



Electrical & Computer Engineering

GRAINGER COLLEGE OF ENGINEERING

Lecture 5 - Nonideal Behavior of Components

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What are components?

- Resistances, inductances, capacitances (“Real components” and 3 “Ideal Components”) Parasitic effects of passive “Real” components (resistors, capacitors, inductors)
- Interfaces : Wires, cables, tracks, bonding wires, conducting parts in materials (Carbon nanotubes) Ferromagnetic cores (“Ferrites”)
- Electromechanically components: motors, relays, switches
Semiconductors (IC and packaging)
- ...

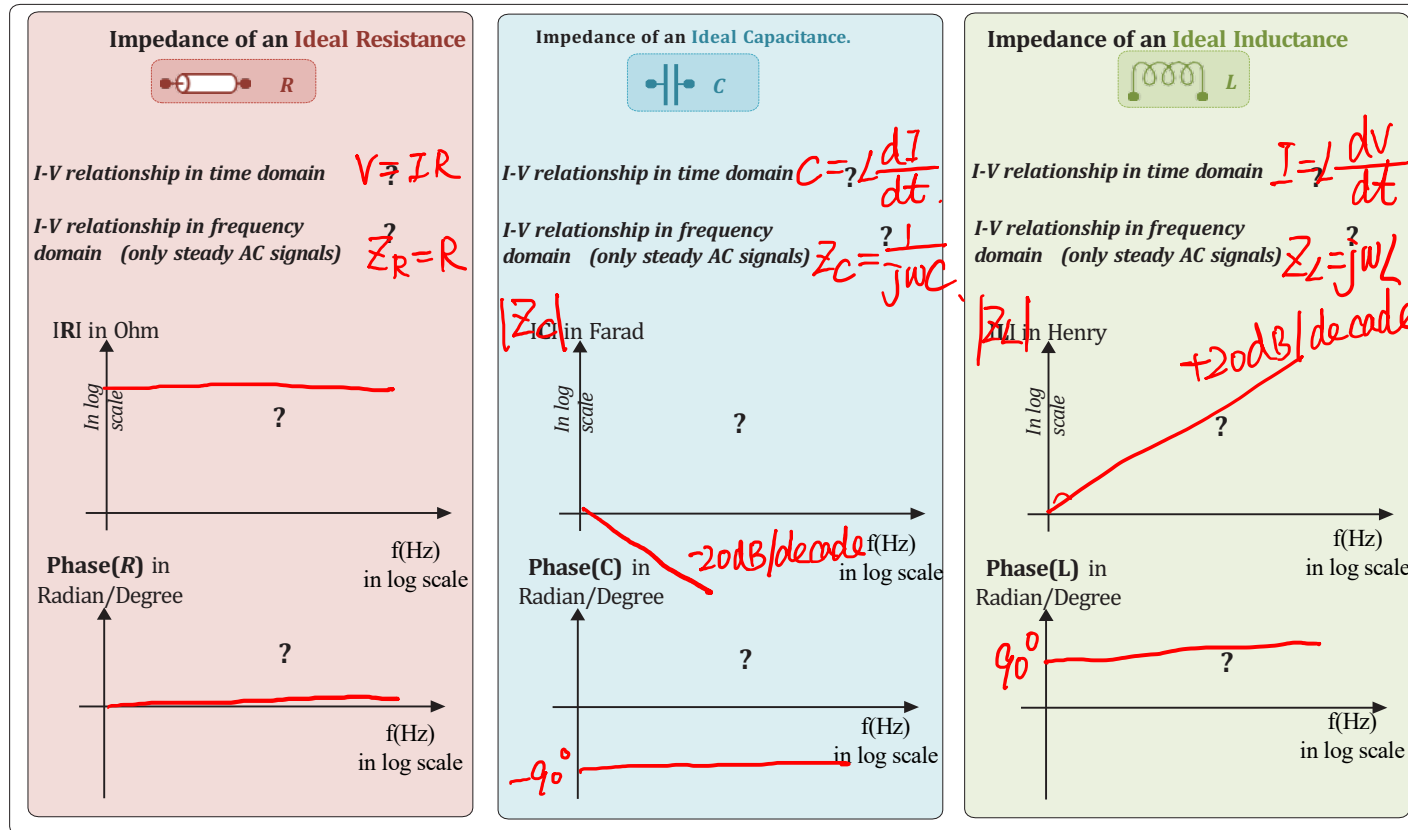


Different kinds of Components

- 5.1 Ideal Components
 - Resistance
 - Inductance
 - Capacitance
- 5.2 ————
 - Effect of Component Leads
- 5.3 Nonideal Components
 - Nonideal Resistor
 - Nonideal Capacitor
 - Nonideal Inductor



5.1 Ideal component – our goal in this first part:



Understands the behavior of the three basic Ideal components: R, L, and C

In other words,

Understand the three “ideal relationships” one can establish between a voltage difference $V(t)$ (between the terminals of a component) and the current $I(t)$ crossing it.

Ideal components - Intro

In this section we will consider a Circuit with Ideal Components

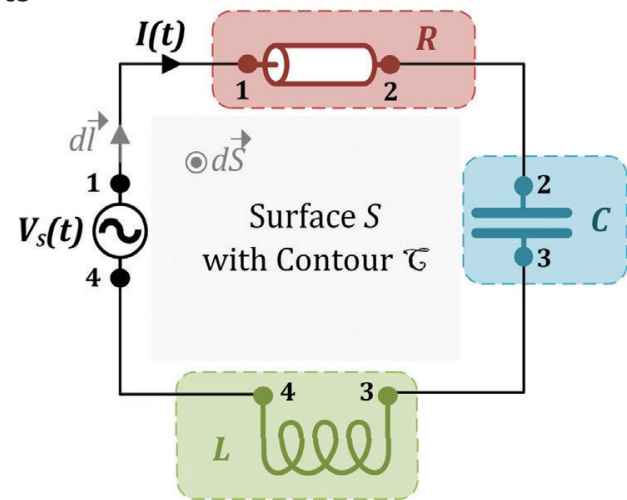
Some assumptions:

- Components are electrically small
 $\lambda = c / f$, with λ the wavelength, c the speed of light in vacuum and f the frequency
- Circuit is electrically small (short wires, no “interfaces” between components)
- Sinusoidal Steady State

$$L < \frac{\lambda}{4}$$

trise / tfall.

t_{prop.}



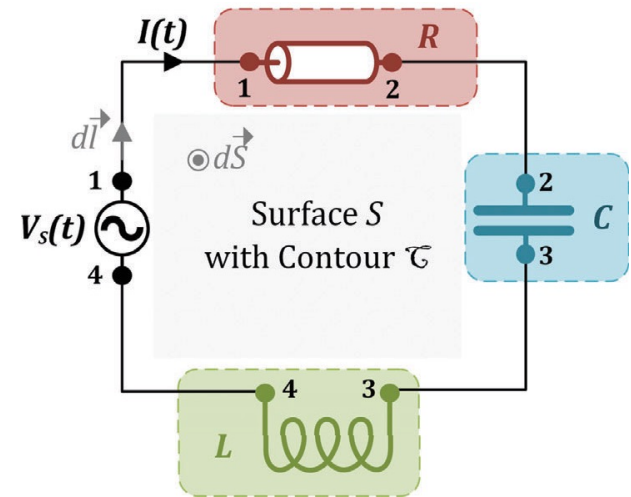
Circuit with Ideal Components

Ideal Components - Intro

In this section we will consider a Circuit with Ideal Components

In the coming slides we will explore the definition of each Ideal components in this circuit:

- The Ideal Resistance,
- The Ideal Capacitance,
- The Ideal Inductance.

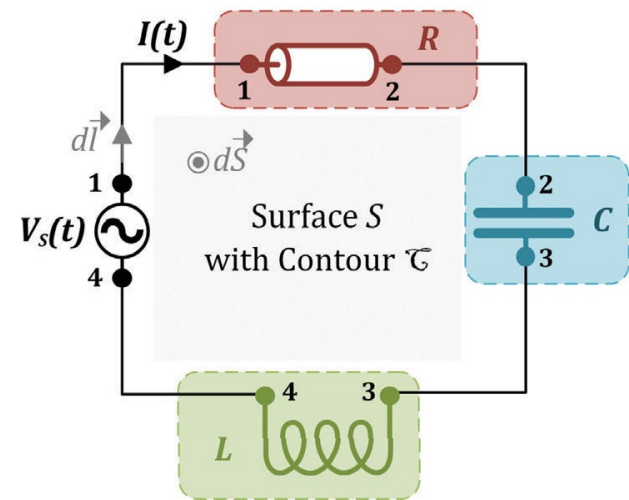


Circuit with Ideal Components

Ideal Components - Intro

Between any two adjacent terminal a and b , the Voltage is defined as:

$$V_{ba}(t) = - \int_a^b \vec{E}(t) \cdot d\vec{l}$$



Circuit with Ideal Components

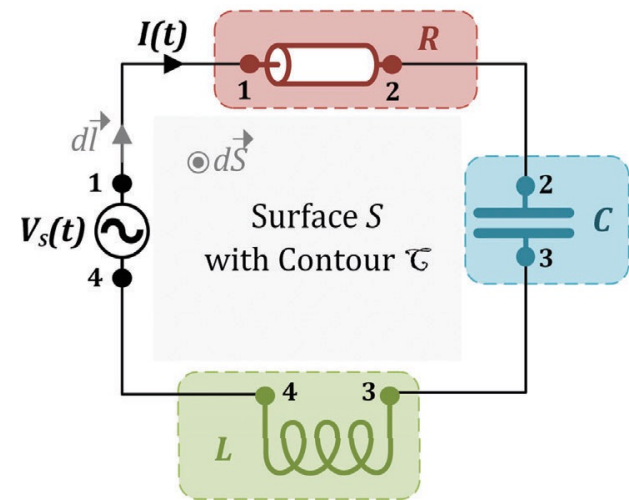
Ideal Components - Intro

Between any two adjacent terminal a and b , the Voltage is defined as:

“a measure (at one point between points b and a) of how closely the electric field $\vec{E}(t)$ aligns with the unit vector $d\vec{l}$, in terms of the directions they point”

$$V_{ba}(t) = - \int_a^b \vec{E}(t) \cdot d\vec{l}$$

“the sum of all contributions along a line between points b and a ”



Circuit with Ideal Components

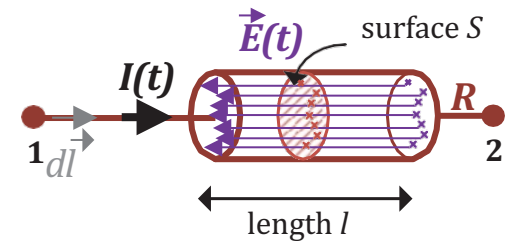
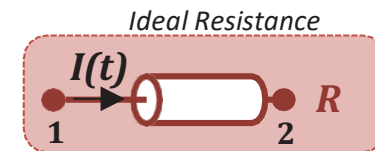
5.1.1 The ideal Resistance

An Electric field is pushing electron inside the resistor.

Resistor are used to reduce current flow.

Inside a conductive material, density of current: $\vec{j}(t) = \sigma \vec{E}(t)$
with σ the electric conductivity of the material and A the area crossed by the Electric Field.

$$V_{21}(t) = - \int_1^2 \vec{E}(t) \cdot d\vec{l}$$



5.1.2 Inductance

Probably the most misunderstood concept in electrical engineering
– Do not confuse ‘inductance’ with ‘inductors’

Current loop $\rightarrow$ inductance!

$$L \rightarrow \psi_B = \oint_C \mathbf{H} \cdot d\mathbf{l} \quad \uparrow \quad L = \frac{\psi_B}{I}$$

$\psi_B = \frac{d}{dt} \oint \mathbf{B} \cdot d\mathbf{S}$

Inductances may be classified as:

- Internal inductance
- External inductance
- Self inductance
- Mutual inductance
- Loop inductance
- Partial inductance

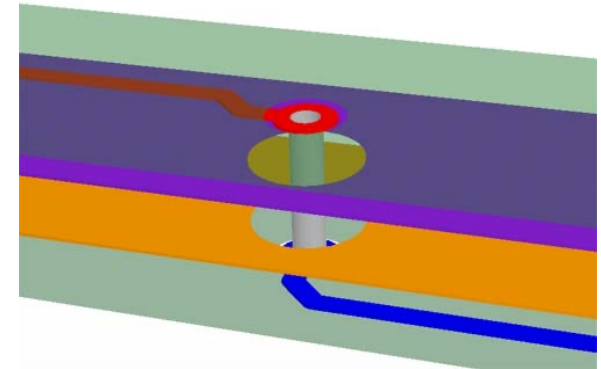
Poll: Do these have inductance?



Ground strap



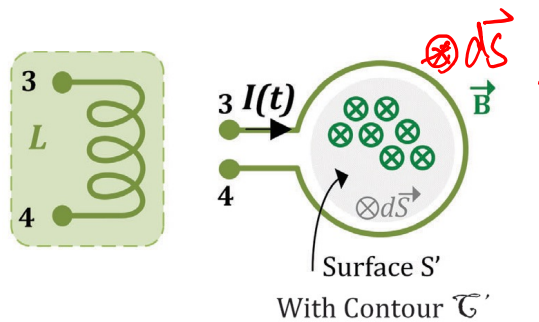
SMT capacitor



PCB via

Yes . after I loop.

The ideal Inductance



Example: Coil
with one loop

(self-) Inductance is the tendency of an electrical conductor to oppose a change in the electric current $I(t)$ flowing through it.

The flow of electric current creates a magnetic field around the conductor. Any change in magnetic field through a circuit induces an **electromotive force (EMF) (voltage)** in the conductors, a process known as electromagnetic induction.

The induced voltage created by the changing current has the effect of opposing the change in current.

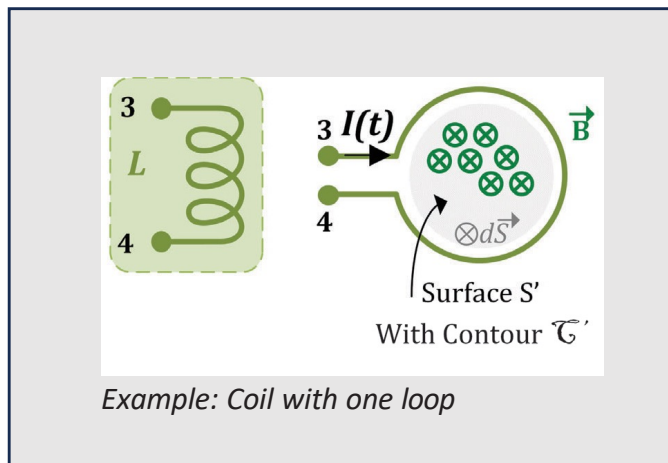
The induced
EMF is

$$V_{34}(t) = \frac{d}{dt} (\text{Flux of } \vec{B}(t) \text{ through } S') = L \frac{dI(t)}{dt}$$

$\iint_S \vec{B} \cdot d\vec{S}$

The inductance L is defined as the flux linkage per unit current.

The ideal Inductance



The inductance L is defined as the flux linkage per unit current.

$$\frac{\psi_B}{I} = L = \frac{\int_{C'(S')} \vec{B}(t) \cdot d\vec{S}'}{I(t)} = \frac{B(t)S'}{I(t)}$$

L depends on geometry

For N superposed turns in a coil, total $L_{total} = NL$

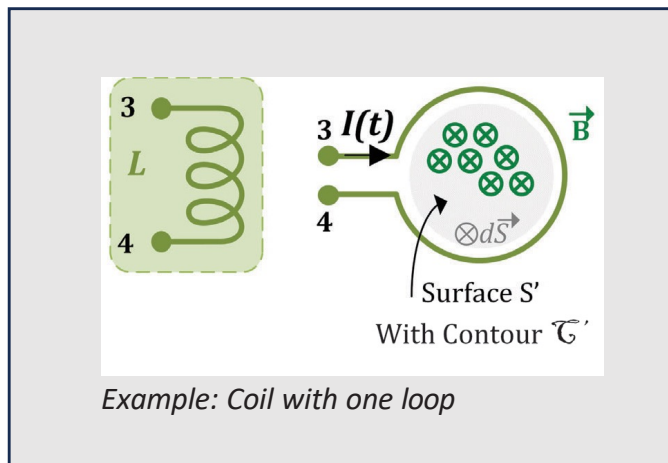
“The **Flux** is the integral over the surface S' . It sums all contributions across the surface S' ”

$$\psi_B = \int_{C'(S')} \vec{B}(t) \cdot d\vec{S}'$$

[Wb]

“a measure (in the Surface S') of how closely the magnetic field $\vec{B}(t)$ aligns with $d\vec{S}'$ (the unit normal vector to the Surface S') in terms of the directions they point”

The ideal Inductance



The inductance L is defined as the flux linkage per unit current.

$$L = \frac{\int_{C'(S')} \vec{B}(t) \cdot d\vec{S}'}{I(t)} = \frac{B(t)S'}{I(t)}$$

L depends on geometry

For N superposed turns in a coil, total $L_{total} = NL$

$$V_{34}(t) = - \oint_{C'} \vec{E}(t) \cdot d\vec{l} = \frac{d}{dt} \int_{S'} \vec{B}(t) \cdot d\vec{S} = L \frac{di(t)}{dt}$$

The ideal Inductance

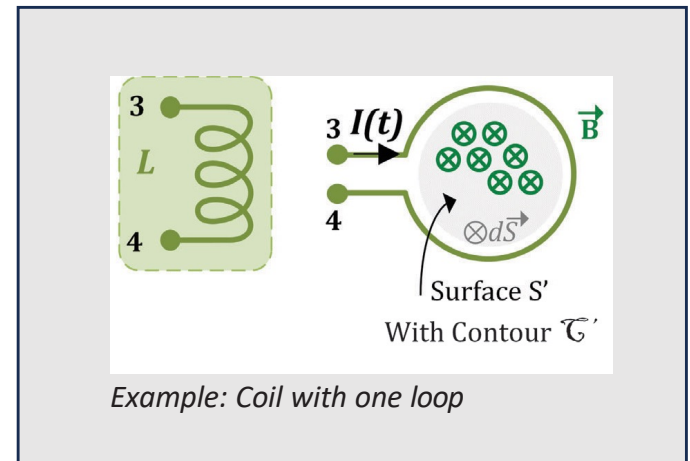
$$V_{34}(t) = - \oint_{C'} \vec{E}(t) \cdot d\vec{l} = \frac{d}{dt} \int_{S'} \vec{B}(t) \cdot d\vec{S} = L \frac{di(t)}{dt}$$

Time domain

$$V_{34}(t) = L \frac{di(t)}{dt}$$

Frequency domain

$$V_{34}(t) = j\omega LI$$



Inductance

- Why inductance is important?
- Internal and external inductance
- Self and Mutual inductances
- Loop and partial inductances
- How to reduce inductance?



