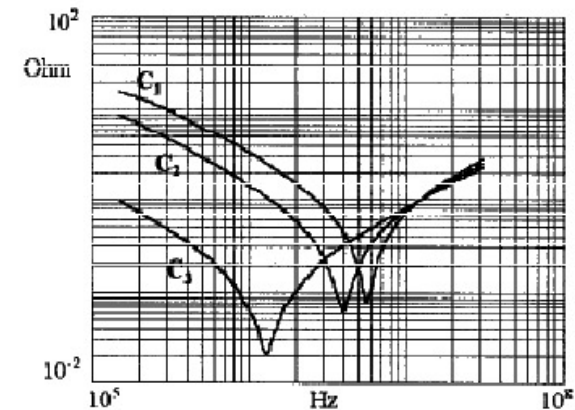
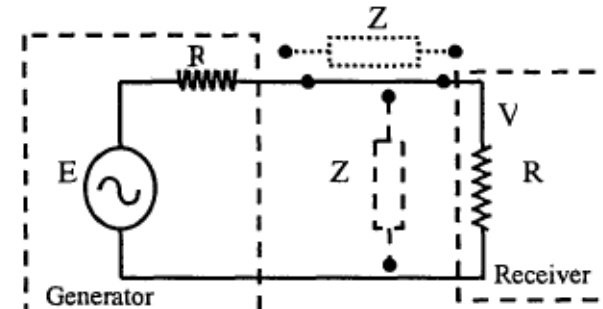
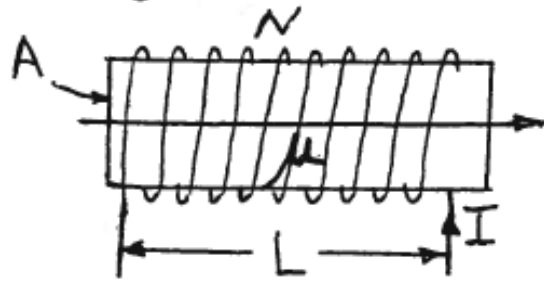


Figure 3: measure of the high frequency impedance of capacitors  $C_1$ ,  $C_2$  and  $C_3$

$C_1=47\text{nF}$ ,  $C_2=220\text{nF}$  and  $C_3 = 1\mu\text{F}$

“Common mode filter project by means of internal impedance measurement”,  
IEEE EMC Symposium, 2000

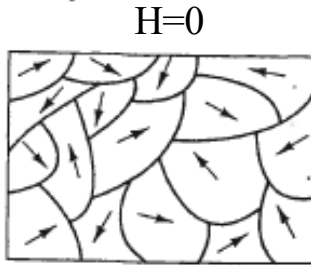
## Inductors with Ferromagnetic Cores



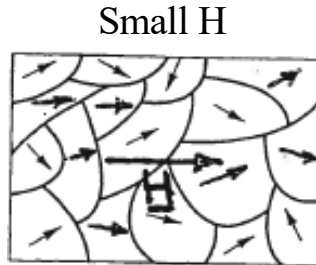
$$B = \mu H = \mu_0 (H + M)$$

$M$ : Magnetization vector

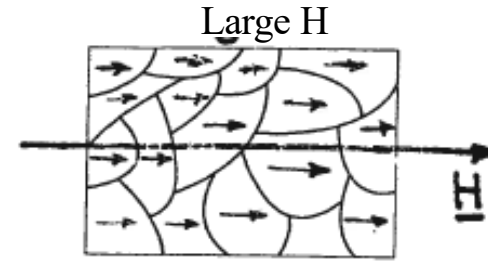
Actual inductance of an inductor can be much less than the stated value since measurement at **low current** and **low frequency**.



