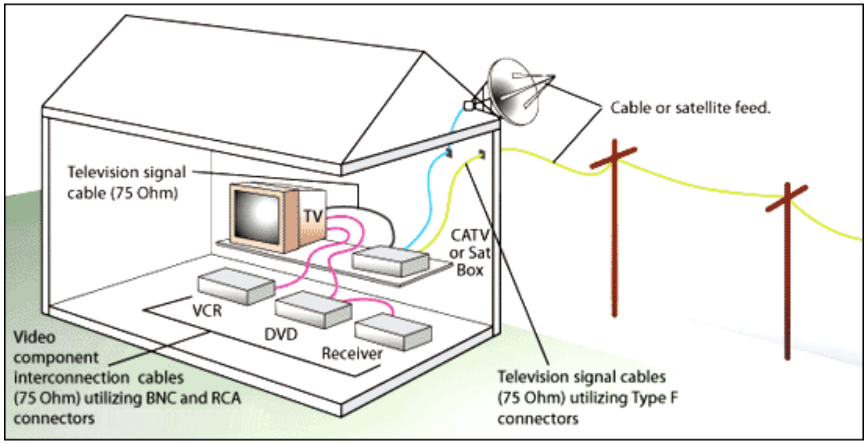
Electrical & Computer Engineering
GRAINGER COLLEGE OF ENGINEERING



Transmission Line Basics

Prof. Victoria Shao

Outlines

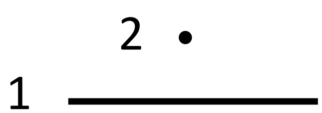
- Transmission Lines in Planar structure.
- Key Parameters for Transmission Lines.
- Transmission Line Equations.
- Per unit length parameters
- Analysis approach for Z_o and T_d
- Loss of Transmission Lines
- Example I: Connector Performance
- Example II: Transmission line on non-ideal GND
- Example III: Rambus and RIMM Module design
- Example IV: EMI resulting from a trace near a PCB edge

3.1 Transmission Lines

Travelling waves on transmission lines has Transverse ElectroMagnetic (TEM) field structure between conductors.

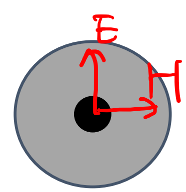
- Cross section shape doesn't change.
- electrical length of the line is long compared with wavelength.
- Cross sectional dimensions are small compared to wavelength.

Example of structures (cross sections) that can support TEM fields given below.

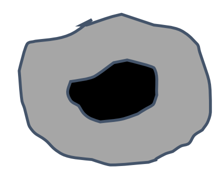


Wire above conducting plane

Parallel conductors

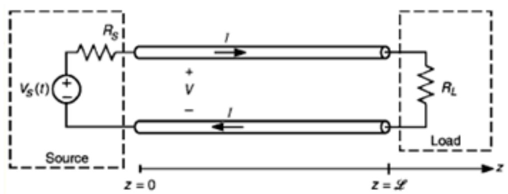


Coaxial cable

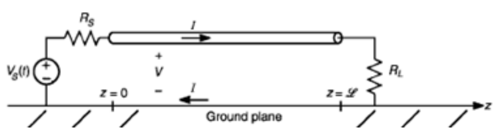


A generic coaxial structure

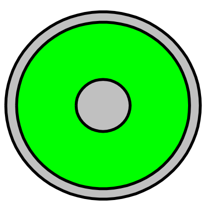
Transmission Lines in Planar structure

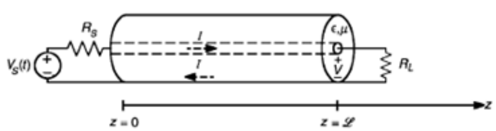


(a) Two wires



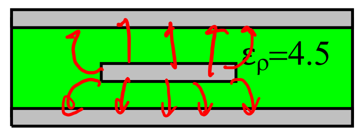
(b) One wire above an infinite ground plane