Why do we care about inductance?

- Why On-chip inductance effects have become increasingly significant?
 - Some global signal and clock wires are routed with large widths and thickness to minimize delays. This decreases the resistance of the wires, making their inductive impedance comparable to the resistive part.

$$Z = R + j\omega L$$

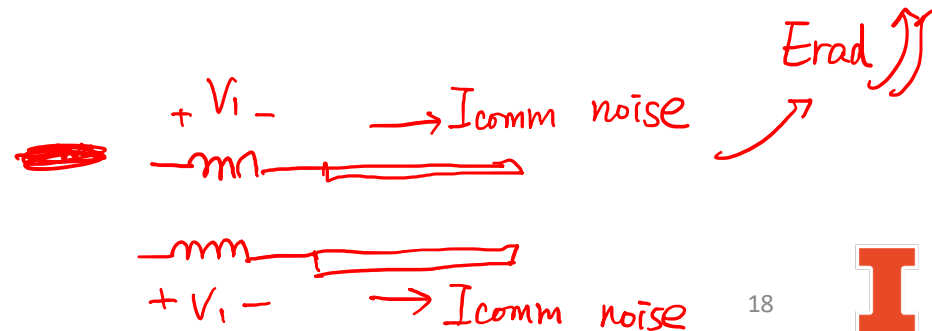
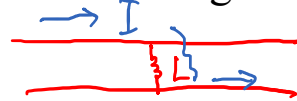
- Copper wires (with low resistivity) has less RC delays compared with aluminum wires (with higher resistivity). This could make the wire inductive reactance larger than the resistance.
- High-frequency components in electrical signals makes the inductance effects more significant.
- Many wires are long and run in parallel, which increases the inductive crosstalk and delay.



Adverse Effects of the Inductance

- Signal quality
 - Inductive discontinuity $\rightarrow$ Overshoot/undershoot, ringing
- $V = L \frac{dI}{dt}$ voltage drop $\rightarrow$ unwanted V_{drop}
 - Voltage drop on traces and ground/power planes
- Crosstalk
 - Distributed mutual inductance in uniform transmission lines and lumped mutual inductance
- Electromagnetic interference (EMI)
 - Generation of common mode noise currents

Z mismatch $\rightarrow$ reflection.



Internal and external inductance

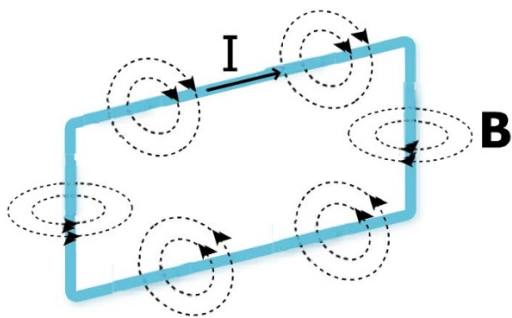
Internal inductance



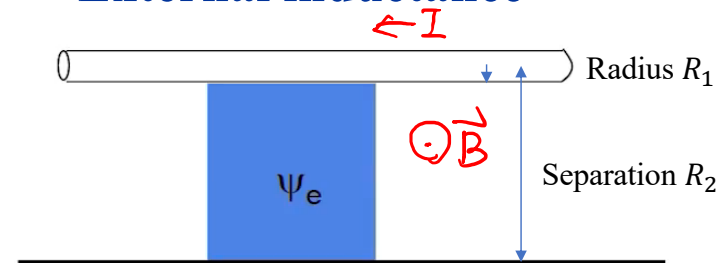
Reference

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{s} \text{ Webers}$$

$$L = \frac{\Psi}{I}$$



External inductance



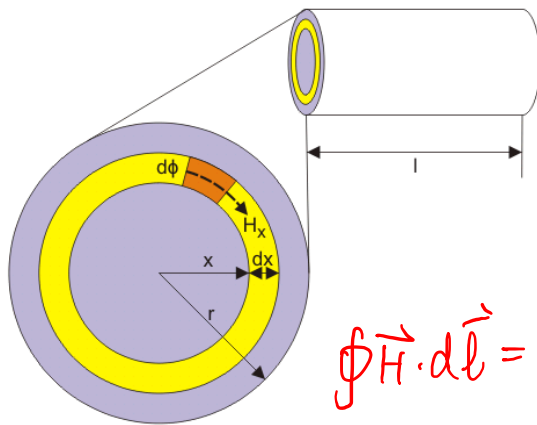
Reference

$$L = \frac{\psi_m}{I} = \frac{2\pi}{\mu_o \ln\left(\frac{R_2}{R_1}\right)} \text{ H/m}$$

$\int \mathbf{B} \cdot d\mathbf{s}$

Internal Inductance

Internal Inductance: due to magnetic flux internal to the wire



$$\oint \vec{H} \cdot d\vec{l} = I_{enc.}$$

$$= J \cdot \frac{r^2}{r_o}$$

☞ Per-unit-length internal inductance at low freq.

$$l_{lf} = l_{i,DC} = \frac{\mu}{8\pi} \text{ H/m } (r \ll \delta) \quad \text{const.}$$

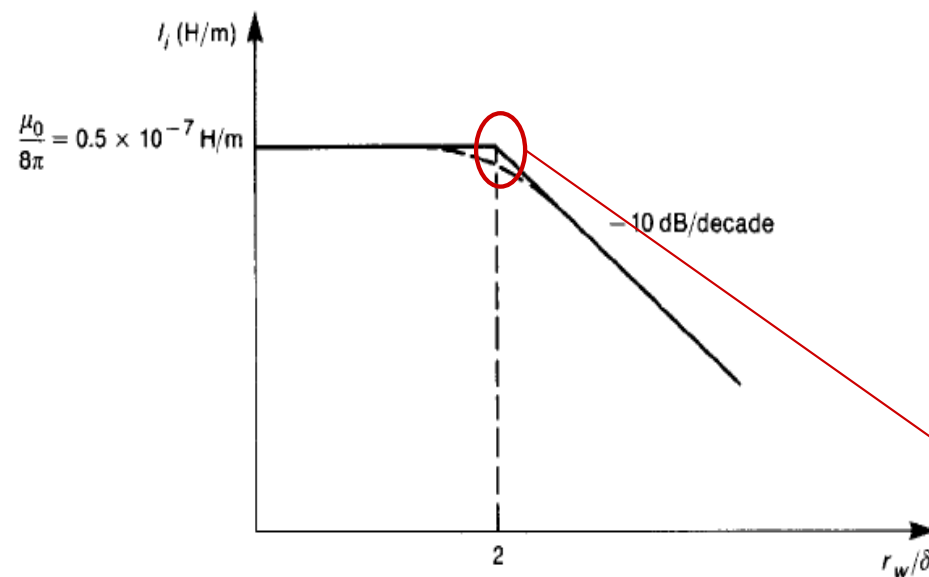
For DC and for low frequencies, internal inductance is a constant

If $\mu_r = 1$, internal inductance $l_{lf} = 50 \text{ nH/m}$

☞ Per-unit-length internal inductance at high freq.

$$l_{hf} = \frac{\mu\delta}{4\pi r} = \frac{1}{4\pi r} \sqrt{\frac{\mu}{\pi\sigma}} \frac{1}{\sqrt{f}} \text{ H/m } (r \gg \delta)$$

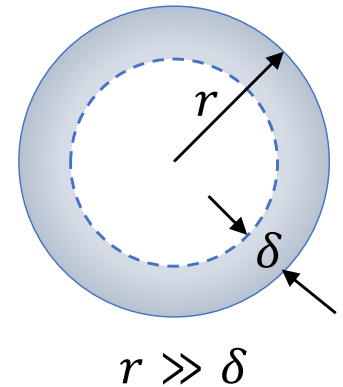
Internal inductance (Low Freq vs. High Freq.)



1. HF inductance decreases at a rate of **10 dB/decade**.

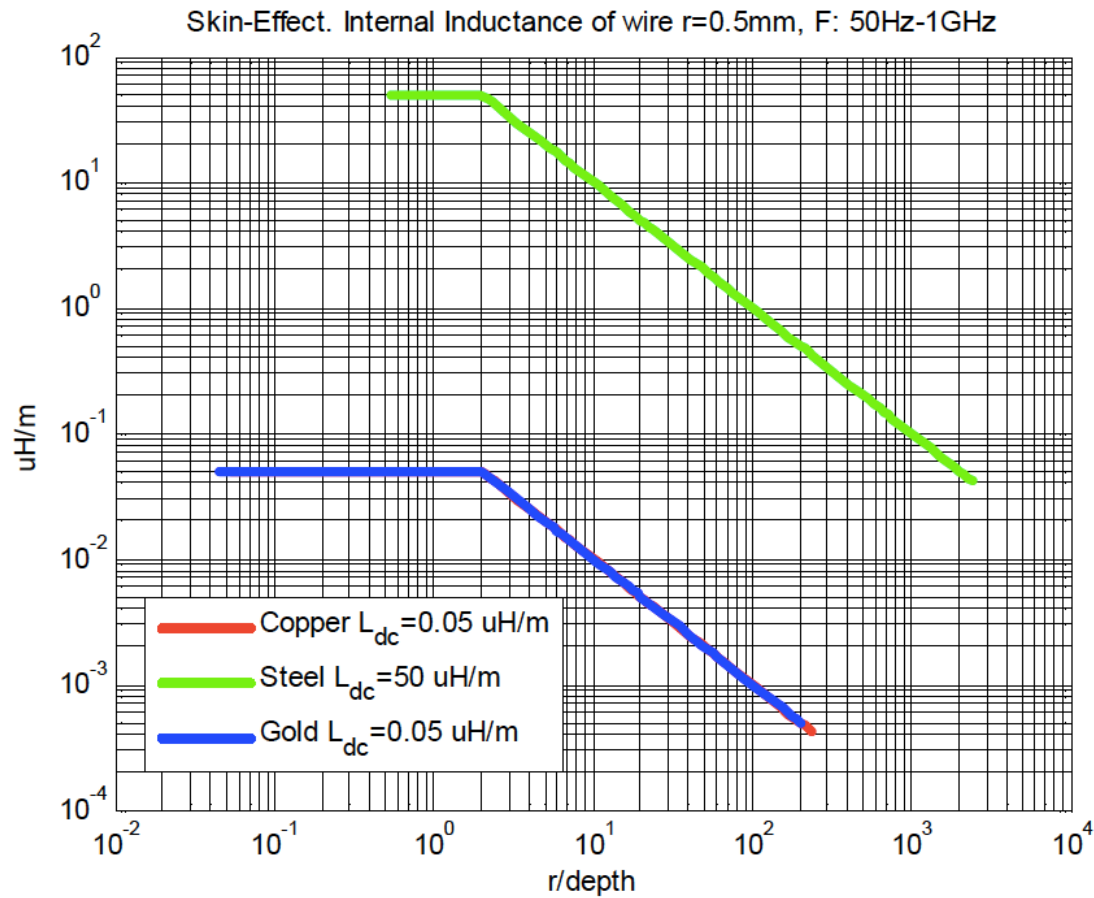
2. The two asymptotes meet at point:

$$r = 2\delta$$



Dependence of the wire per-unit-length internal inductance on frequency

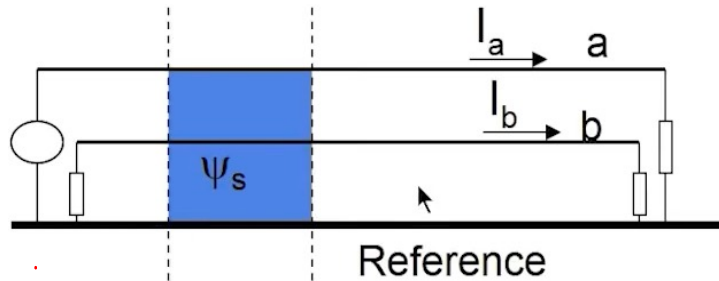
Illustration of the dependence of the per-unit-length Internal Inductance of a wire on frequency (skin effect)



Self and Mutual inductances

Self inductance

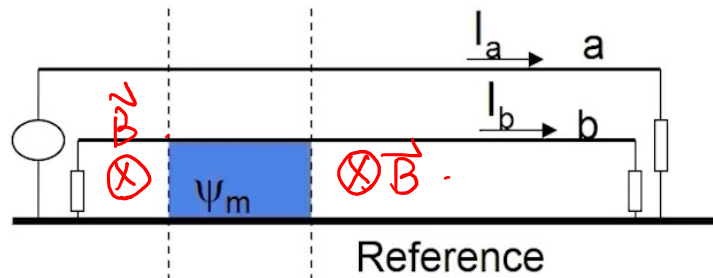
$L_{\text{internal}} + L_{\text{external}}$



$$L_s = \frac{\psi_s}{I_a} \text{ H/m}$$

Self inductance = internal inductance
+ external inductance

Mutual inductance

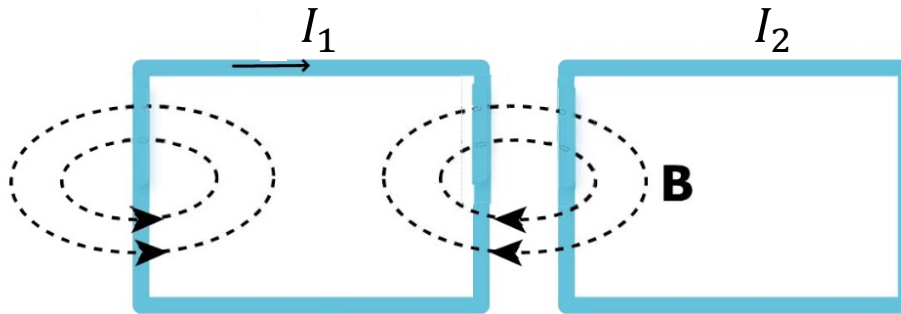


$$L_m = \frac{\psi_m}{I_a} \text{ H/m}$$

How much magnetic flux is induced in the inner loop (loop b and reference) from current I_a ?

The magnetic field and mutual inductance drop off rapidly with increasing the spacing between the two wires.

Mutual Inductance



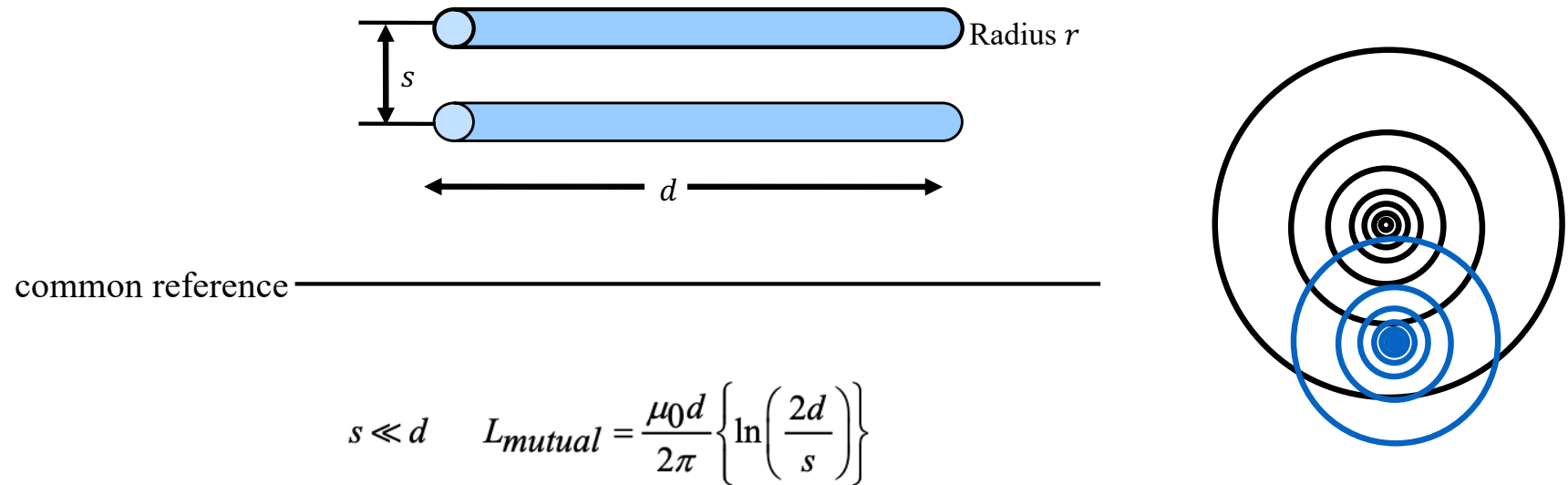
Two current loops with no common reference

$$L_{21} = \frac{\psi_{21}}{I_1} \text{ H}$$

$\int \int_{S_2} \vec{B} \cdot d\vec{S}$

ψ_{21} : flux inside loop 2 due to current I_1

First-order Equation for Mutual Inductance



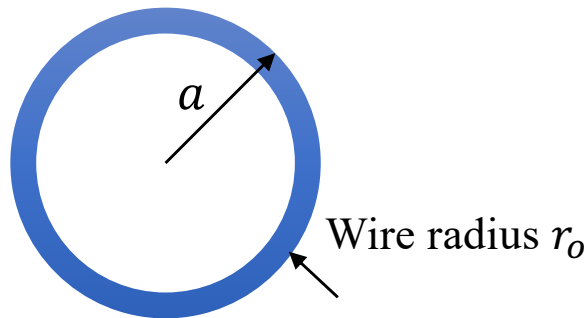
$$s \ll d \quad L_{mutual} = \frac{\mu_0 d}{2\pi} \left\{ \ln \left(\frac{2d}{s} \right) \right\}$$

$$s < d \quad L_{mutual} = \frac{\mu_0 d}{2\pi} \left\{ \ln \left(\frac{2d}{s} \right) - 1 + \frac{s}{d} - \left(\frac{s}{2d} \right)^2 \right\}$$