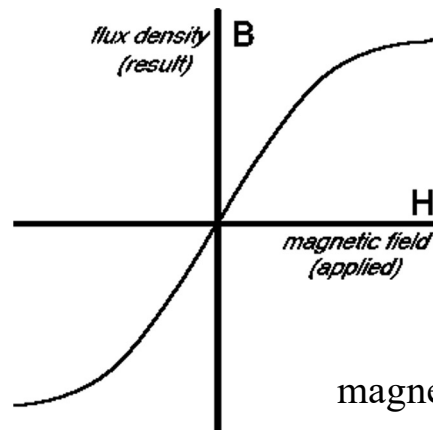
$M=0$



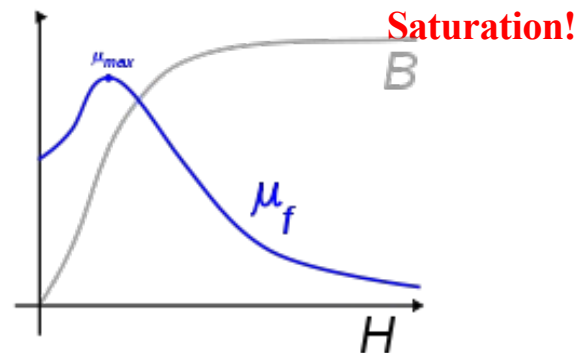
Small  $M$



$M$  Max.

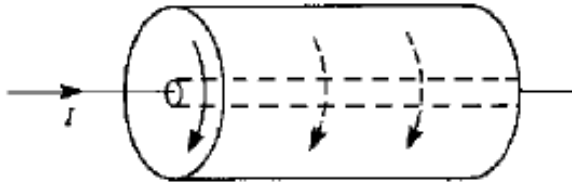


magnetic saturation



## Ferrite beads

A cylindrical ferrite core formed around a wire.



$$\mu_r(f) = \mu'_r(f) - j\mu''_r(f)$$

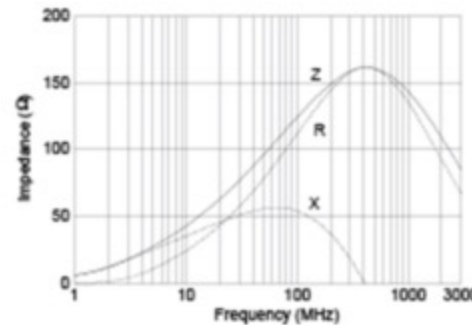
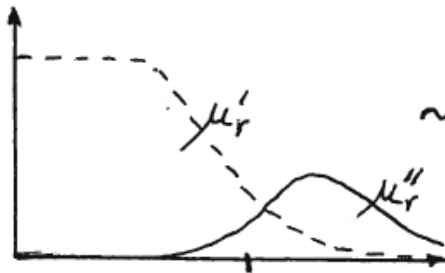
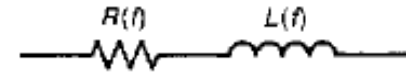
The ferrite bead will add an inductance proportional to the complex permeability of the ferrite core

$$L_{bead} = \mu_0 \mu_r(f) K \quad (K: \text{const. dep. on geo.})$$

$$Z_{bead} = j\omega L_{bead} = j\omega \mu_0 \left[ \mu'_r(f) - j\mu''_r(f) \right] K$$