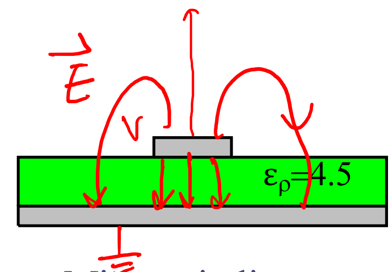


(c) Coaxial cable

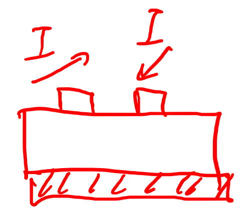
Typical wire-type transmission lines

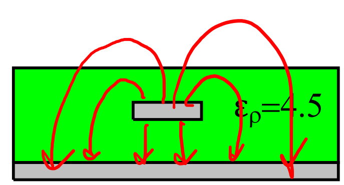


Stripline

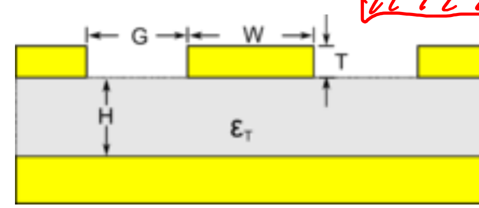


Microstrip line





Embedded Microstrip line



Coplanar

Typical printed circuit board (PCB) structures

3.2 Key Parameters for Transmission Lines

V^+ / I^+

1. Relation of V / I : Characteristic Impedance $Z_0 \rightarrow$ **Reflection** $\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$
2. Velocity of Signal: Effective dielectric constant ε_r $\rightarrow$ **Time delay** $T_D = \frac{L}{v_p}$.
3. Attenuation: Conductor loss $\alpha_c \rightarrow$ **Loss**

Dielectric loss $\epsilon_r = \epsilon_r' - j\epsilon_r''$

$\frac{V^{ref}}{V^{inc}}$

Eeff.

$\tan\theta = 0.002$.

Fr4.

Fr4.

material σ, ε, μ.

cable. R.

Lossless case

air

Speed $c_o = \frac{1}{\sqrt{\epsilon_o \mu_o}} = \frac{1}{\sqrt{lc}}$

per unit length

Characteristic impedance $Z_o = \sqrt{\frac{l}{c}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$.

dielectric $\Rightarrow$ Speed $v_p = \frac{c}{\sqrt{\epsilon_r \mu_r}} = \frac{1}{\sqrt{lc}}$

$= \frac{1}{\sqrt{\mu\varepsilon}}$.

Time delay in various mediums

$$T_d = \frac{\mathcal{L}}{v_p}$$

$\mathcal{L}$ → Total Length

v_p → velocity of propagation

1. Free space

$$v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, \mathrm{m}/s$$

Td per meter: $3.3 ns/\mathrm{m}$

2. PCB (FR-4)

$$v_p = \frac{v_0}{\sqrt{\mu_r \varepsilon_r}} = \frac{v_0}{\sqrt{\varepsilon_r}} \approx 1.38 \times 10^8 \, \mathrm{m}/s$$