- Notice the change in mutual inductance with rod separation

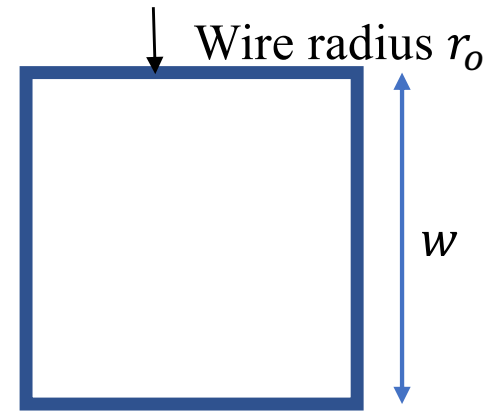
Loop and partial inductances

inductance of a loop



$$L_{loop} \approx \mu_o \mu_r \left[\ln \left(\frac{8a}{r_o} \right) - 2.0 \right] \text{ H}$$

Simple Round Loop ($r_o \ll a$)



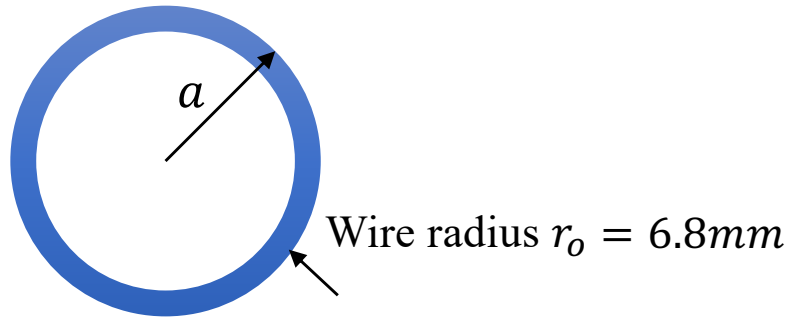
$$L = \frac{2\mu_o w}{\pi} \left(\ln \frac{w}{r_o} - 0.774 \right) \text{ H} \quad (r_o \ll w^2)$$

Simple Square Loop

Loop inductance is directly influenced by loop AREA and less influenced by conductor size!

Loop and partial inductances

Poll: inductance of a loop



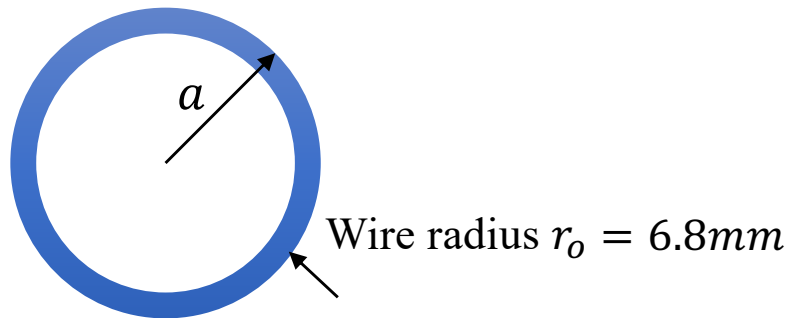
$$L_{loop} \approx a\mu_o\mu_r \left[\ln \left(\frac{8a}{r_o} \right) - 2.0 \right] \text{ H}$$

The inductance of a 5-cm radius loop is

- a) about 130 nH
- b) about 13 uH
- c) undefined

Loop and partial inductances

Poll: inductance of a loop



The inductance of a 5-cm radius loop is

- a) about 130 nH
- b) about 13 uH
- c) undefined

$$L_{loop} \approx a\mu_o\mu_r \left[\ln\left(\frac{8a}{r_o}\right) - 2.0 \right] \text{ H}$$

$$L_{loop} \approx 0.05 \times 4\pi \times 10^{-7} \left[\ln\left(\frac{8 \times 0.05}{0.0068}\right) - 2.0 \right] \approx 130 \text{ nH}$$

Low/high inductance decoupling capacitor connection

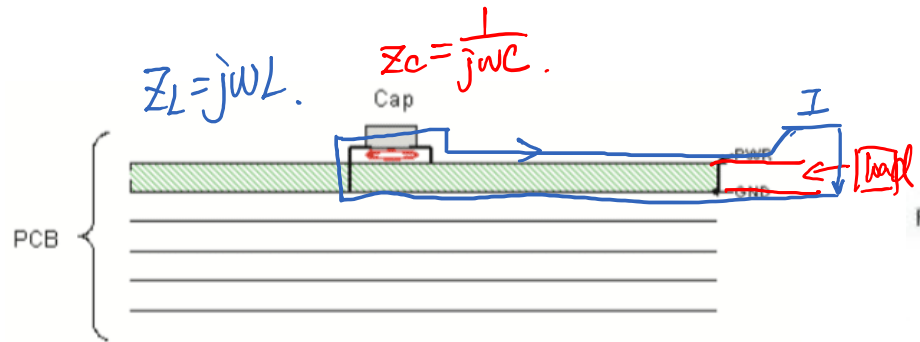


Fig 1. Low inductance connection with capacitor mounted on top of board.

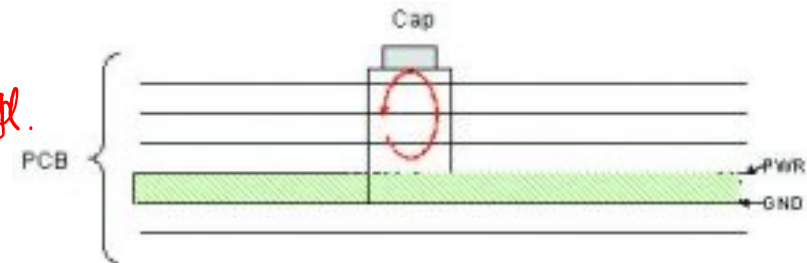


Fig 2. High inductance connection with capacitor mounted on top of board.

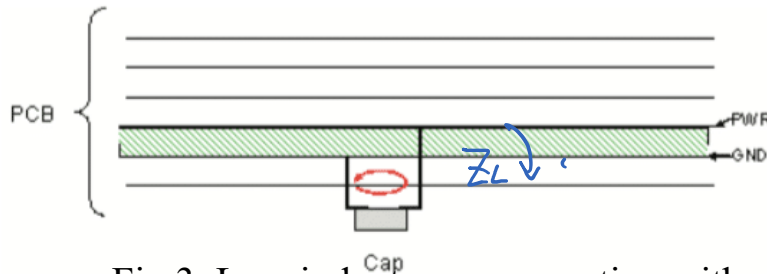


Fig 3. Low inductance connection with capacitor mounted on bottom of board.

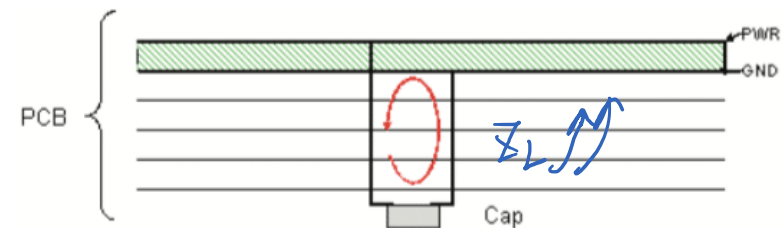
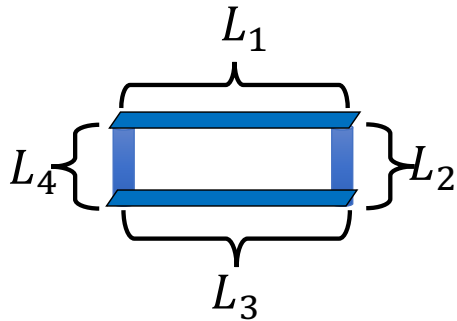


Fig 4. High inductance connection with capacitor mounted on top of board.

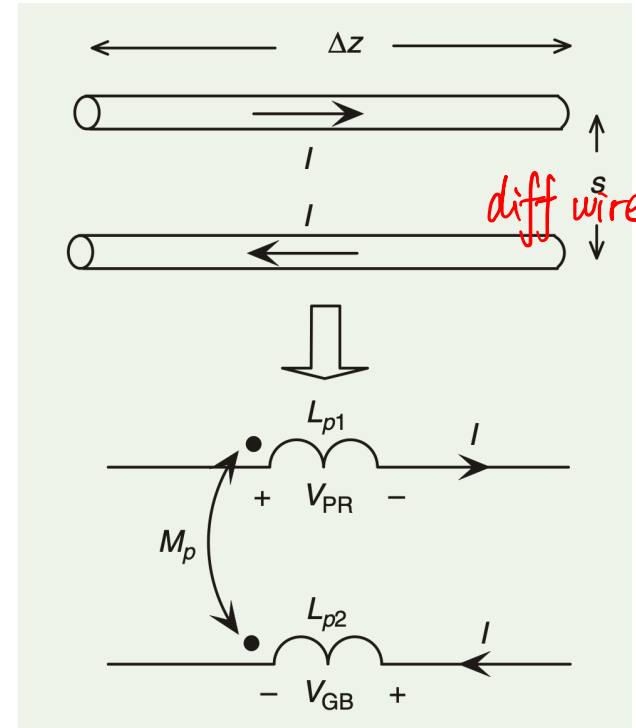
Partial inductances

Use partial inductance to find the total inductance



In analogy to

$$L_{total} = \underbrace{L_{11} + L_{22} + L_{33} + L_{44}}_{\text{self } L.} - \underbrace{2L_{13} - 2L_{24}}_{\text{mutual } L.}$$



diff wire
M J J.

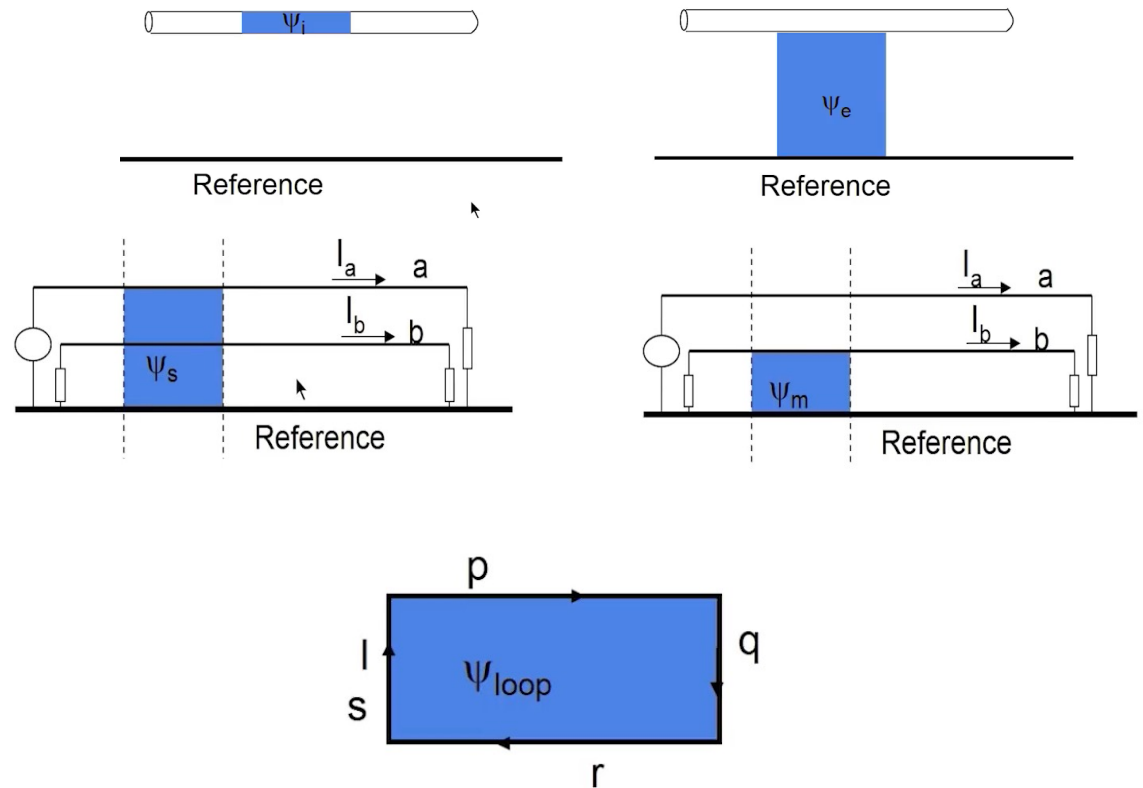
$$V_{PR} = L_{p1} \frac{dI}{dt} - M_p \frac{dI}{dt} = (L_{p1} - M_p) \frac{dI}{dt}$$

$$V_{GB} = L_{p2} \frac{dI}{dt} - M_p \frac{dI}{dt} = (L_{p2} - M_p) \frac{dI}{dt}$$

Inductance

- Internal inductance
- External inductance
- Self inductance
- Mutual inductance
- Loop inductance
- Partial inductance

Current loop $\rightarrow$ inductance!



How to reduce inductance?

- Reducing loop area is most importance
 - Run the wire close to ground plane or return wire
- Reducing the length of wire
- Use large surface area of planes

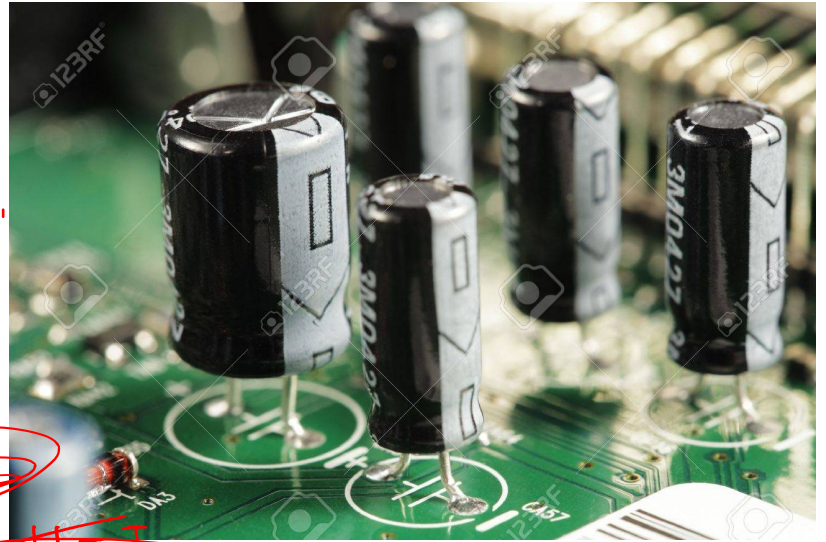
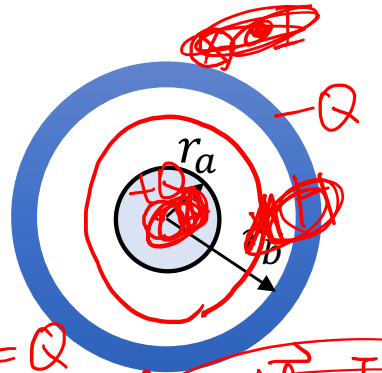
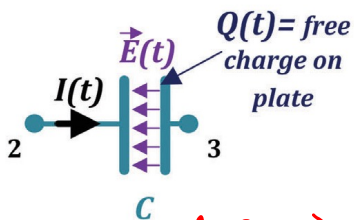
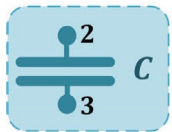
Eg. for round conductors, diameter increase from 1mm to 10mm, but L decreases from $1.5 \mu H/m$ to $1.0 \mu H/m$.

Rule of thumb: Inductance of a conductor approximately $1 \mu H/m \sim 1.5 \mu H/m$
($1 nH/mm \sim 1.5 nH/mm$)

1 meter earth wire has about 126Ω impedance at 20 MHz compared with less than 1 ohm at 60 Hz.

Inductance is everywhere!!!

Capacitance $C = \frac{Q}{V}$ farads



Handwritten red notes:

$$\oint \vec{E} \cdot d\vec{s} = Q$$

$$E = \frac{Q}{2\pi\epsilon_0 r}$$

$$\oint \vec{E} \cdot d\vec{l} = V$$

Capacitance is the ability of a structure to hold charge (electrons) for a given voltage

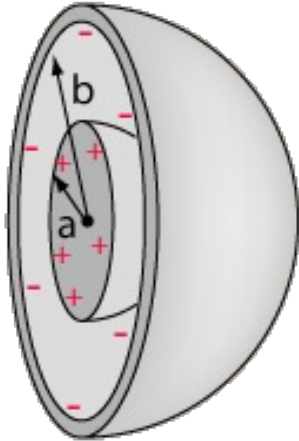
Consider a capacitor as a *bucket* holding lots of electrons!

For coaxial cables, per unit length $C = \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)}$

Handwritten red note:

$$V = -\int \vec{E} \cdot d\vec{l}$$

3 Capacitance



Spherical Capacitor

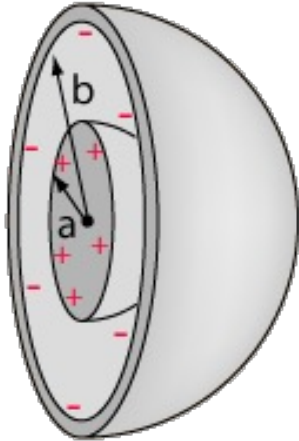
$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad a < r < b$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$$

$$C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0}{\left[\frac{1}{a} - \frac{1}{b} \right]}$$

Q: Does an isolated charged sphere have capacitance?