$$= \omega \mu_0 \mu''_r(f) K + j\omega \mu_0 \mu'_r(f) K$$

$$= R(f) + jX(f)$$



$$Z = R + jX$$

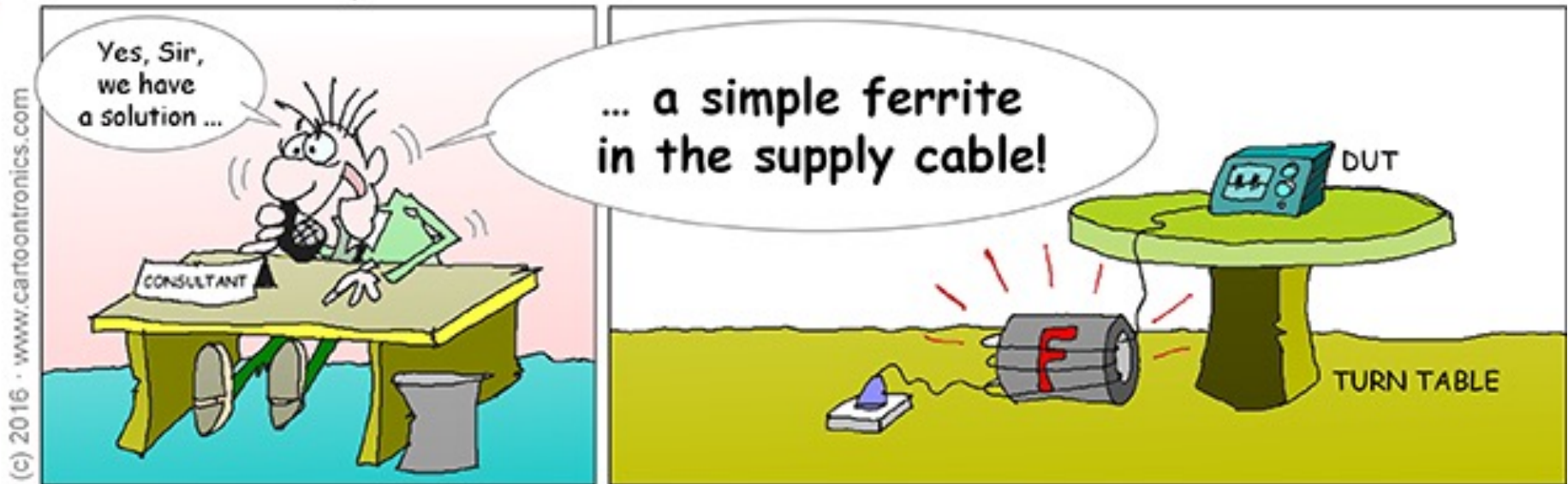


A ferrite bead at the end of a Mini USB cable

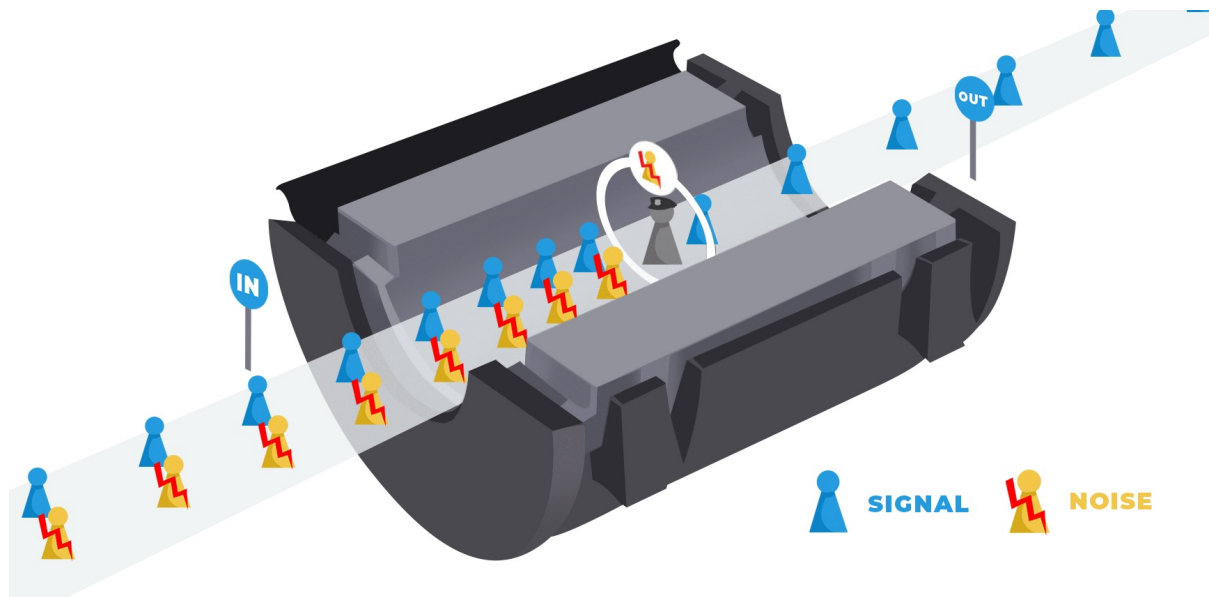
## Ferrite beads

**Cartoontronics Inc.**

Pass the test! by Arturo Mediano



## Ferrite beads



Ferrite beads are generally used for high frequency EMI noise suppression  
Disadvantage: Voltage drop and heat dissipation