$$\boxed{\varepsilon_r = 4.7}$$

Td per meter: $7.2 ns/\mathrm{m}$

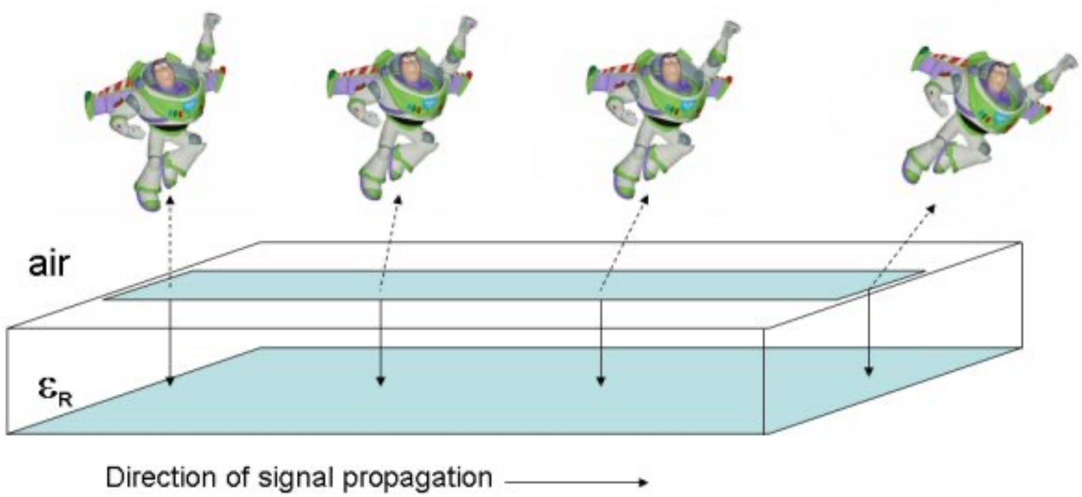
3. Microstrip Line

$$v \approx \frac{v_0}{\sqrt{\mu_r \varepsilon_r}} = \frac{v_0}{\sqrt{\varepsilon_r}} \approx 1.78 \times 10^8 \, m/s$$

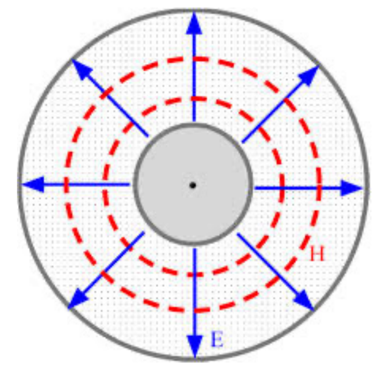
$$\boxed{\varepsilon_r \approx (1 + 4.7)/2 = 2.85}$$

Td per meter: $5.6 ns/m$

3.3 Transmission Line Equations



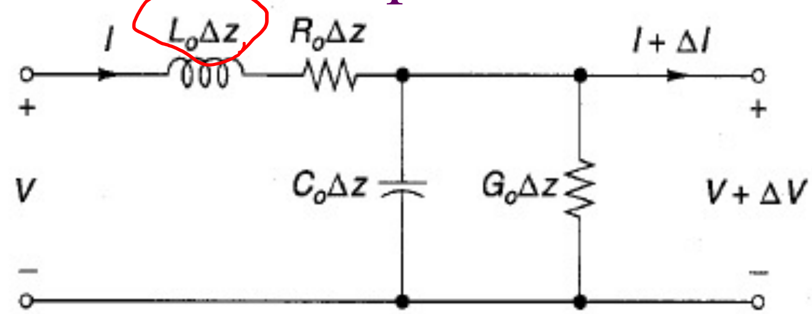
Quasi-TEM in microstrip



TEM in coaxial cable

How to calculate ϵ_{eff}?

Transmission Line Equations



R_0 = resistance per unit length(Ohm/ cm)

G_0 = conductance per unit length (S/ cm)

L_0 = inductance per unit length (H / cm)

C_0 = capacitance per unit length (F/ cm)

KVL : $\frac{dV}{dz} = -(R_0 + j\omega L_0)I$

KCL : $\frac{dI}{dz} = -(G_0 + j\omega C_0)V$

V. I phasors.

Telegrapher's equations

Solve 2nd order D.E. for V and I

Transmission Line Equations

In Lossless TL $R = 0, G = 0$

$$\frac{\partial V}{\partial z} = -L_o \frac{\partial I}{\partial t}$$
$$\frac{\partial I}{\partial z} = -C_o \frac{\partial V}{\partial t}$$

$$\Rightarrow \quad \frac{1}{L_o C_o}\frac{\partial^2 \mathrm{V}}{\partial z^2} = \frac{\partial^2 V}{\partial t^2}$$