3 Capacitance



Spherical Capacitor

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad a < r < b$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$$

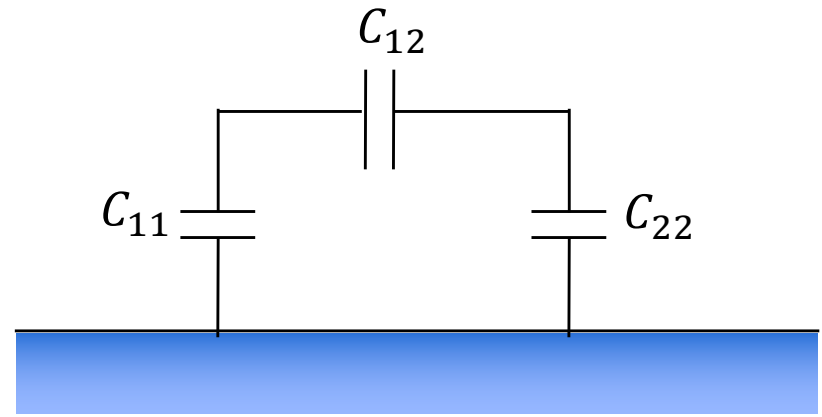
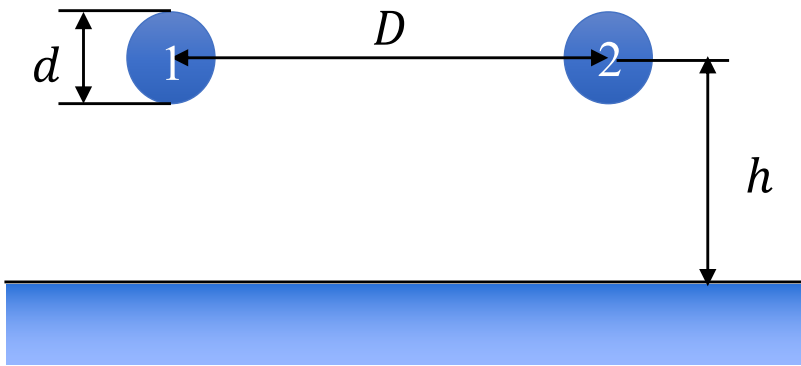
$$C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0}{\left[\frac{1}{a} - \frac{1}{b} \right]}$$

Q: Does an isolated charged sphere have capacitance?

Self capacitance

$$a \rightarrow R \quad \text{and} \quad b \rightarrow \infty \quad C = 4\pi\epsilon_0 R$$

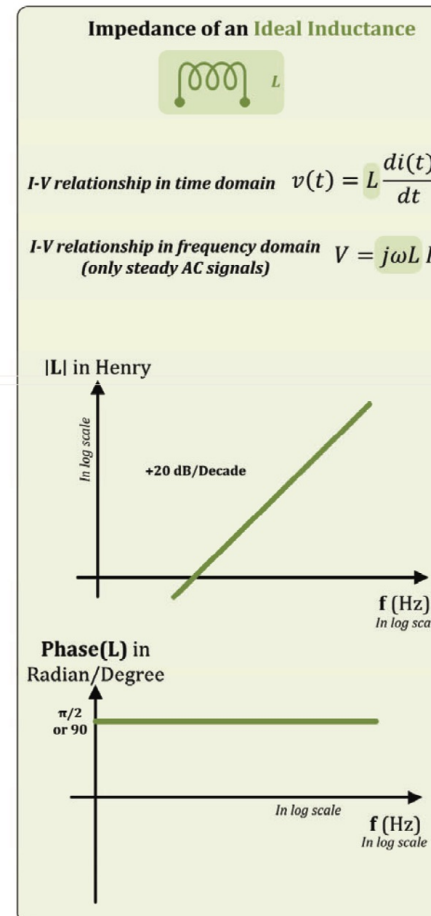
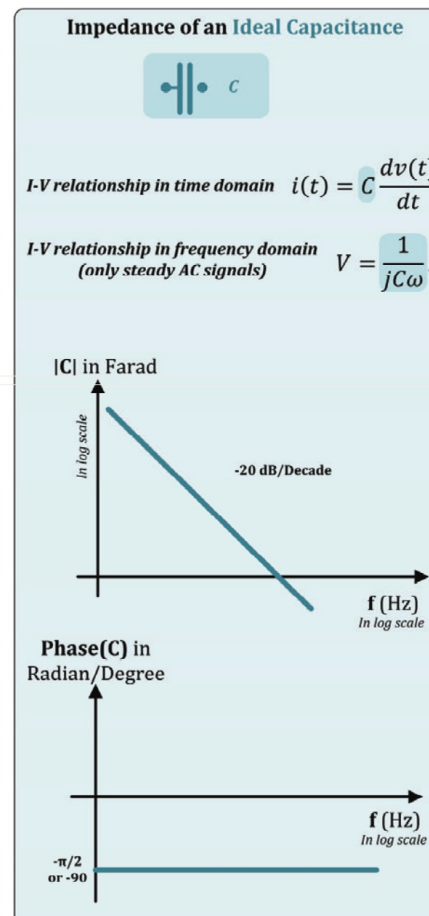
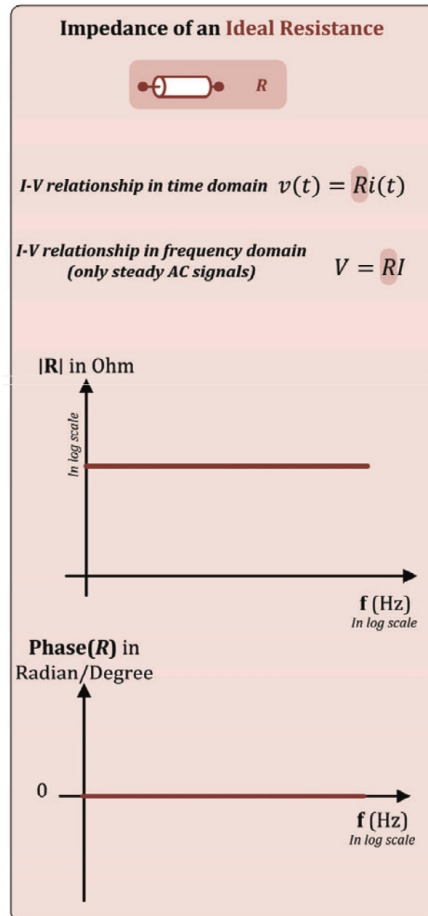
Self and mutual capacitance



d=1 mm, h=infinity		d=1 mm, h=2 mm	
D=5 mm	$C_{12}=12$ pF/m	D=5 mm	$C_{12}=5$ pF/m
D=10 mm	$C_{12}=10.3$ pF/m	D=10 mm	$C_{12}=2$ pF/m
D=20 mm	$C_{12}=7.5$ pF/m	D=20 mm	$C_{12}=0.5$ pF/m

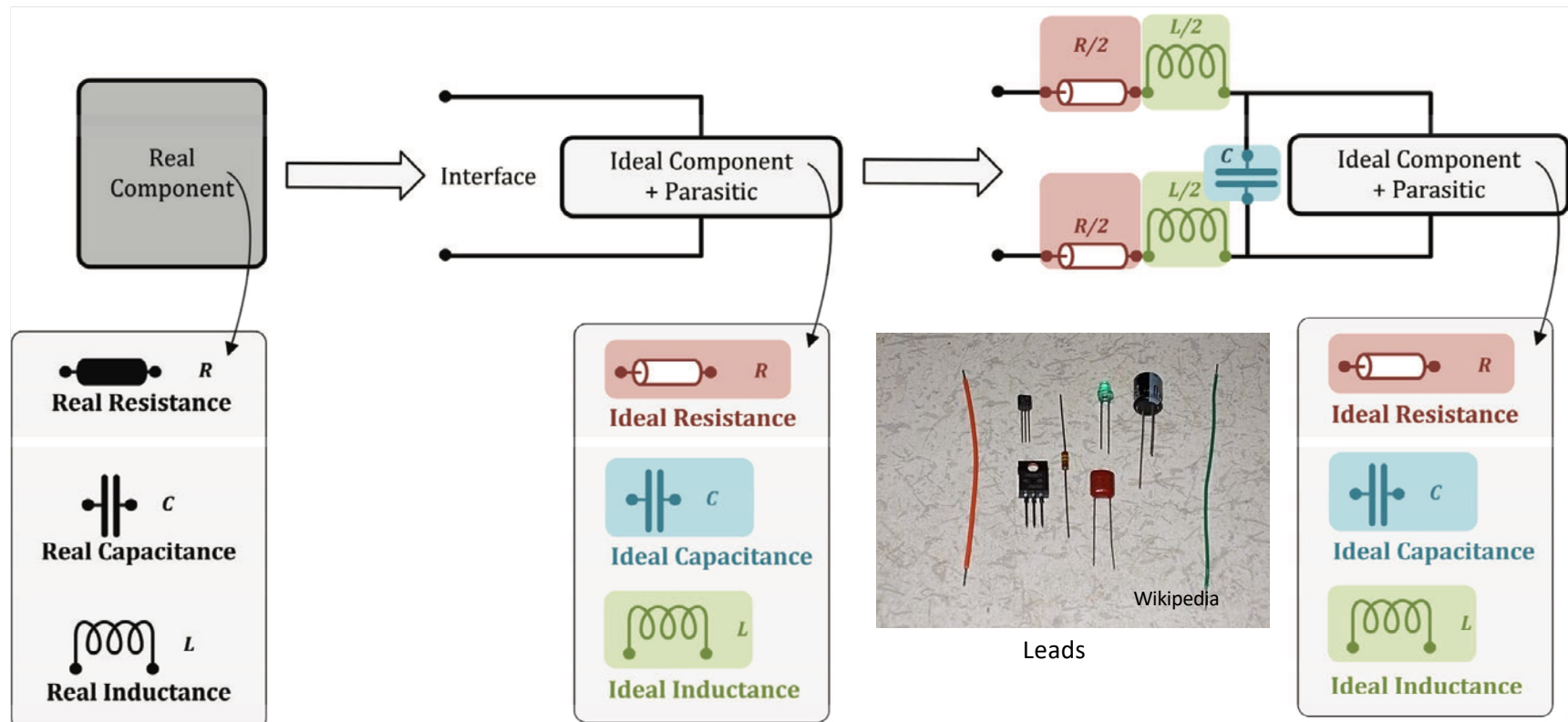
Coupling capacitance C_{12} is reduced with the proximity of reference plane

Ideal Components recap

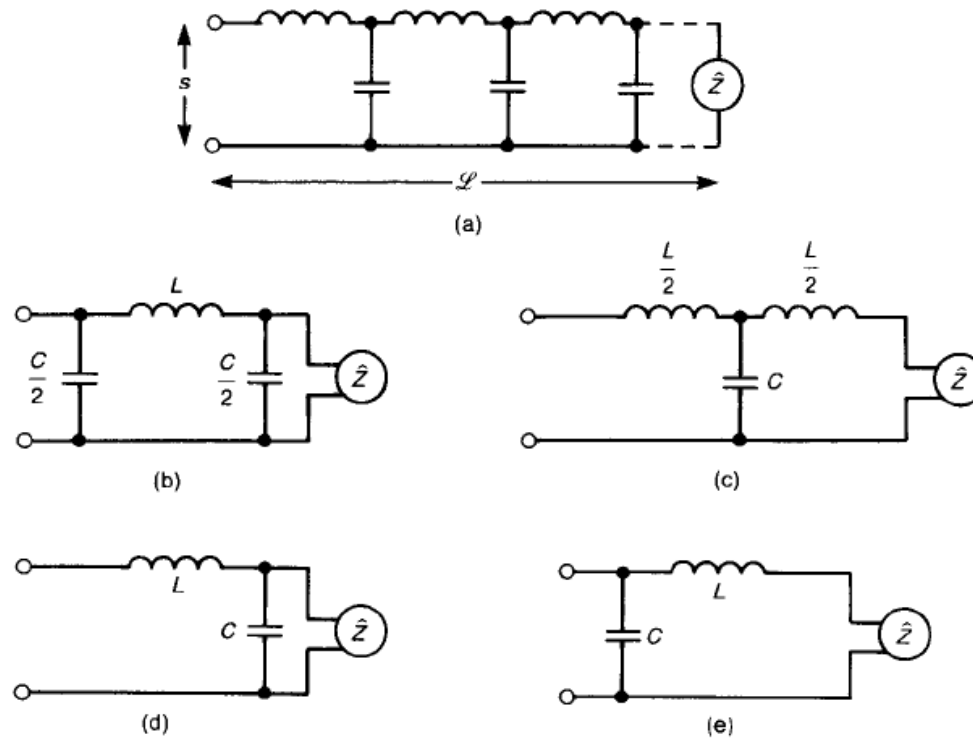


- We have three circuit elements: R, L and C.
- These are IDEAL components: they are mathematical “perfect things”
- We will use them as building blocks to describe REAL components

5.2 Effect of Component Leads



Effect of Component Leads

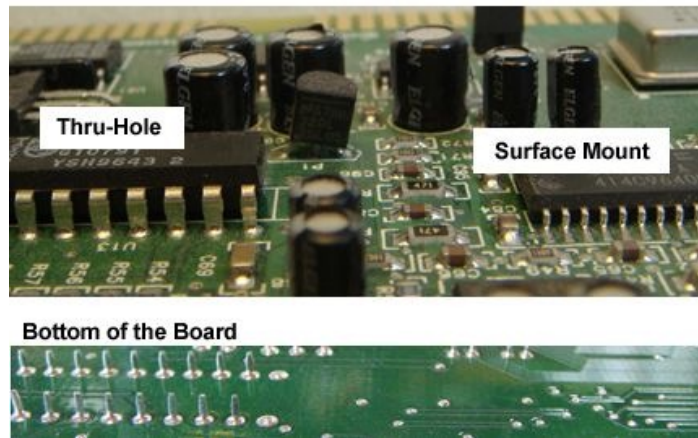


Equivalent circuits of component leads

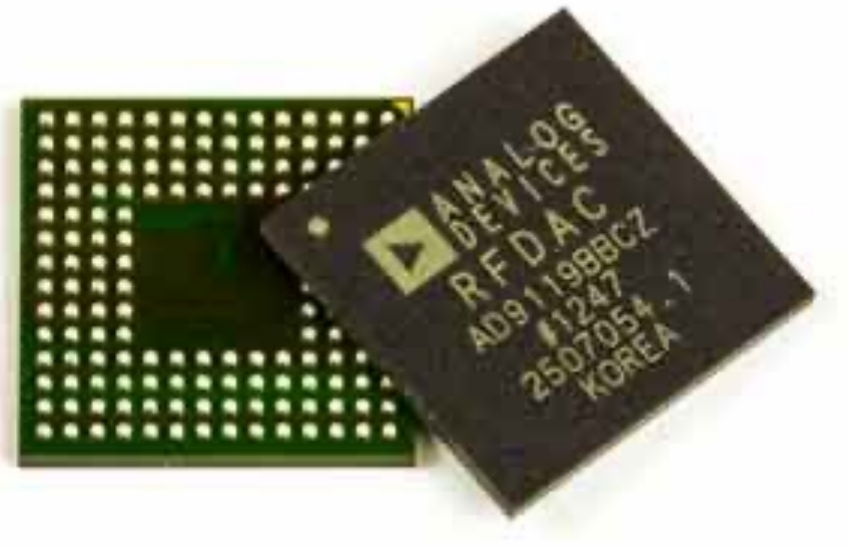
Effect of Component Leads

SMT: surface-mount technology

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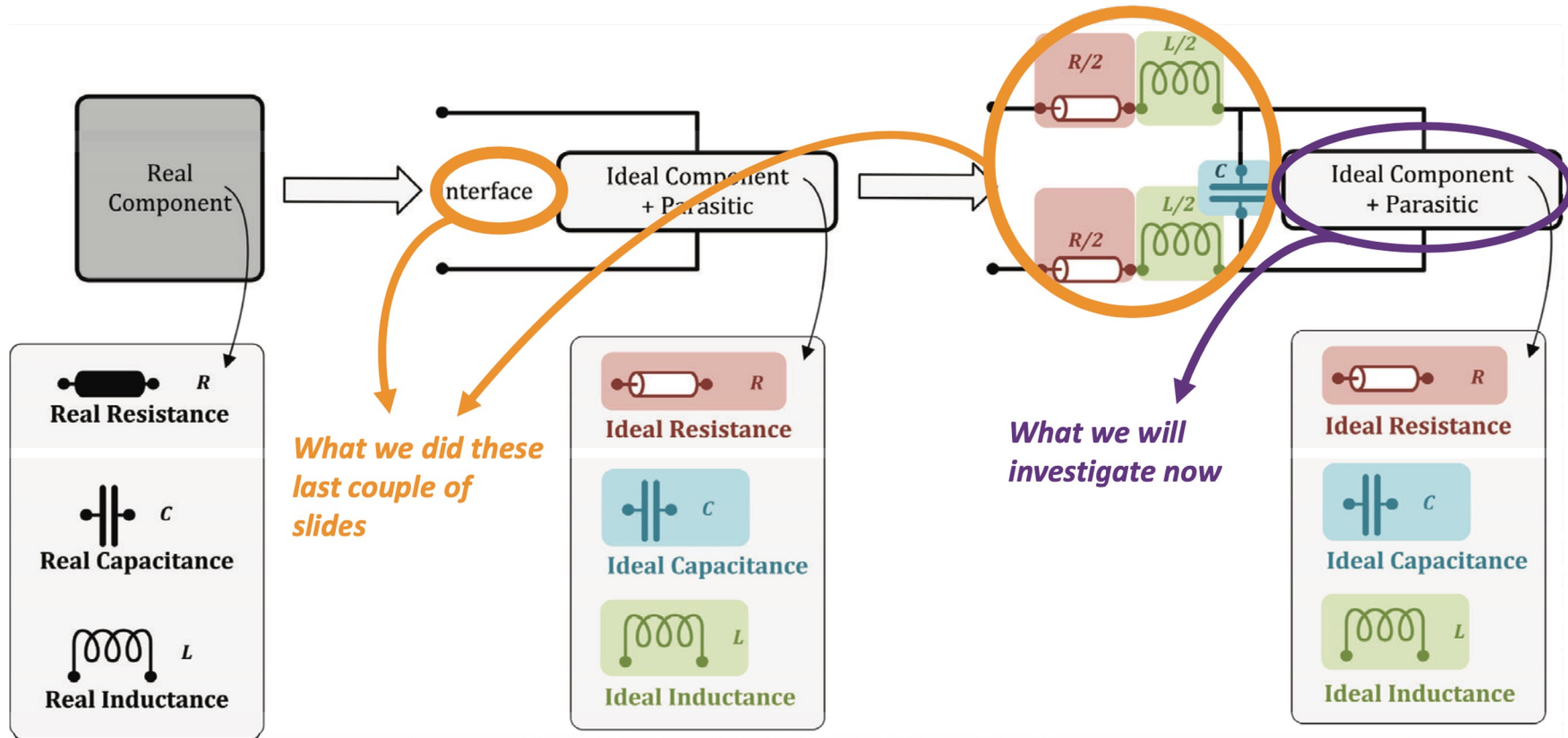