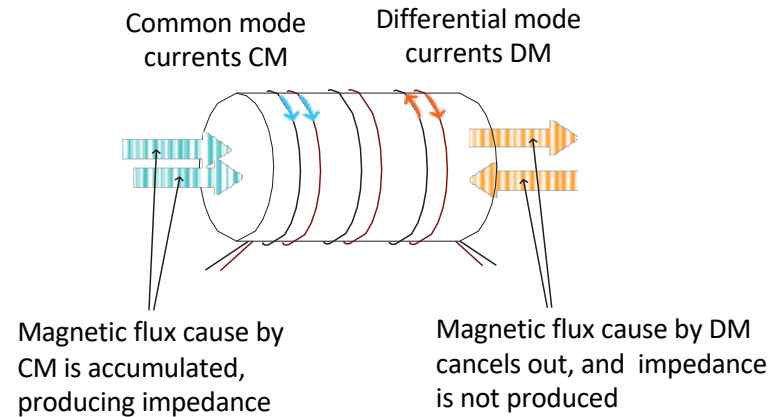
## Example: Common mode chokes



The final objective of a common mode choke is **to block the conduction of the electromagnetic interferences** while the lower frequency range 'operational signals' are not affected.

Common mode inductors are wound with two or three windings of equal numbers of turns. The number of windings is the same as the number of phases.

They are made of **ferromagnetic materials**.