Wave equation

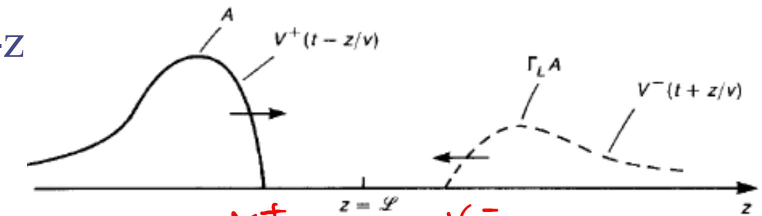
The above wave equation has a solution of the form $V(z,t) = V(0, t \mp \frac{z}{v})$, which represent a wave travelling in forward direction (z direction) and a wave travelling in backward direction (-z direction), without attenuation ($R = 0$) and distortion ($G = 0$).

$$v^2 \frac{\partial^2 \mathrm{V}}{\partial z^2} = \frac{\partial^2 V}{\partial t^2}$$

Wave equation. Comparing, we get $v = \frac{1}{\sqrt{L_o C_o}}$ $= c.$

Transmission Line Equations with Loss

Two wave components with amplitudes
V+ and V- traveling in the direction of +z and -z



forward backward

$$V = V^+ \boxed{e^{-\gamma z}} + V^- \boxed{e^{+\gamma z}}$$

$$I = \frac{1}{Z_0}\left(\underbrace{V^+ e^{-\gamma z}}_{I^+} - \underbrace{V^- e^{+\gamma z}}_{I^-}\right) = I^+ + I^-$$

$$Z_0 = \frac{V^+}{I^+} = -\frac{V^-}{I^-}$$

Where propagation constant and characteristic impedance are

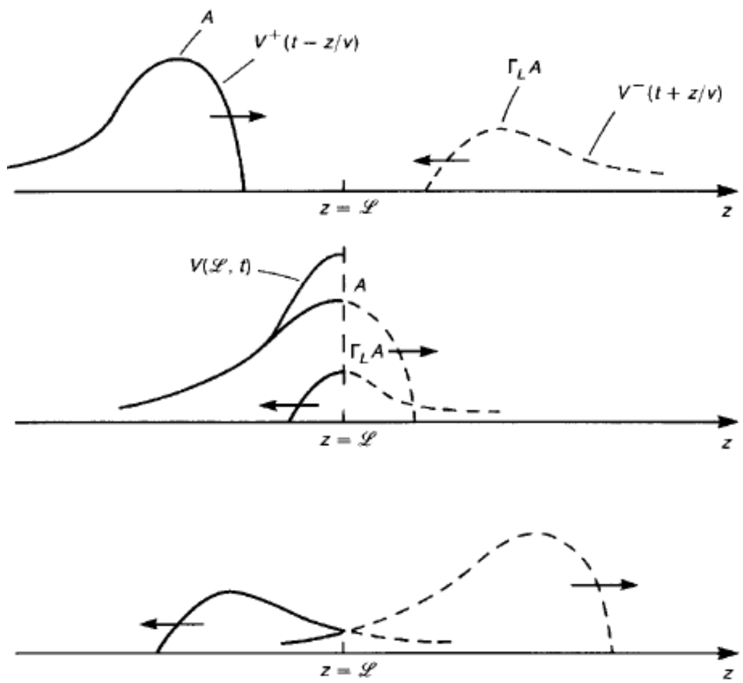
$$\gamma = \sqrt{(R_0 + j\omega L_0)(G_0 + j\omega C_0)} = \alpha + j\beta$$

phase.

attenuation

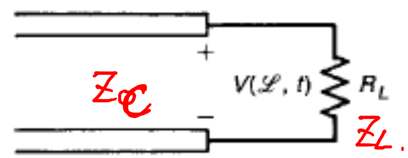
$$Z_0 = \frac{V^+}{I^+} = \frac{V^-}{I^-} = \sqrt{\frac{R_0 + j\omega L_0}{G_0 + j\omega C_0}}$$

Reflection



Reflection of waves at a termination

When?