Surface mount vs. Thru-Hole

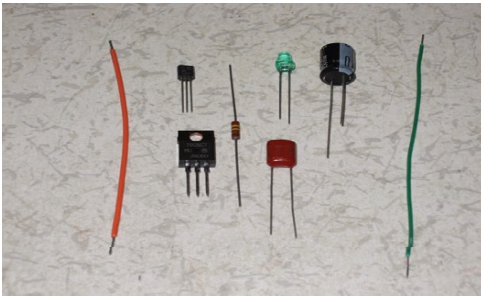


Top and bottom view of a BGA (Ball grid array)

5.3 Nonideal R, L, and C Components

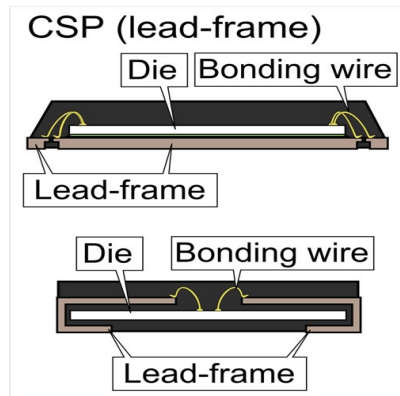


5.3.1 Nonideal Resistors



Several kinds of lead wires

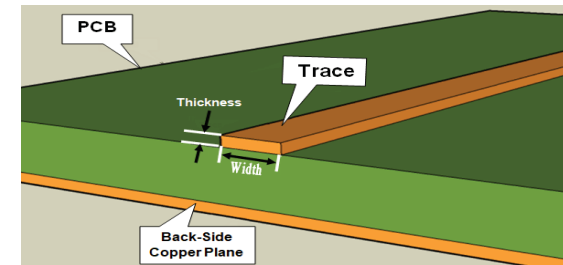
- A lead wire is a metal wire connected from the electric pole of an electronics part or an electronic component.
- The lead wire is a coated copper wire, a tinned copper wire or another electrically conductive wire used to connect two locations electrically.



Lead frame of a surface-mount technology IC package



Cables and multi strand wires

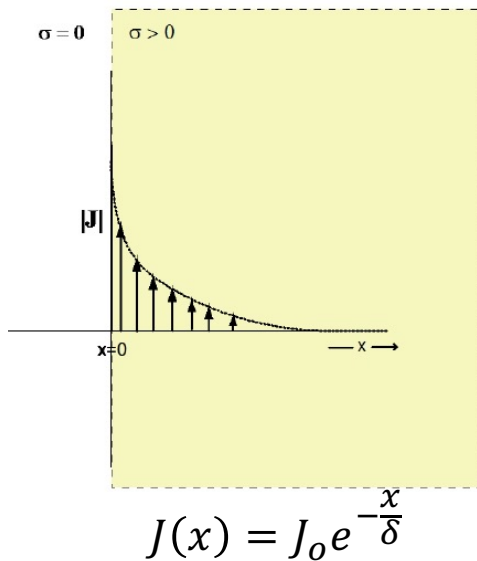


PCB trace

*Ref: pictures: Wikipedia article "Lead (Electronic)" June 2022 -
https://en.wikipedia.org/wiki/Lead_%28electronic%29
** Ref: PCB design world
<https://pcbdesignworld.com/article/pcb-track-width-and-track-resistance-importance-calculation-and-design-tips>

Skin Depth

High-frequency electric fields and currents decay exponentially with distance from the surface of a good conductor.



$$I = \int_0^d J(x) dx = \int_0^d J_0 e^{-\frac{x}{\delta}} dx = J_0 \delta (1 - e^{-\frac{d}{\delta}})$$

Skin depth δ

The skin depth δ is the distance over which the amplitude, $E_0 e^{-\alpha x}$ of the wave decreases by $1/e$ ($\sim 37\%$).

$$\delta = 1/\alpha$$

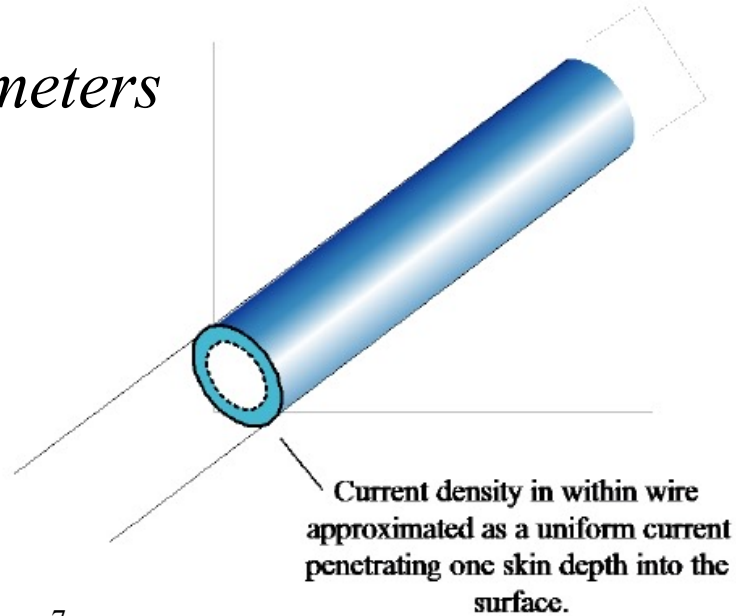
α is the attenuation constant.

For good conductors ($\frac{\sigma}{\omega\epsilon} \gg 1$),

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

Skin Depth

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \text{ meters}$$



copper $\sigma = 5.8 \times 10^7 \text{ S/m}$

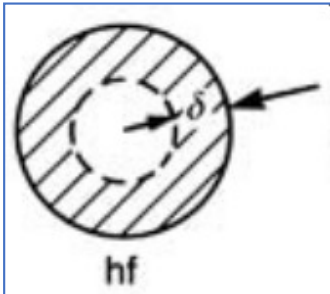
f	δ	f	δ
50Hz	9.3mm	1MHz	2.6mils
1kHz	2.09 mm	10MHz	0.82 mils
10kHz	0.66 mm	100MHz	0.26 mils
100kHz	0.21 mm	1GHz	0.0823 mils

Wire resistance

Per-unit-length resistance at low freq.

$$r_{lf} = r_{DC} = \frac{1}{\pi \sigma r_w^2} \quad (r_w \ll \delta)$$

Per-unit-length resistance at high freq.



Influence of skin effect

$$r_{hf} = \frac{1}{\sigma [\pi r_w^2 - \pi (r_w - \delta)^2]}$$

Reduced cross-section area

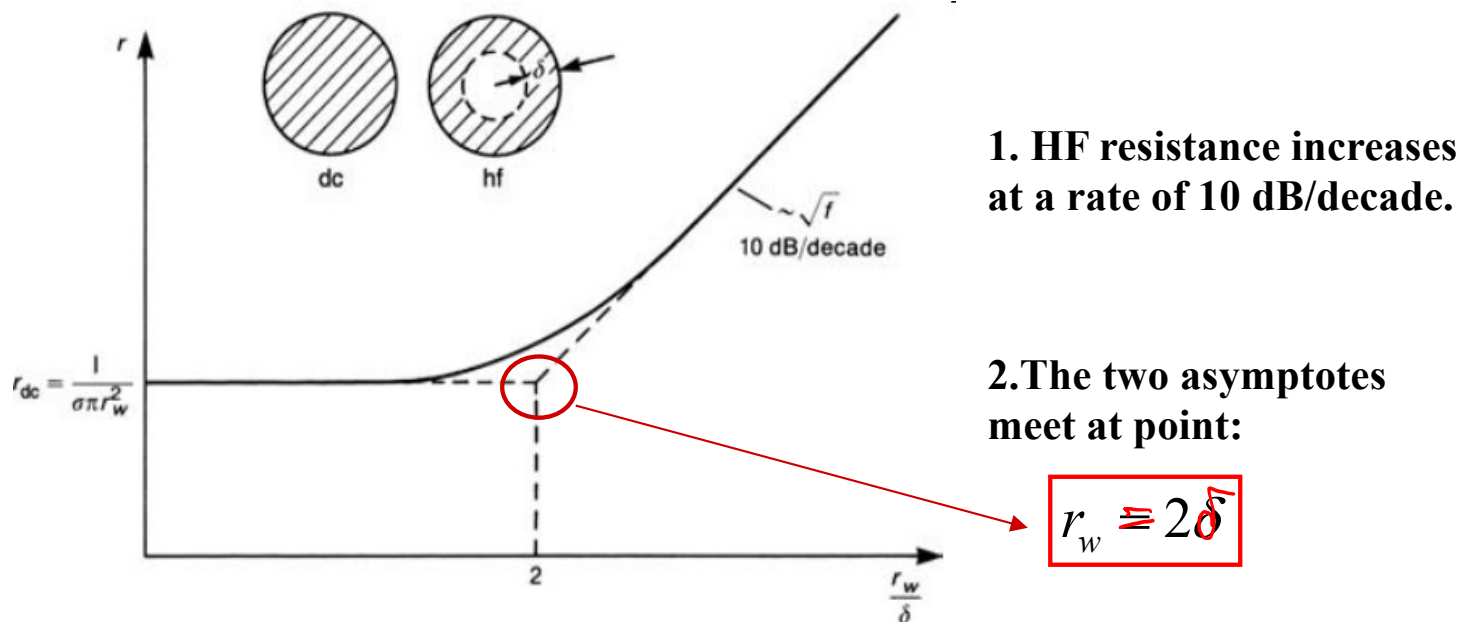
$$= \frac{1}{\sigma (2\pi r_w \delta - \pi \delta^2)}$$

$$\frac{r_{hf}}{r_{lf}} = \frac{r_w}{2\delta}$$

Increased resistance at high freq.

$$\approx \frac{1}{\sigma 2\pi r_w \delta} = \frac{r_w}{2\delta} r_{DC} = \frac{1}{2r_w} \sqrt{\frac{\mu_0}{\pi \sigma}} \sqrt{f} \quad (r_w \gg \delta)$$

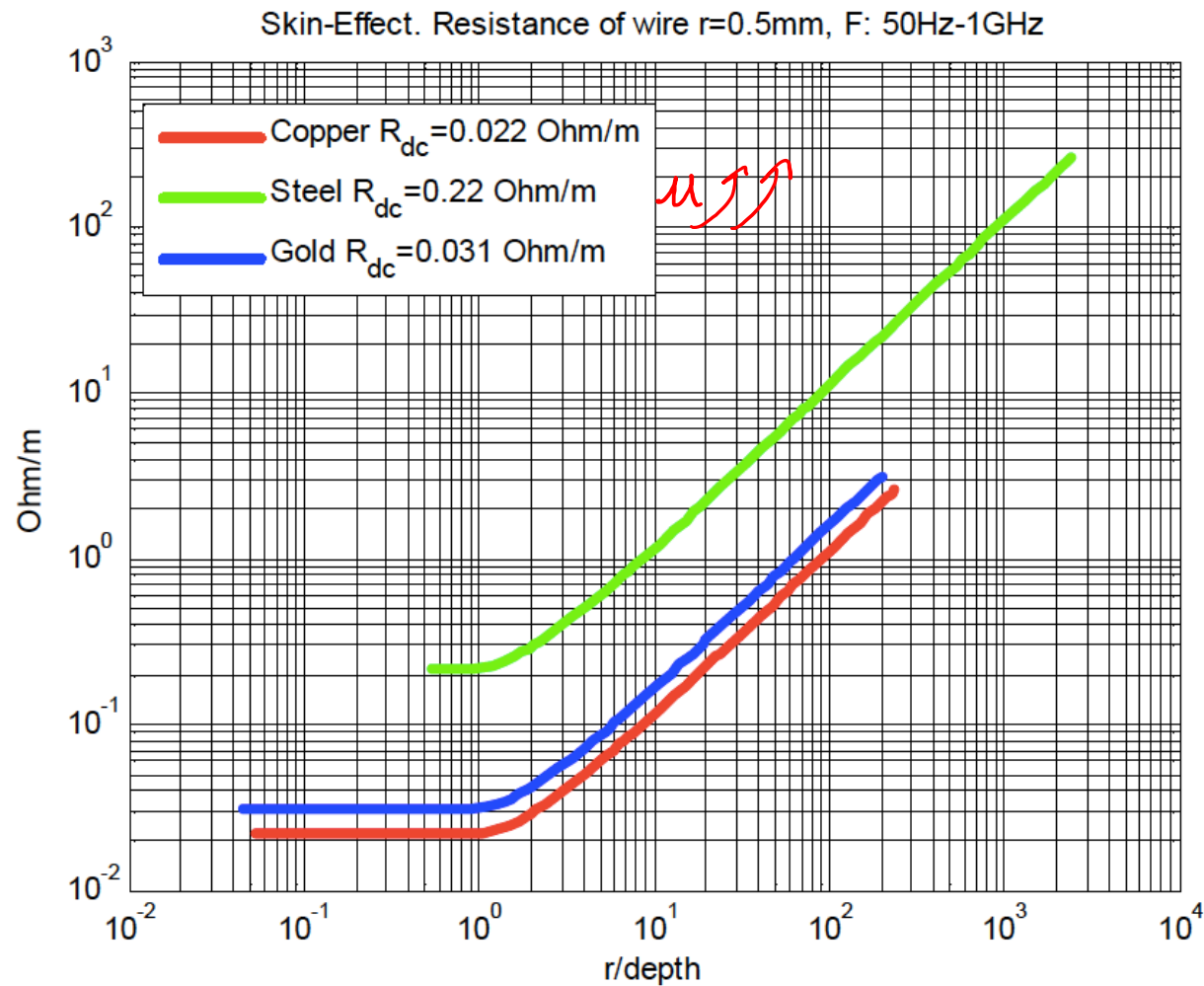
Wire resistance asymptotic dB plot



Dependence of the wire per-unit-length resistance on frequency

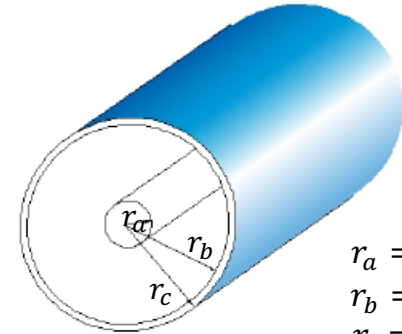
5.3 Nonideal Components — Nonideal Resistors

Illustration of the dependence of the per-unit-length resistance of a wire on frequency (skin effect)



Resistance per unit length of a coaxial cable

At 60 Hz



$$\begin{aligned} r_a &= 0.5 \text{ mm} \\ r_b &= 4.9 \text{ mm} \\ r_c &= 5.0 \text{ mm} \end{aligned}$$

$$\delta_{60\text{Hz}} = \frac{1}{\sqrt{\pi f \mu_o \sigma}} = \frac{1}{\sqrt{\pi(60)(4\pi \times 10^{-7})(5.8 \times 10^7)}} = 8.53 \text{ mm}$$

$$R_{\text{inner conductor}} = \frac{1}{\sigma A} = \frac{1}{(5.8 \times 10^7) \pi (0.5 \times 10^{-3})^2} = 22.0 \text{ m}\Omega/\text{m}$$

$$R_{\text{outer conductor}} \approx \frac{1}{\sigma A} = \frac{1}{(5.8 \times 10^7) 2\pi (5 \times 10^{-3})(0.1 \times 10^{-3})} = 5.5 \text{ m}\Omega/\text{m}$$

$$R_{\text{total}} = 22.0 + 5.5 = 27.5 \text{ m}\Omega/\text{m}$$

Resistance per unit length of a cable

At 1 GHz

$$\delta_{1GHz} = \frac{1}{\sqrt{\pi f \mu_o \sigma}} = \frac{1}{\sqrt{\pi(10^9)(4\pi \times 10^{-7})(5.8 \times 10^7)}} = 0.0021 \text{ mm}$$

$$R_{inner \text{ conductor}} = \frac{1}{\sigma A} = \frac{1}{(5.8 \times 10^7)2\pi(0.5 \times 10^{-3})(2.1 \times 10^{-6})} \approx 2.61 \text{ } \Omega/m$$