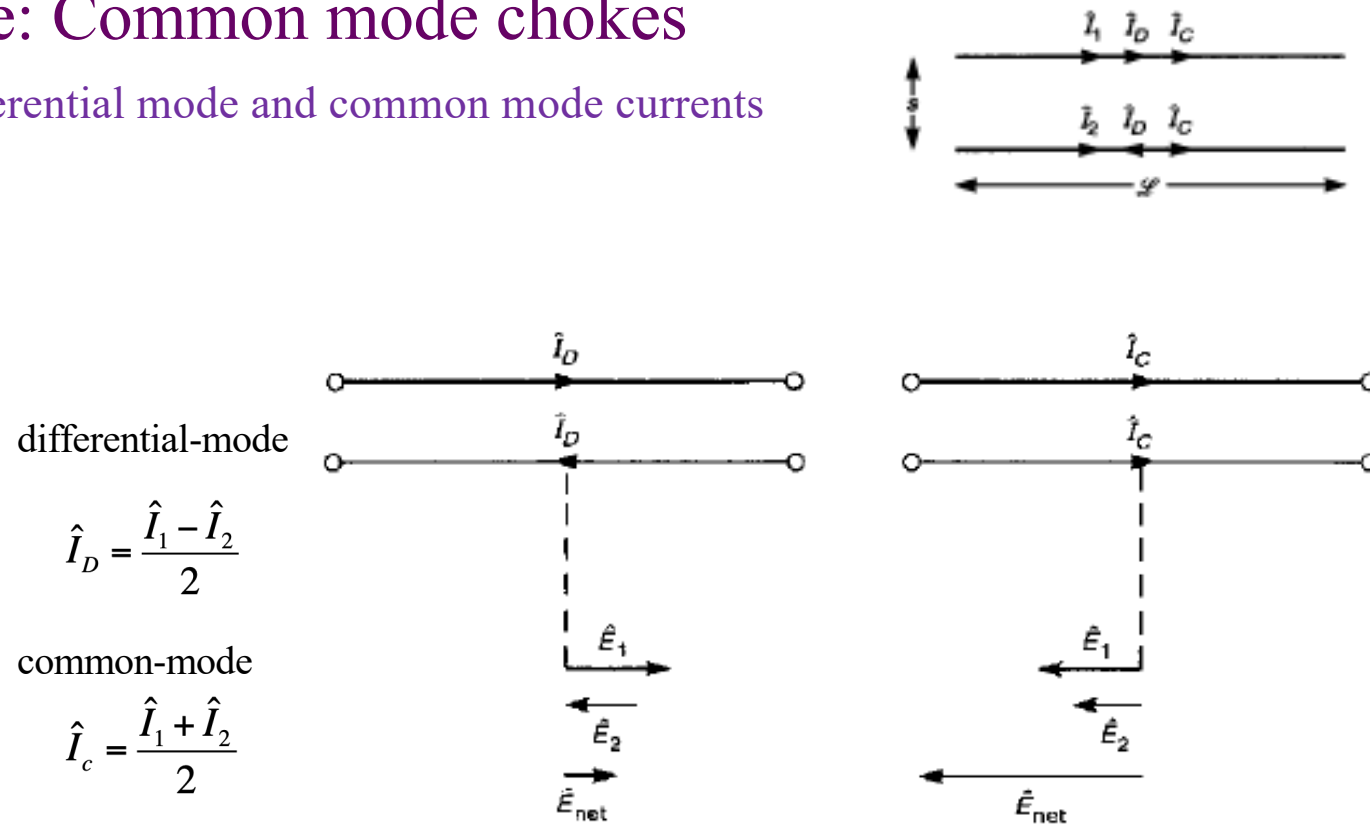


A common mode choke is used for its inductive behavior and also for a **transformation of the stored energy into heat**.

The amount of common mode current that is transformed in heat by the common mode choke and the frequency range of efficiency is first determined by the material itself and then by the overall design.

## Example: Common mode chokes

Review: Differential mode and common mode currents



relative radiated emission potential of (a) differential-mode currents and (b) common-mode currents.

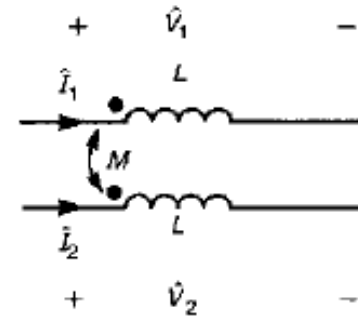
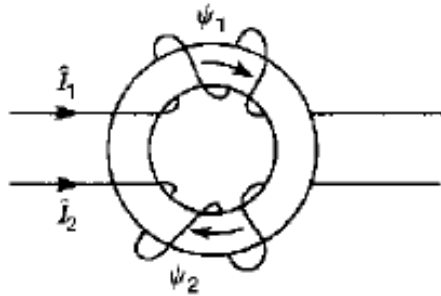
## Poll

- Which current mode dominate the radiated emission in the far field region?
  - A. Common mode current
  - B. Differential mode current



## Common mode chokes

### Common Mode Current Rejection



$$B_1 = \mu \frac{N}{l} I_1 \rightarrow \Psi_1 = B_1 A = \mu \frac{NA}{l} I_1 \rightarrow L_1 = \frac{\Psi_1}{I_1} = \mu \frac{NA}{l} = L_2 \quad \text{assume that the windings are identical}$$

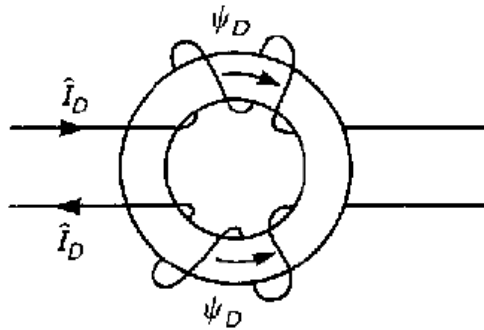
$$\text{For high } \mu \quad \Psi_{12} = \Psi_1 \rightarrow M_{12} = \frac{\Psi_{12}}{I_1} = L_1 = M_{21}$$