Mismatched lines

i.e., $\mathbf{R_L \neq Zc}$

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c}.$$

$$V^- = \Gamma_L V^+$$

Major aspect of degrading signal integrity!

3.4 The per-unit-length Parameters R, L, G, C.

Homogeneous structure

TEM wave structure is like the DC (static) field structure

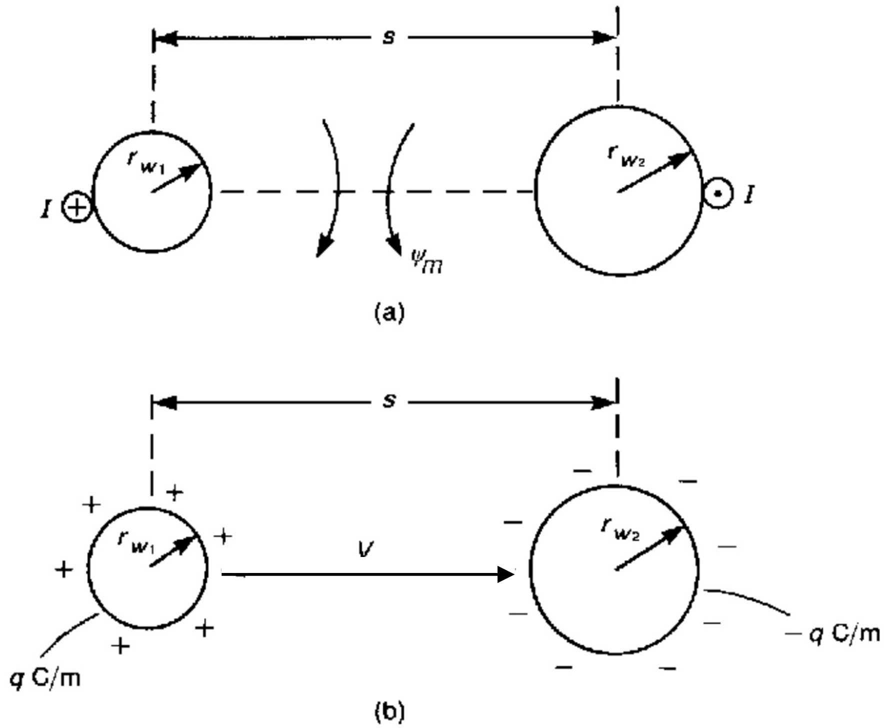
$$lc = \mu\varepsilon$$

$$lg = \mu\sigma$$

homogeneous surrounding medium

So, if you can derive how to get the *l*, *g* and *c* can be obtained by the above two relations.

Analysis approach (Wires in air)