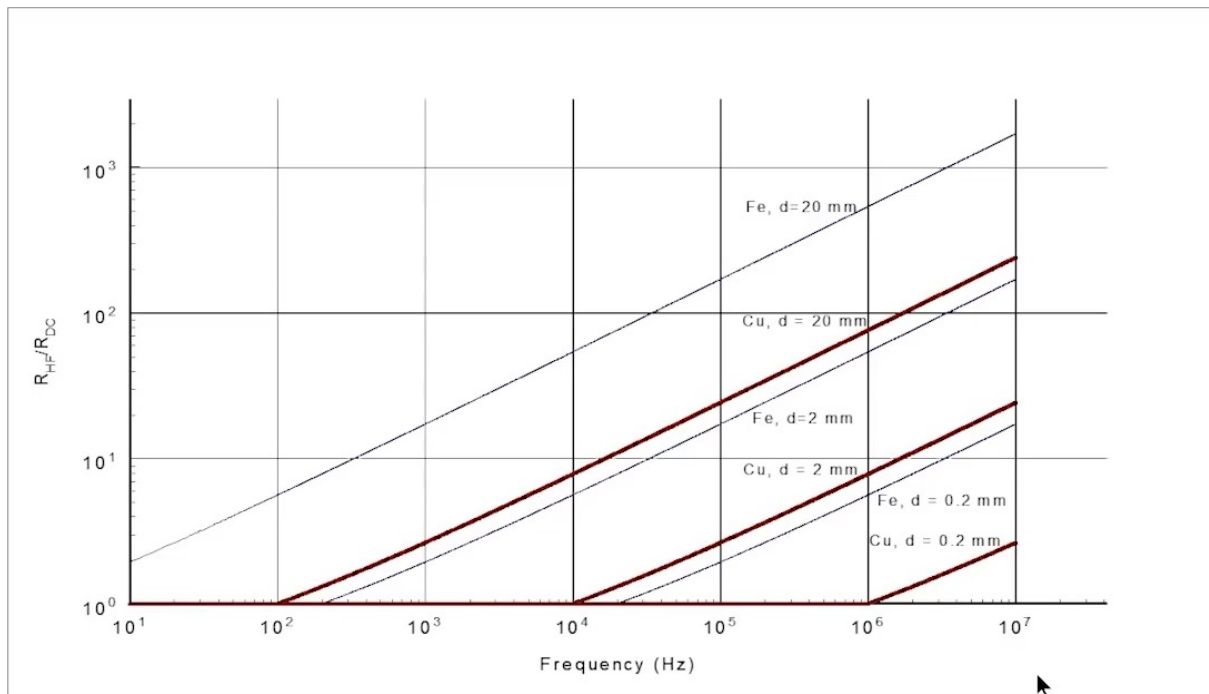
$$R_{outer \text{ conductor}} \approx \frac{1}{\sigma A} = \frac{1}{(5.8 \times 10^7)2\pi(5 \times 10^{-3})(2.1 \times 10^{-6})} \approx 0.261 \Omega/m$$

$$R_{total} = 2.61 + 0.261 = 2.871 \text{ } \Omega/m$$



Wire resistance

$$\frac{r_w}{2\sigma} = r_{hf}/r_{DC} \text{ for copper and iron wires}$$

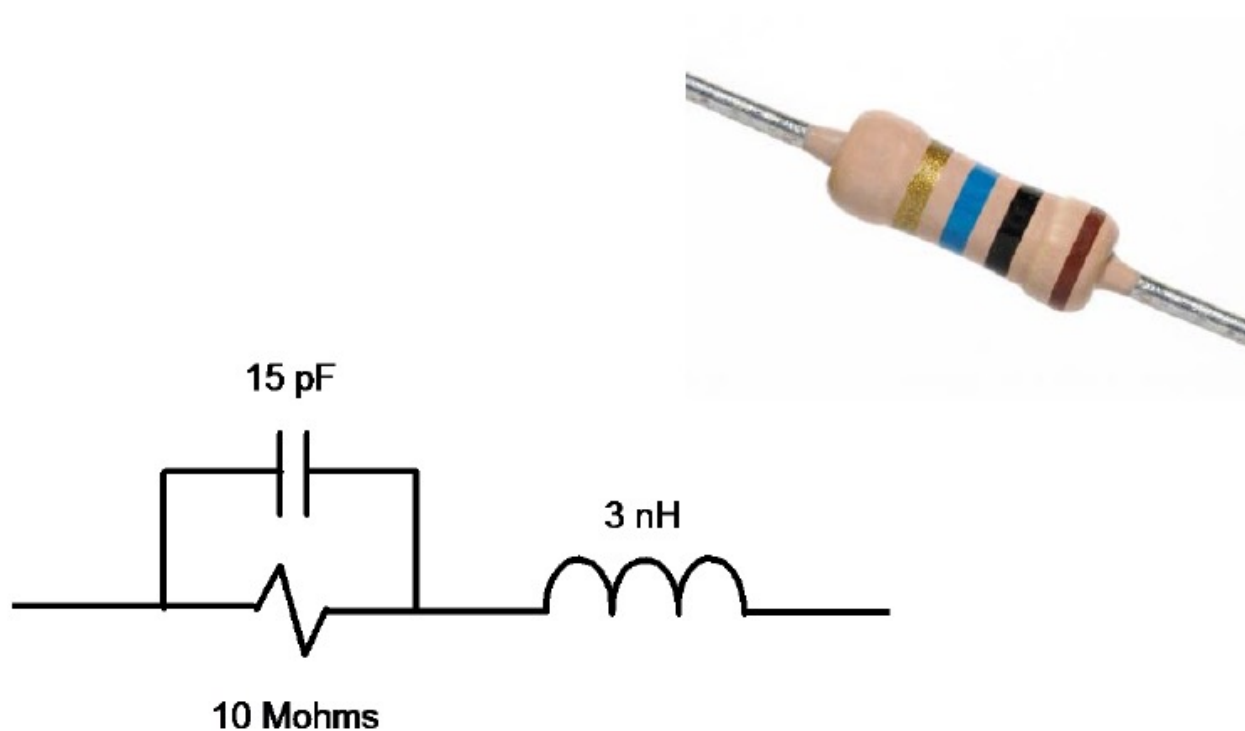


Stranded wire

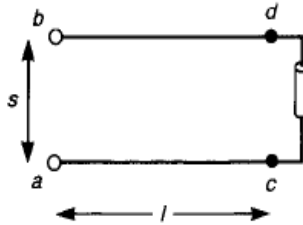


Solid wire

Do Resistors Have Capacitance and Inductance?

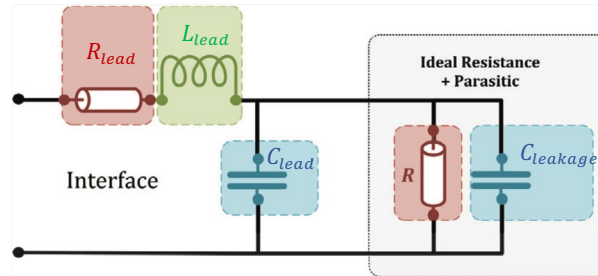


The nonideal resistor including the effects of the leads

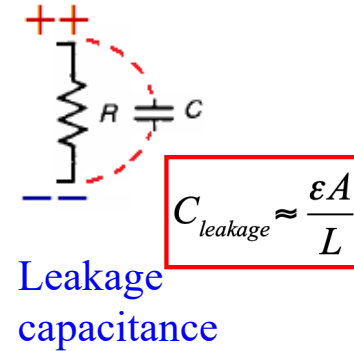


Typically

$$R_{lead} \ll R$$

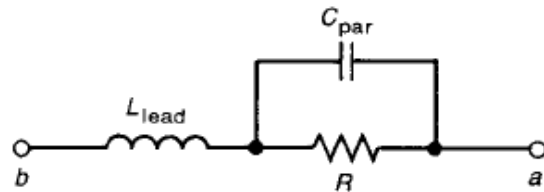
$$C_{lead} \ll C_{leakage}$$


Lumped backward Γ
model of leads



Parasitic capacitance

$$C_{par} = C_{lead} + C_{leakage}$$



Equivalent circuit

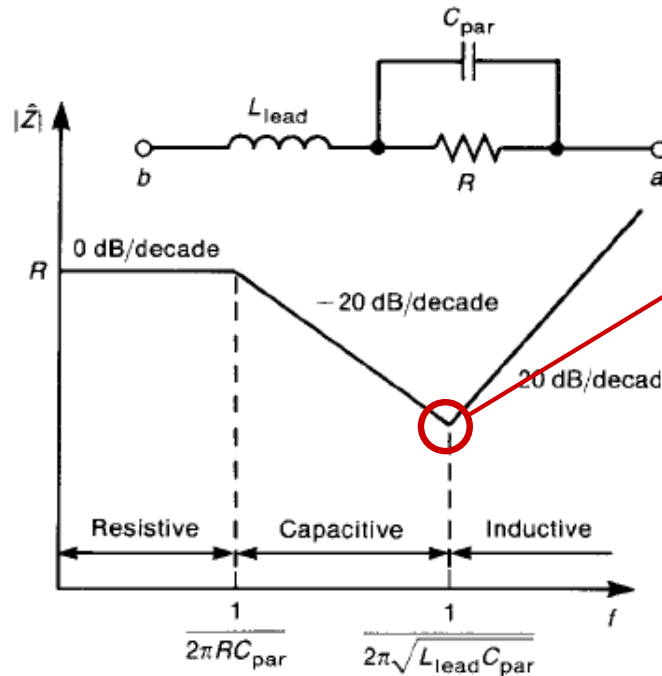
Total Impedance:

$$\hat{Z}(j\omega) = L_{lead} \frac{1/L_{lead} C_{par} - \omega^2 + j\omega/RC_{par}}{j\omega + 1/RC_{par}}$$

Typical values are $C_{par} = 1 \sim 2$ pF. $L_{lead} = 14$ nH

lead lengths of 0.5 in., lead separation of 0.25 in., and lead wires (20-gauge with $r_w = 16$ mils)

The nonideal resistor including the effects of the leads

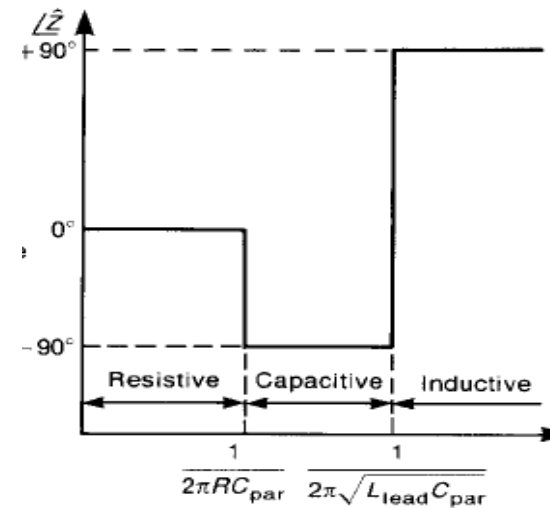


Resonate freq.

$$f = \frac{1}{2\pi \sqrt{L_{lead} C_{par}}}$$

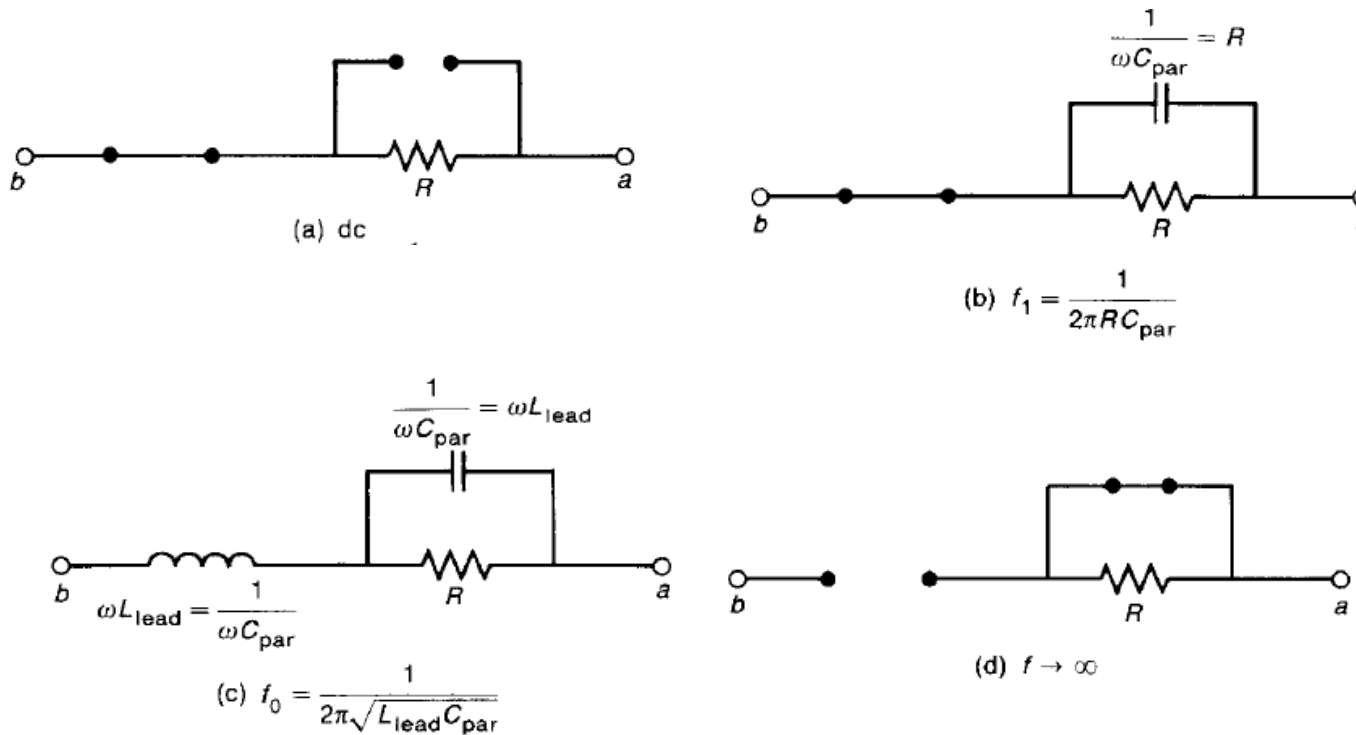
Magnitude bode plot of the impedance variation with frequency

$$\hat{Z}(j\omega) = L_{lead} \frac{1/L_{lead} C_{par} - \omega^2 + j\omega/RC_{par}}{j\omega + 1/RC_{par}}$$



Phase bode plot of the impedance variation with frequency

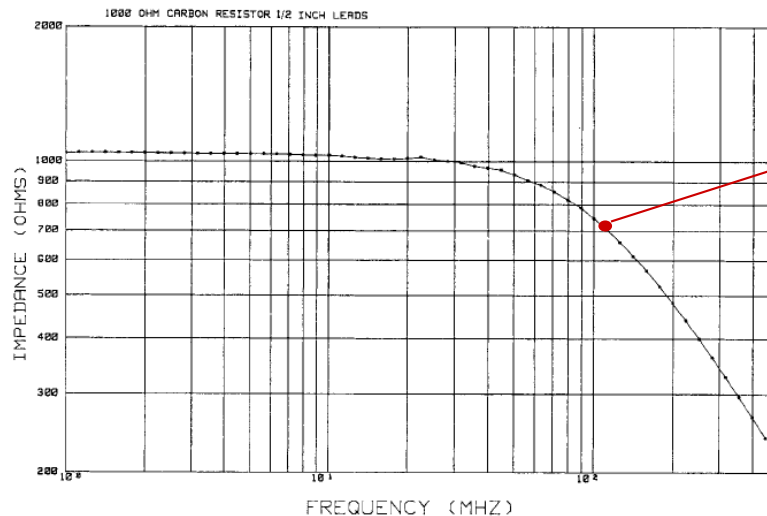
Simplification of the equivalent circuit of a resistor



At various frequencies

Nonideal resistor: Example

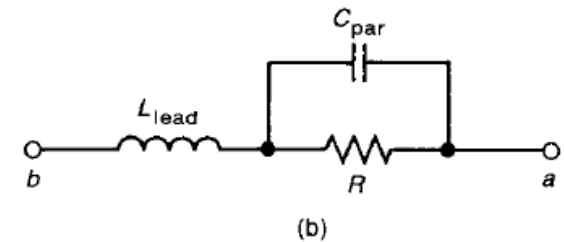
Example: Measure the impedance of a 1k Ω , 1/8W carbon resistor having 0.5in (1 in = 2.54cm) lead lengths, and 0.25in lead separation over a frequency range of 1–500 MHz.



$$f_1 = \frac{1}{2\pi RC_{par}} \approx 120 \text{ MHz}$$

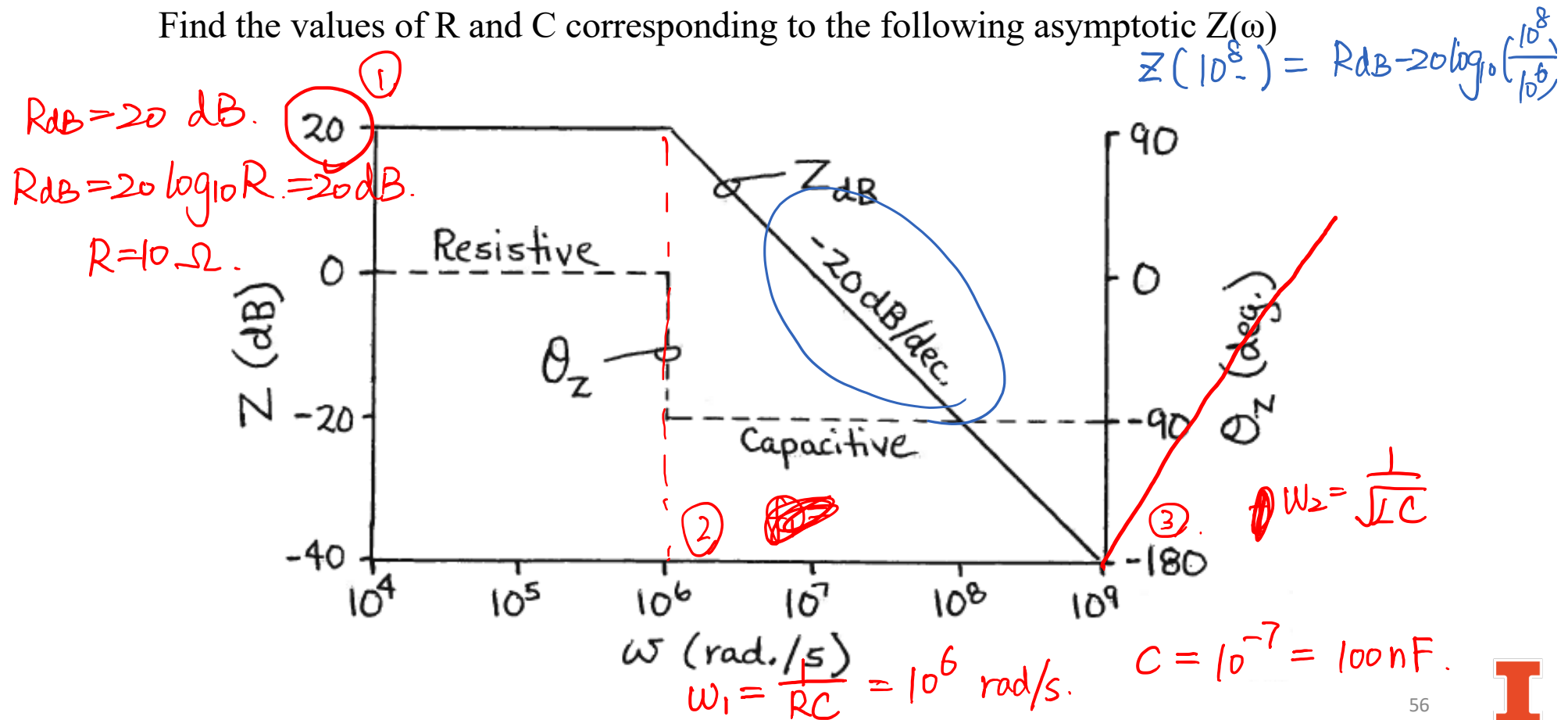
$R = 1.05 \text{ k}\Omega$, $C_{par} = 1.2 \text{ pF}$, and $L_{lead} = 14 \text{ nH}$.