$$V_1 = j\omega L I_1 + j\omega M I_2$$

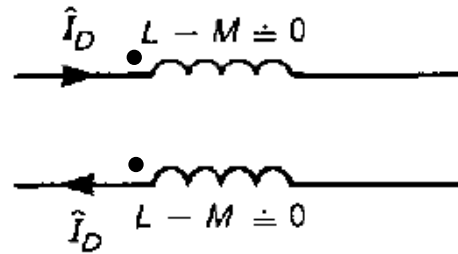
$$\text{impedance of one winding} \quad \therefore Z_1 = \frac{V_1}{I_1} = \frac{j\omega(LI_1 + MI_2)}{I_1} \quad \leftarrow \text{Depends on } I_1 \text{ and } I_2$$

## Common mode chokes

### Common Mode Current Rejection



Differential-mode currents

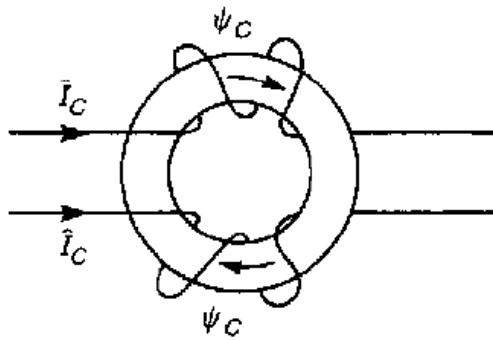


$$I_2 = -I_1$$

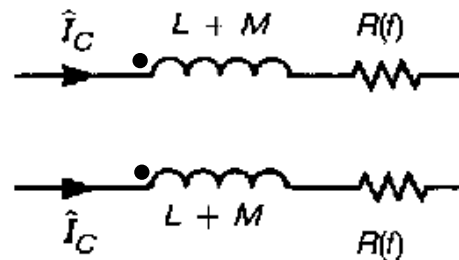
$$Z_D = j\omega(L - M)$$

For ideal case,  $L = M$

$$\therefore Z_D = 0$$



Common-mode currents



$$I_2 = I_1$$

$$Z_c = j\omega(L + M)$$

For ideal case,  $L = M$

$$\therefore Z_D = j\omega 2L$$

Note: 1.  $L=M$  requires  $\mu \rightarrow \infty$  2. Since  $I_C \ll I_D$ , saturation rarely occurs for  $I_C$

# Differences Between Ferrite Beads and Common-Mode Chokes

| Feature         | Ferrite Bead                                                                                  | Common-mode Choke                                                                             |
|-----------------|-----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|
| Primary Purpose | Suppresses high frequency noise                                                               | Suppresses common-mode noise                                                                  |
| Function        | Acts as a high-frequency resistor, absorbing high-frequency noise                             | Presents high impedance to common-mode signals and low impedance to differential-mode signals |
| Application     | Used in power supply lines, signal lines, or specific conductors for high-frequency filtering | Used in power cables, Ethernet, USB, HDMI for common-mode EMI filtering                       |
| Construction    | Typically a ferrite core through which a single conductor passes                              | Typically a ferrite core with multiple windings (two or more conductors)                      |



## Summary

- All components (when connected to a circuit) have resistance, capacitance, and inductance.
- The behavior of a component at high frequencies is usually much different than the normal (low frequency) behavior
- The inductance of a low-inductance device is generally determined by the connection and is not a property of the device itself.