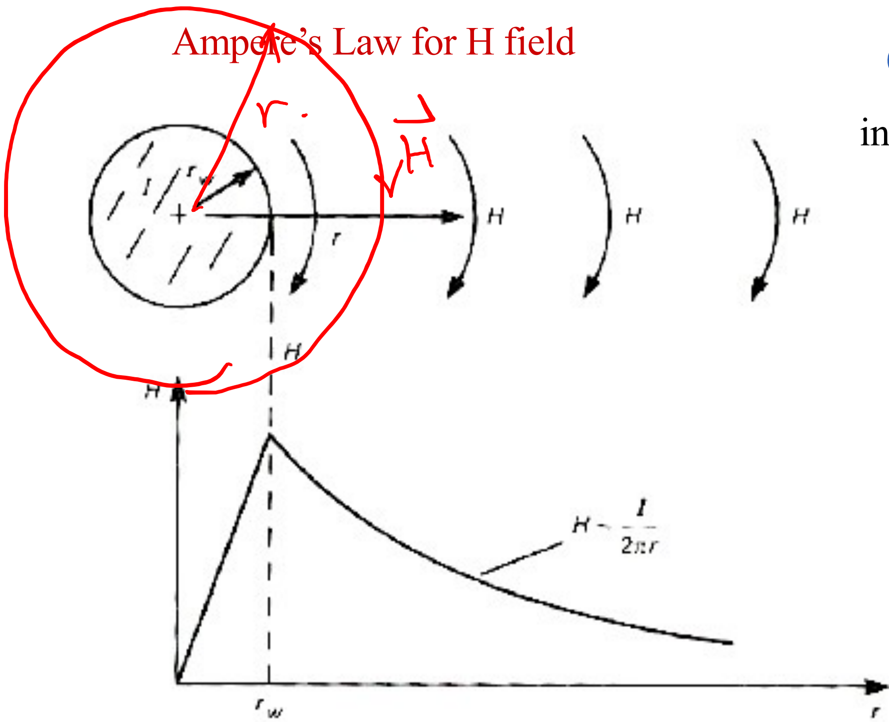
Determination of the per-unit-length parameters of a two-wire line
(a) inductance; (b) capacitance

L = ? (by $L = \Psi / I$)

C = ? (by $C = Q / V$)

3.4.1 The per-unit-length Parameters (l)

Ampere's Law for H field



The magnetic field about a current-carrying wire.

$$\oint_c H \cdot dl = \int_S J \cdot ds + \frac{d}{dt}\int_S D \cdot ds = I.$$

I_c I_d.

in the transverse plane,

$$\oint_{c_{xy}} H_T \cdot dl = \int_{S_{xy}} J_z \cdot ds + \frac{d}{dt}\int_{S_{xy}} D_z \cdot ds$$

Conduction current I

$$I = \int_S J \cdot ds$$

Displacement current I_d

$$I_d = \frac{d}{dt}\int_S D_z \cdot ds = 0$$

$$\Downarrow$$

$$H_T(r) = \frac{I}{\oint_c dl} = \frac{I}{2\pi r} \quad A/m.$$

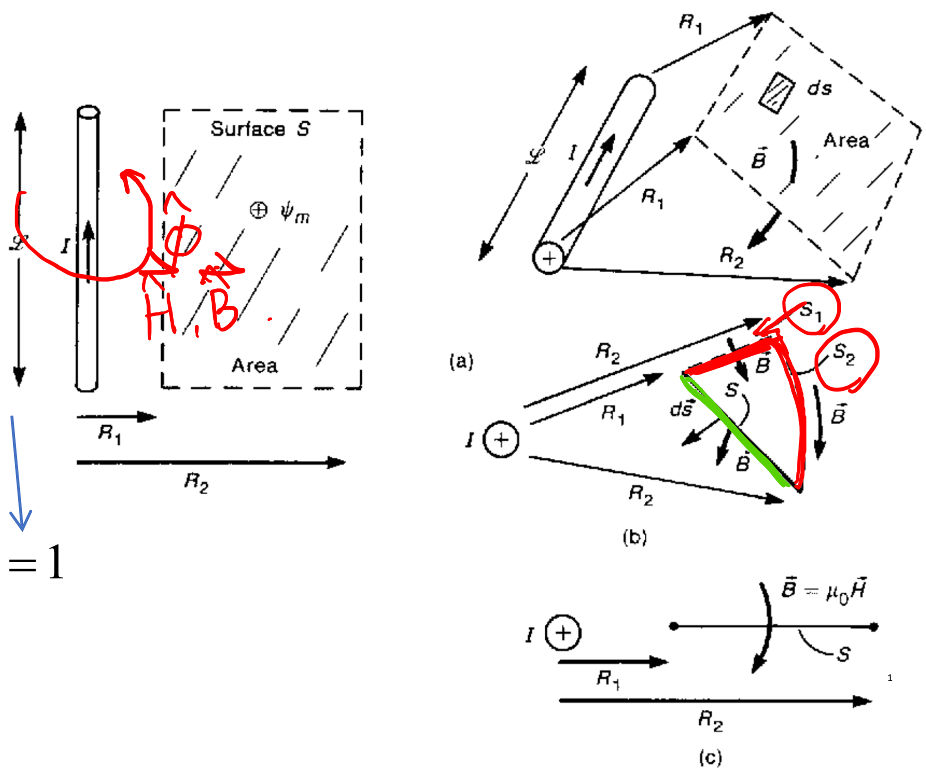
$$B_T = \mu_0 H_T = \frac{I\mu_0}{2\pi r} \quad T.$$