$$f_2 = \frac{1}{2\pi\sqrt{L_{lead}C_{par}}} \approx 1.23 \text{ GHz}$$



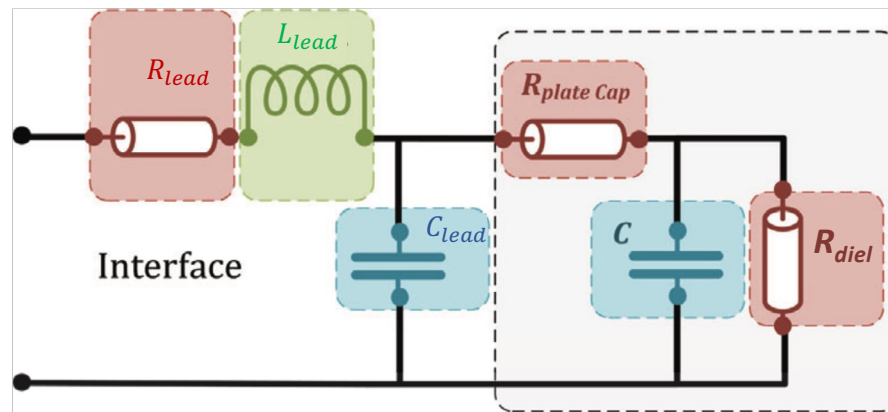
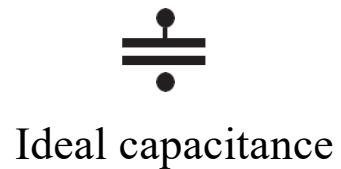
Quiz: RC Bode Plot

Find the values of R and C corresponding to the following asymptotic $Z(\omega)$



5.3.2 Nonideal Capacitors

Do Capacitors Have Resistance and Inductance?



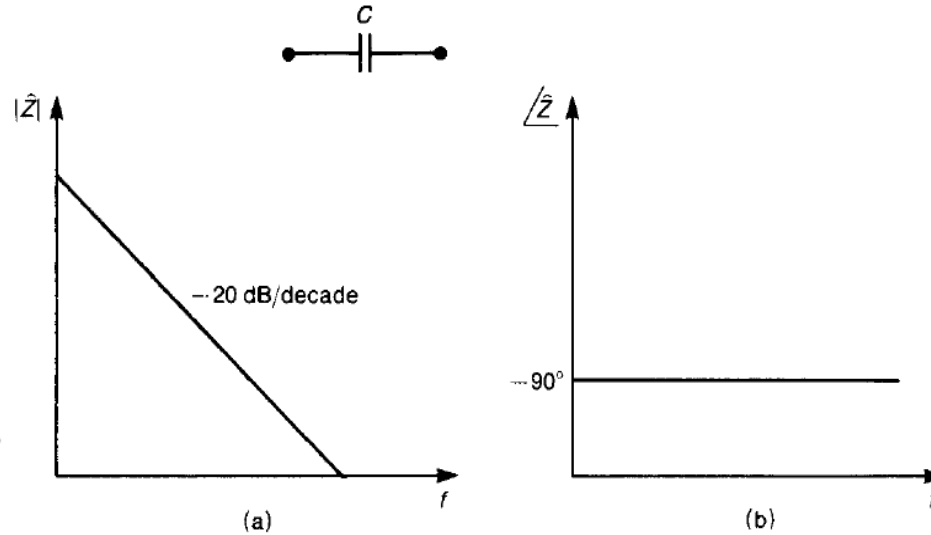
Interface with Real Capacitance

Nonideal Capacitors

Ideal capacitor C

$$\hat{Z}(j\omega) = \frac{1}{j\omega C}$$

$$= \frac{1}{\omega C} \angle -90^\circ$$



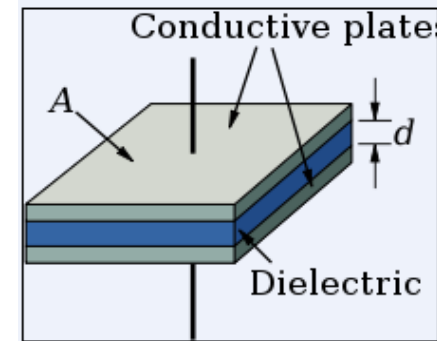
Nonideal capacitor parameters

R_{diel} : loss in the dielectric, large value

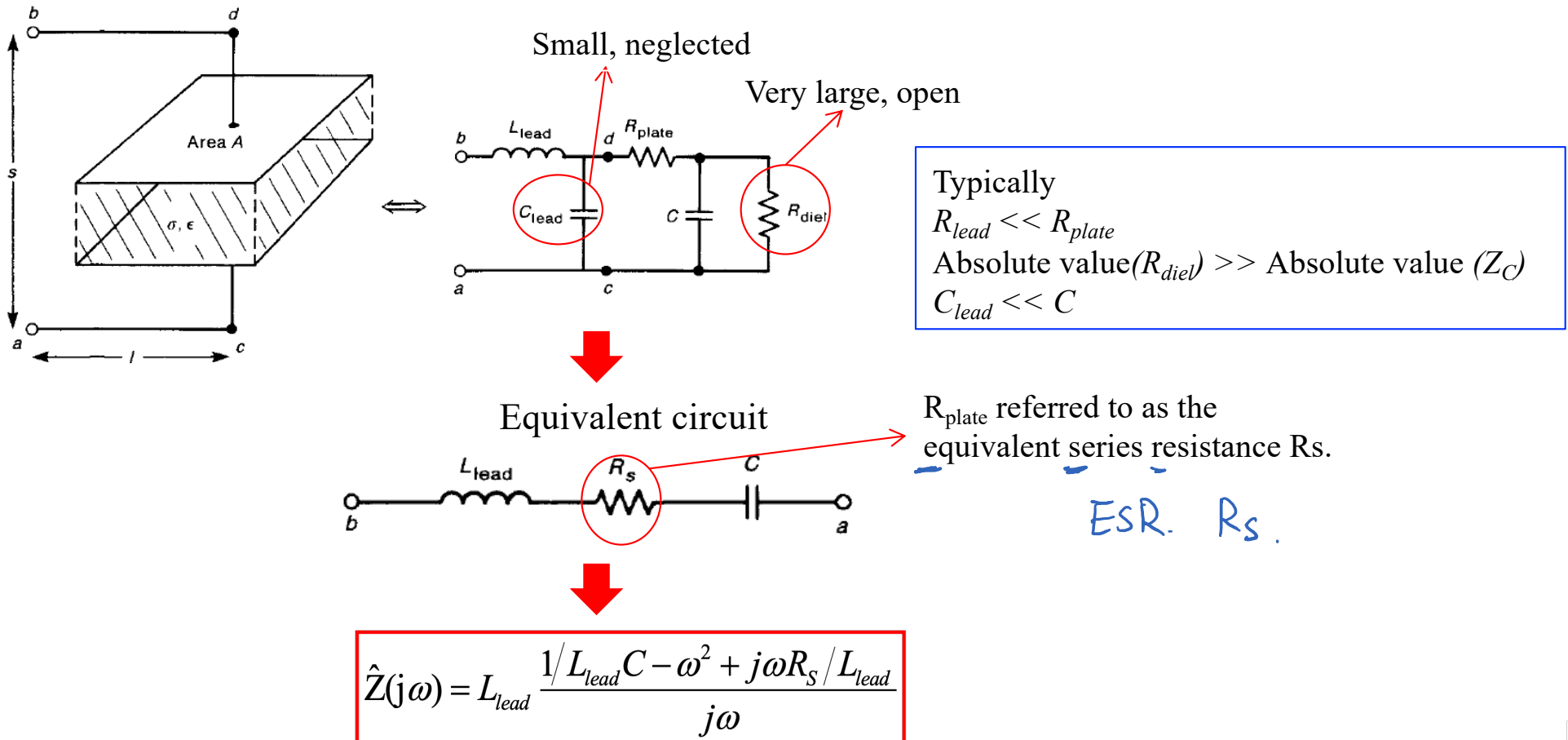
R_{plate} : resistance of the plates, small value

L_{lead} : inductance of the leads attached to the capacitor

C_{lead} : capacitance of the leads attached to the capacitor

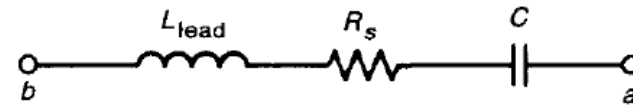


Equivalent model of nonideal capacitors



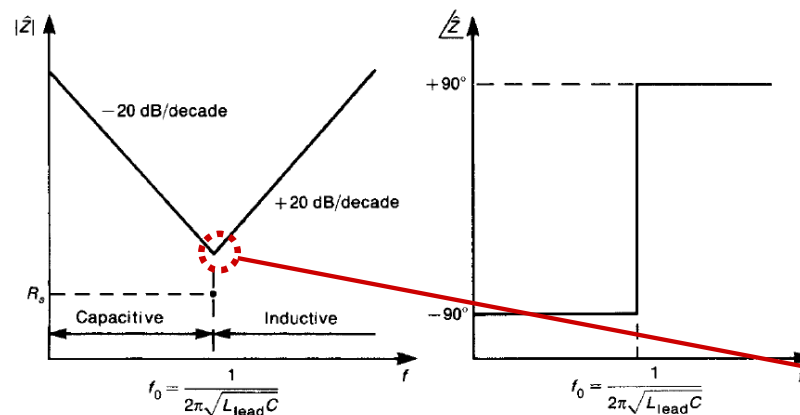
Equivalent model of nonideal capacitors

$$\hat{Z}(j\omega) = L_{lead} \frac{1/L_{lead}C - \omega^2 + j\omega R_S/L_{lead}}{j\omega}$$



1. At DC $\rightarrow$ open circuit

2. As frequency increases, the impedance of the capacitor dominates and decreases linearly with frequency at a rate of - 20 dB/ decade.



3. self-resonant frequency of the capacitor

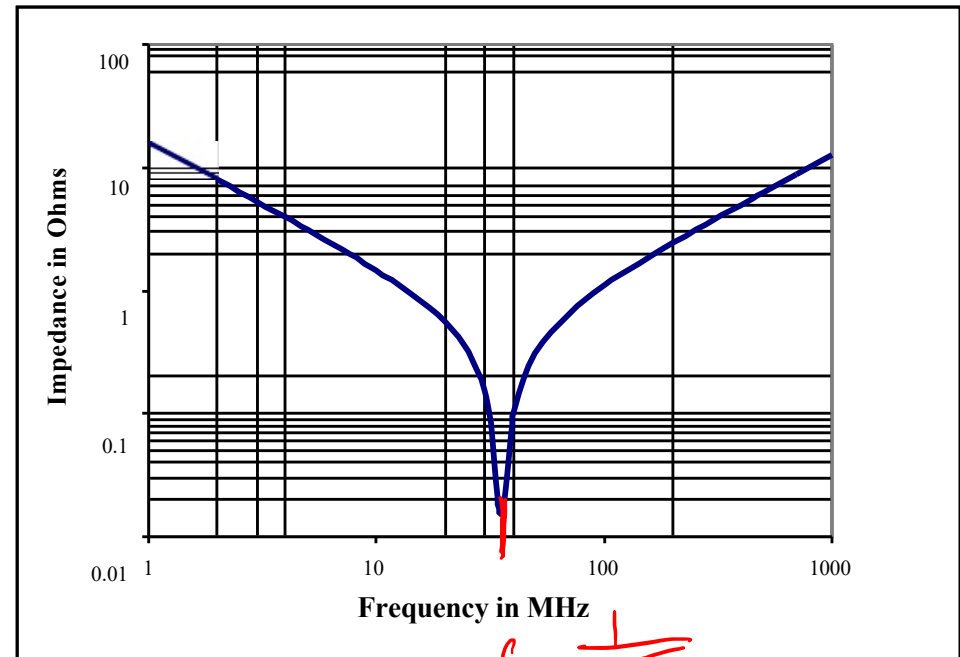
$$f_0 = \frac{1}{2\pi\sqrt{L_{lead}C}}$$

① $L_{lead} \uparrow \rightarrow f_0 \downarrow$

4. For higher frequencies the magnitude of the impedance of the inductor dominates and the impedance increases at a rate of +20 dB/ decade

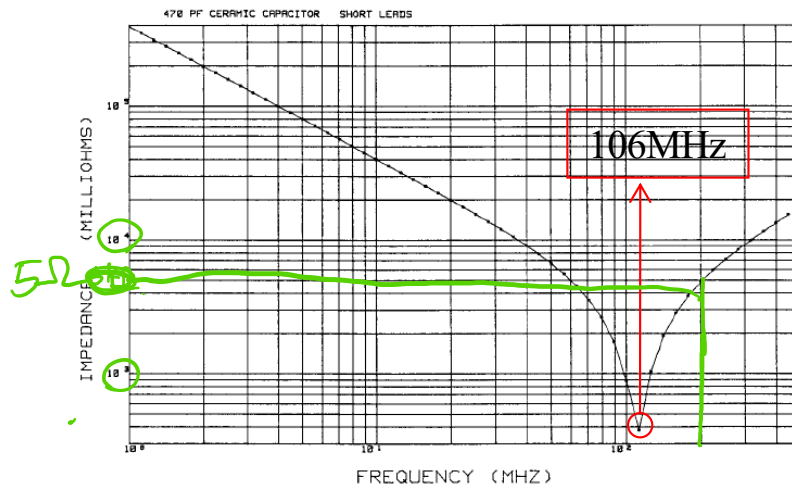
② For a fixed lead length and spacing, the larger the capacitance value the lower the self-resonant frequency

Impedance of a 0.01- μ F Capacitor



$$f = \frac{1}{2\pi LC}$$

Lead lengths

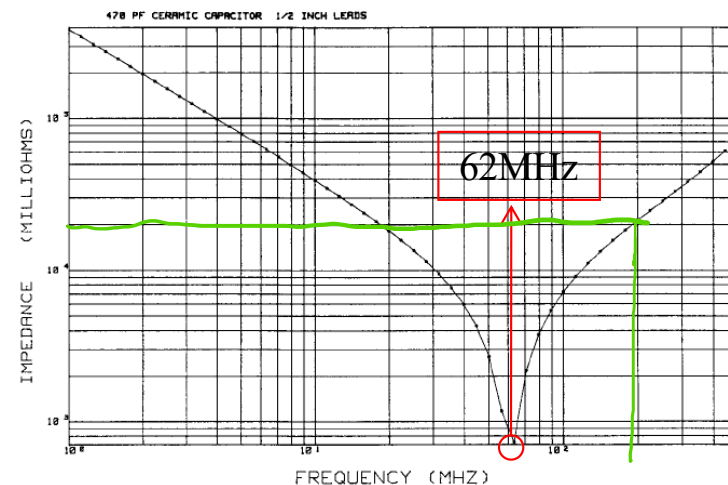


Impedance of a 470-pF ceramic capacitor with very short lead lengths

Lead small

Q: If one is interested in providing a low impedance to shunt a 200MHz signal, which one do you choose?

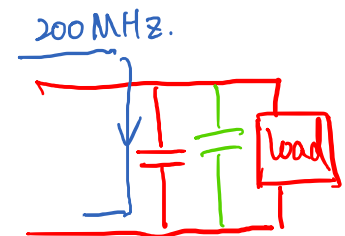
A: Neither. The 470-pF capacitor with either lead length will give an impedance larger than expected.