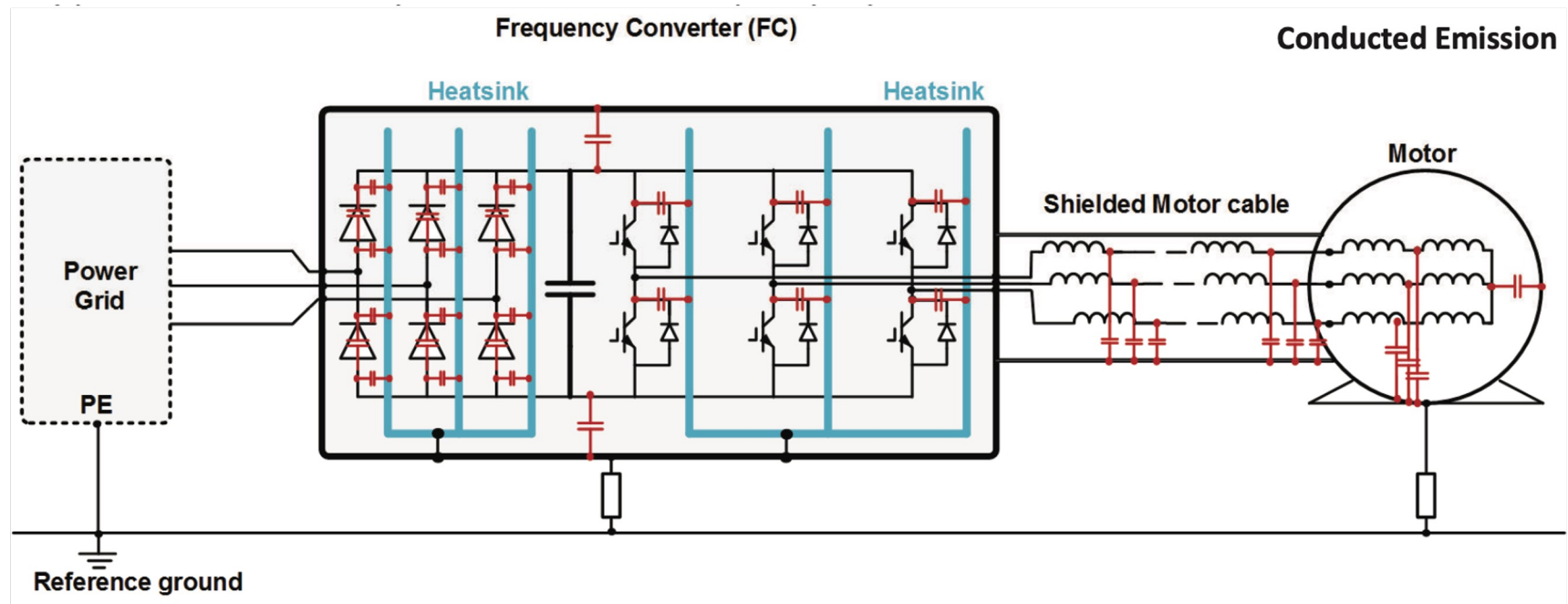
# References

- **Introduction to Electromagnetic Compatibility**, 2<sup>nd</sup> Edition, by Paul Clayton
- **Real and Ideal Components**, by Dr. Anne Roc'h, TU/e, the Netherlands