3.4.1 The per-unit-length Parameters (l)

$\oiint_S \vec{B}\cdot d\vec{s} = 0$

$[T], [Wb/m^2]$

Gauss's Law for H field



$l = 1$

$$\Psi_m = \int_S \overline{B}_T \cdot d\overline{s}$$

$$= \int_{S_1} \overline{B}_T \cdot d\overline{s} + \underbrace{\int_{S_2} \overline{B}_T \cdot d\overline{s}}_{0} + \underbrace{\int_{S_{end}} \overline{B}_T \cdot d\overline{s}}_{0}$$

$$= \int_{r=R_1}^{R_2} \frac{\mu_0 I}{2\pi r} dr \times \boxed{1\,m}$$

length $=1m$.

$$= \frac{\mu_0 I}{2\pi} \ln\left(\frac{R_2}{R_1}\right) \quad (\text{in } [\text{Wb}])$$

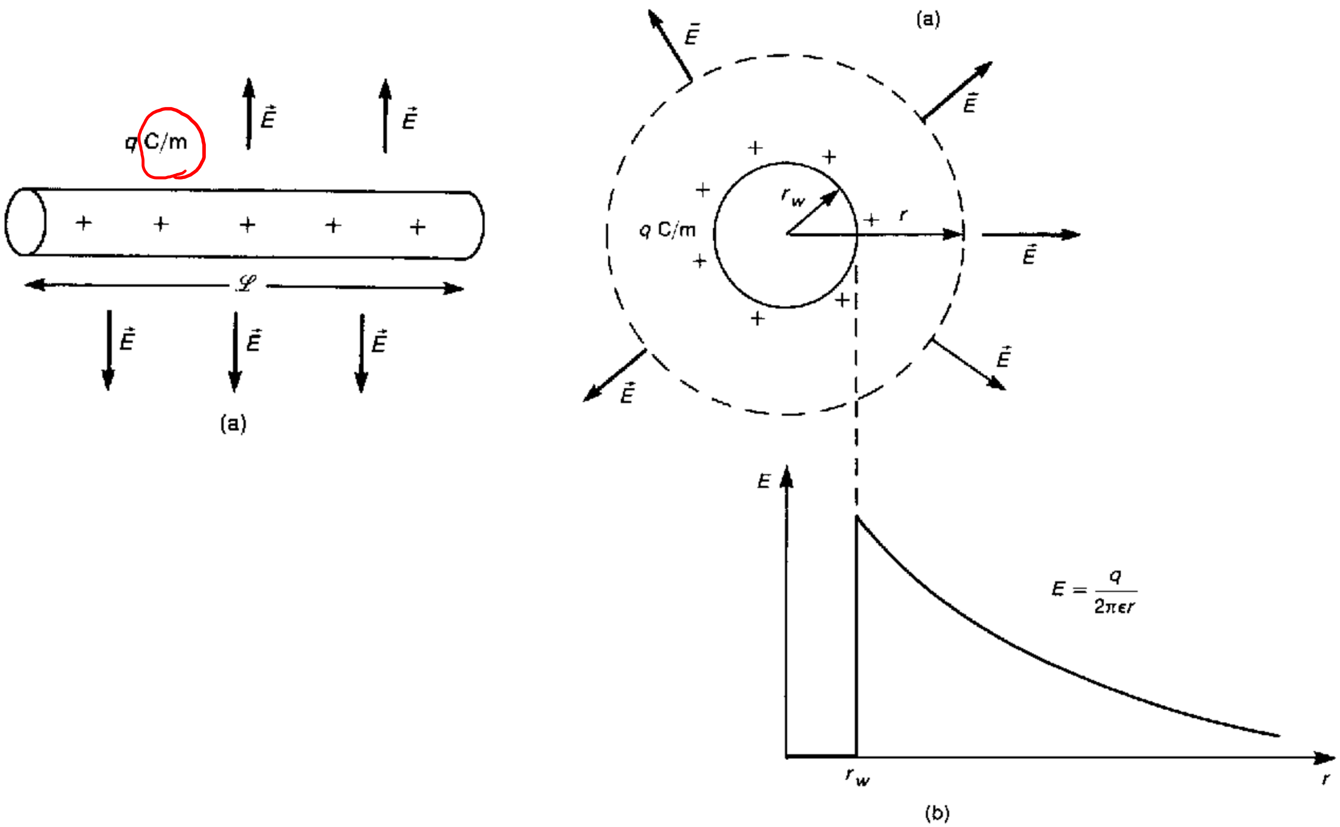
$$\boxed{l = \frac{\Psi_m}{I} = \frac{\mu_0}{2\pi} \ln\left(\frac{R_2}{R_1}\right)}$$