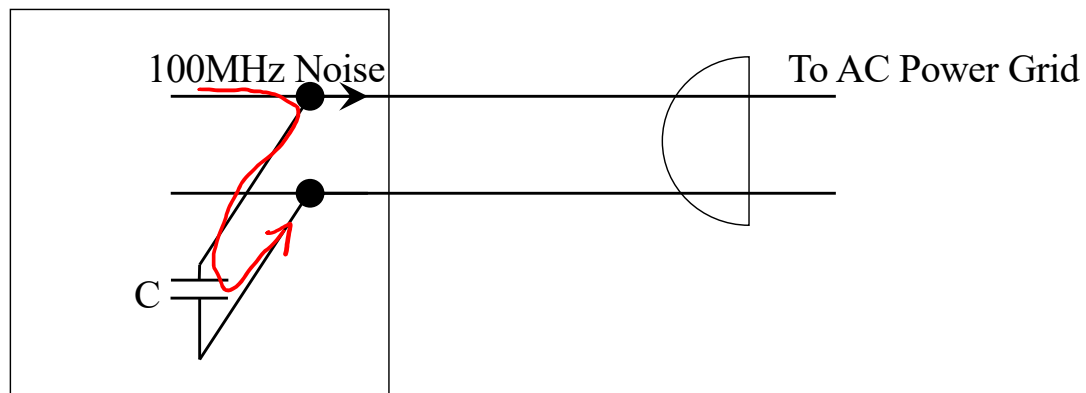
Impedance of a 470-pF ceramic capacitor with 1/2 in. lead lengths

Lead. large.

$$f = \frac{1}{2\pi\sqrt{LC}}$$



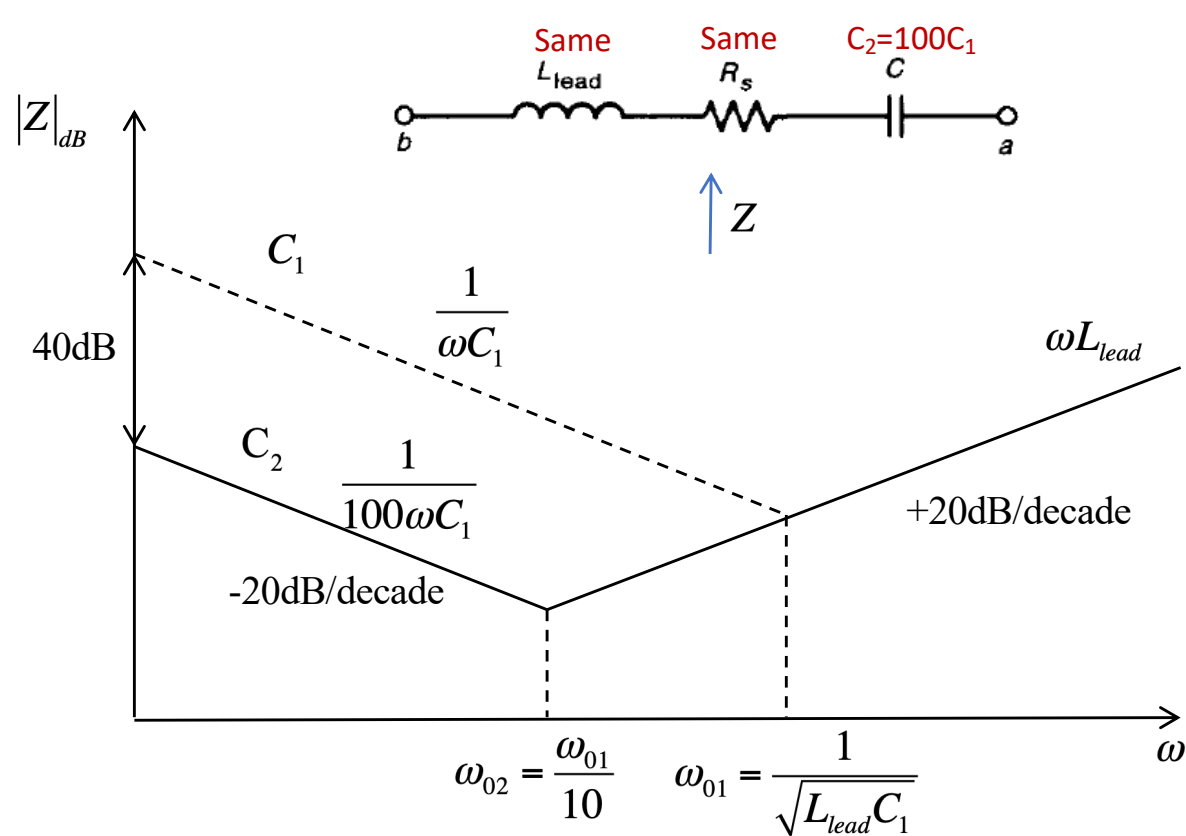
Noise suppression via shunt capacitance



$$C_1 = 100\text{pF}$$

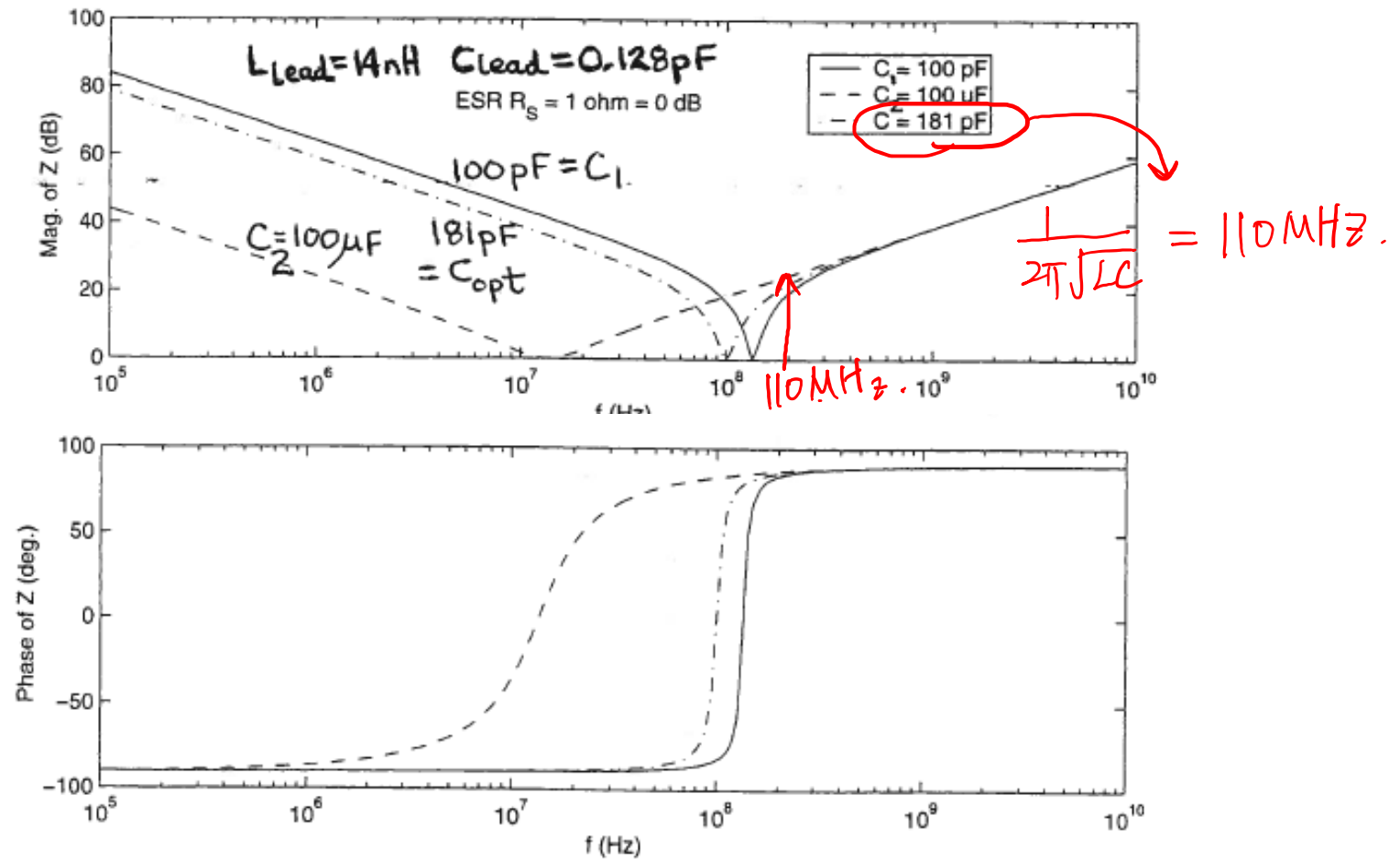
$$C_2 = 0.01\mu\text{F}$$

Bold plot of C1 and C2 capacitor with same leads



$$Z_{ideal} = \frac{1}{j\omega C}$$

Optimum shunt capacitor value C @ 10MHz

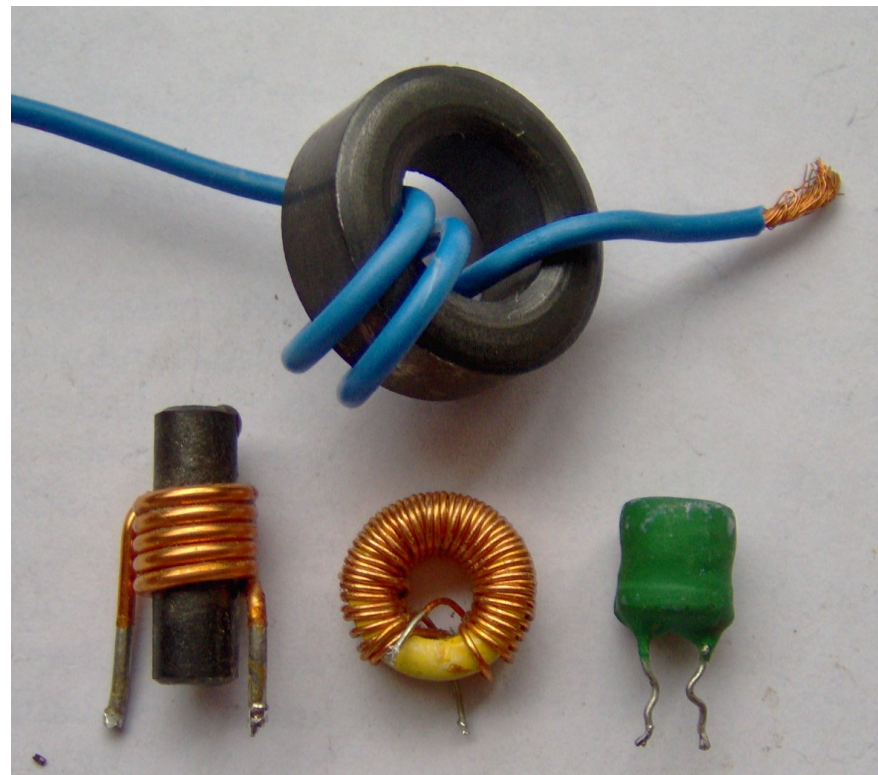


5.3.3 Nonideal Inductors

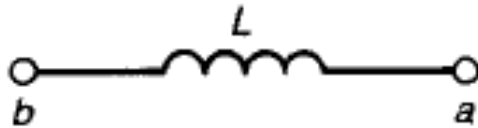
Do Inductors Have Resistance and Capacitance?

Inductors are an electrical/electronic component that opposes a change in current due to its self-inductance.

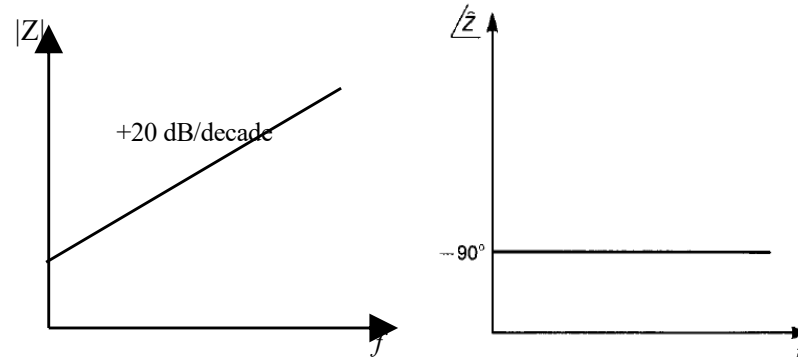
It consists of a number of turns of fine wire of copper wound on a core of some material (air, iron, powdered iron, or ferrite) in spiral form.



Nonideal Inductors



Ideal conductor: L



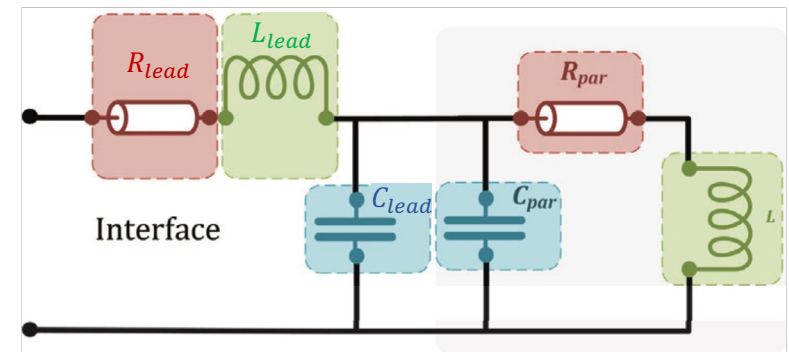
Nonideal inductor parameters

R_{par} : Series self-resistance of the inductor's windings

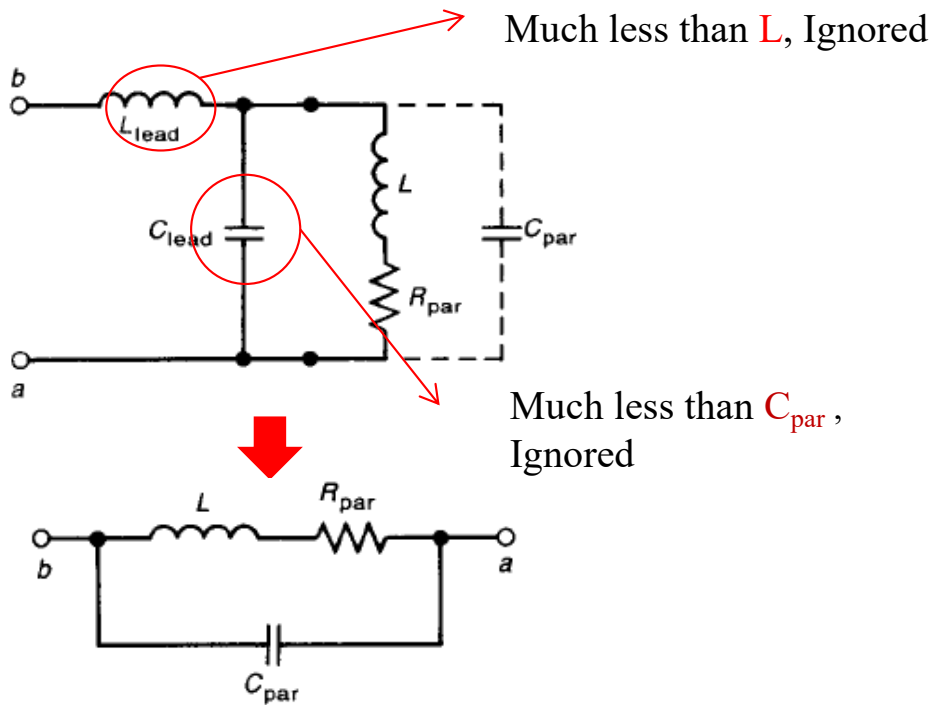
C_{par} : Parallel parasitic capacitance

L_{lead} : Lumped inductance of the leads, usually much less than L , neglected

C_{lead} : Lumped capacitance of the leads, usually much less than C_{par} , neglected



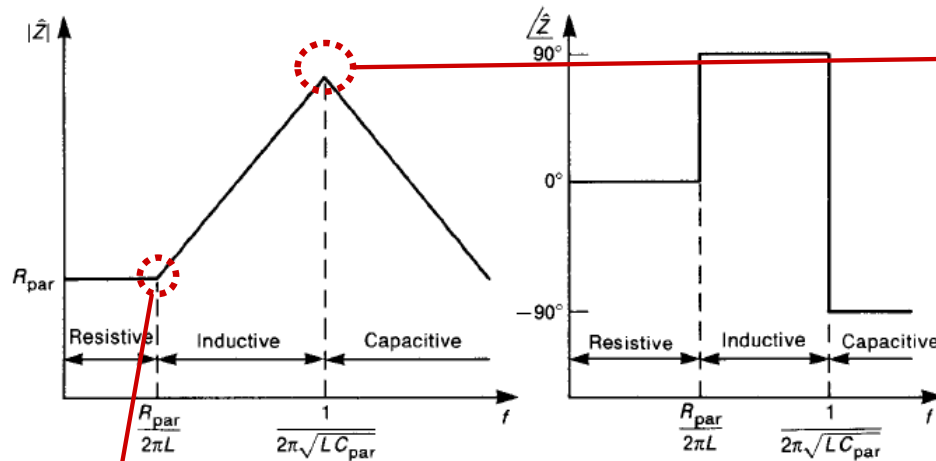
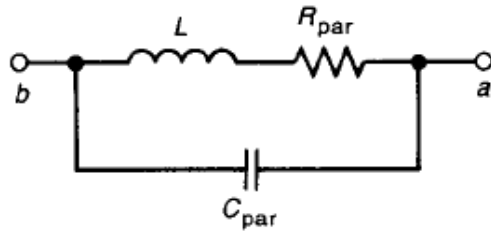
Equivalent model of nonideal inductors



Typically
 $R_{\text{lead}} \ll R_{\text{par}}$
 $L_{\text{lead}} \ll L$
 $C_{\text{lead}} \ll C$

$$\hat{Z}_L(j\omega) = R_{\text{par}} \frac{1 + j\omega L/R_{\text{par}}}{1 - \omega^2 LC_{\text{par}} + j\omega R_{\text{par}} C_{\text{par}}}$$

Frequency response of the impedance of a nonideal inductor



$$f_1 = \frac{R_{\text{par}}}{2\pi L}$$

2. As frequency increases, when $f > f_1$, the inductor impedance dominates, and increases at a rate of 20 dB/decade.,

1. DC: $Z = R_{\text{par}}$

3. Self-resonant frequency:

$$f_0 = \frac{1}{2\pi\sqrt{LC_{\text{par}}}}$$

For a fixed lead length and spacing, the larger the inductance value the lower the self-resonant frequency

4. As frequency increases, the impedance of the capacitor dominates, and decreases linearly with frequency at a rate of -20 dB/decade.