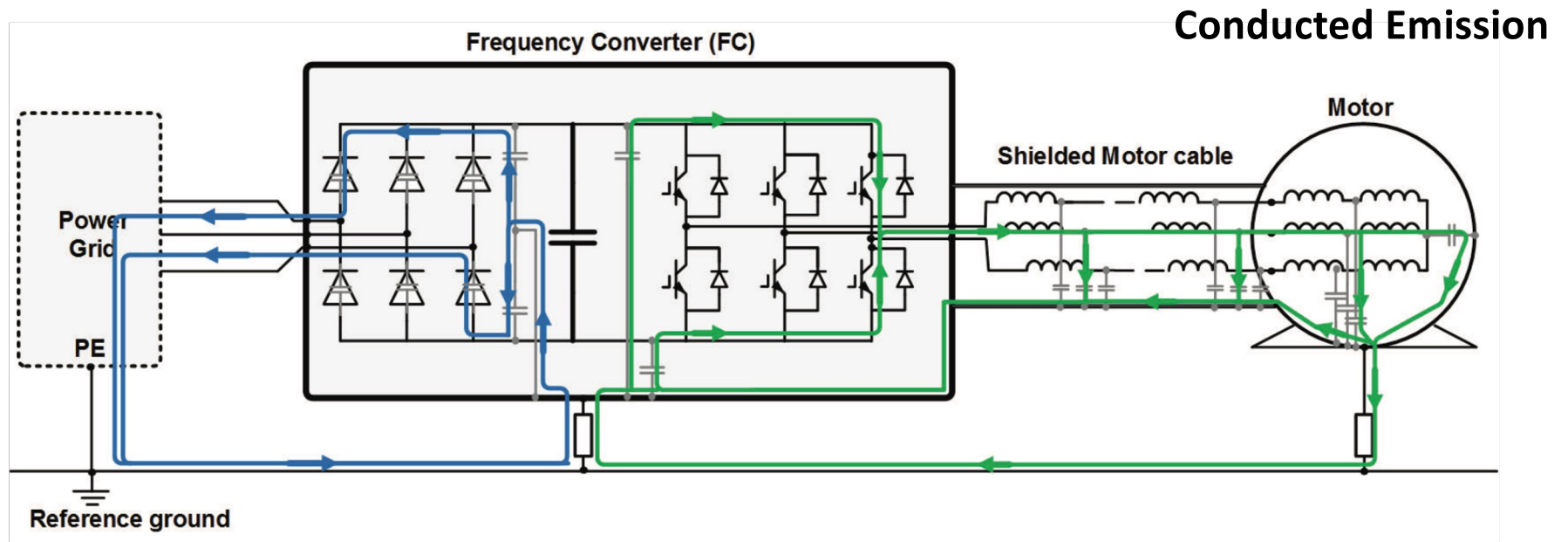
# Propagation of EMI in ASD

Key parasitic in an ASD. They are installation and frequency dependent.

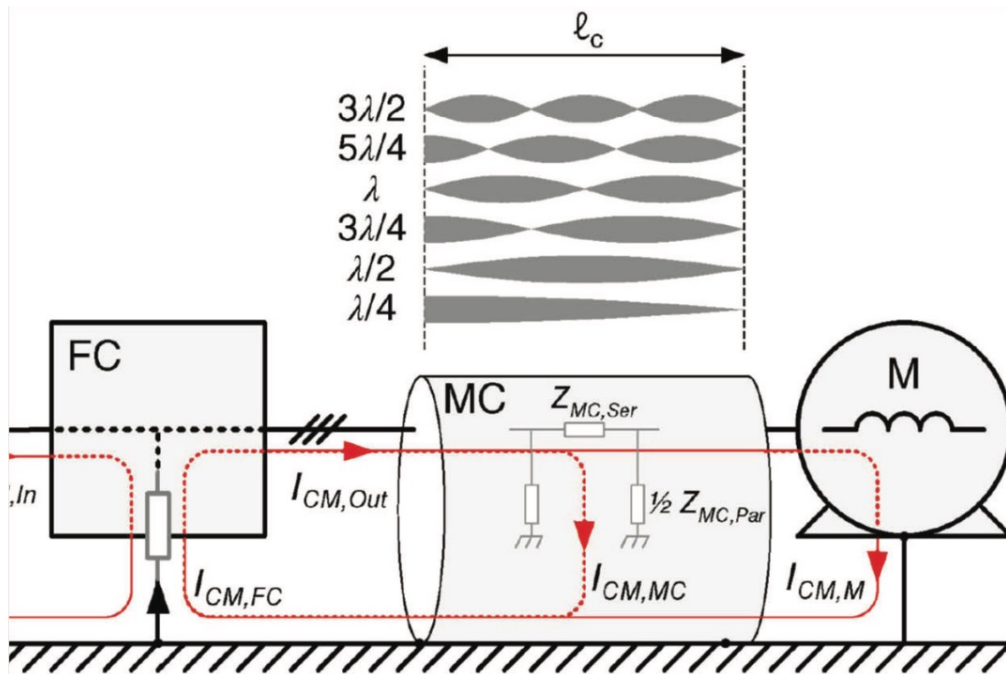


# Propagation of EMI in ASD

Simplified overview of CM propagation in an ASD



## Propagation of EMI in ASD



For wavelength  $\lambda/4$

Current starts to form standing waves in the cable.

The cable is radiating.