Per unit length

Illustration of a basic subproblem of determine magnetic flux of a current through a surface: (a) dimension of the problem; (b) use of Gauss' law; (c) an equivalent but simpler problem

3.4.2 The per-unit-length Parameters (c):

$C = \frac{Q}{V}.$



The electric field about a charge-carrying wire

1) from Gauss's law

Gauss's Law for E field

$$\oint_S D \cdot ds = \iiint_v \rho_v \, dv$$

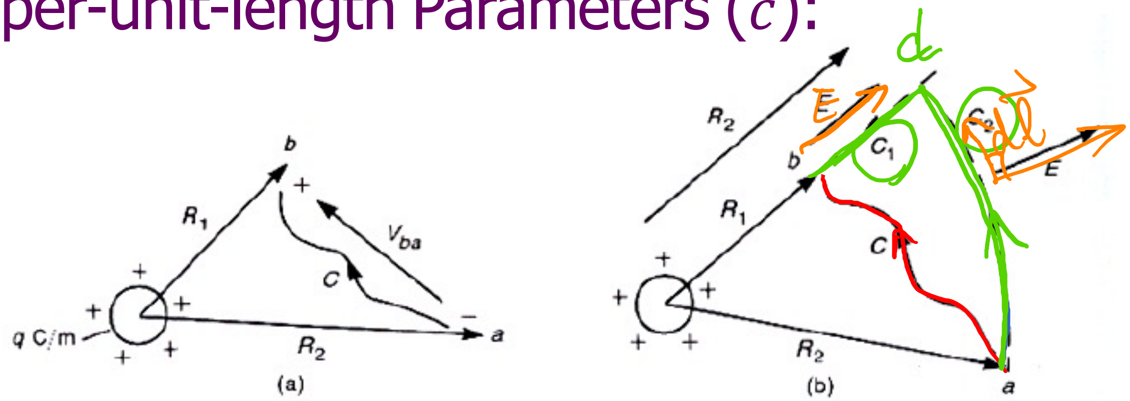
$$\oint_s \varepsilon_0 \vec{E}_T \cdot d\vec{s} = Q_{total}$$

$$\therefore \quad \vec{E}_T = \frac{q \times 1m}{\varepsilon_0 \oint_s ds} = \frac{q}{2\pi\varepsilon_0 r}$$

$\nabla \times \vec{E} = 0$ curl free

Electrostatic

3.4.2 The per-unit-length Parameters (c):



Determining the voltage between two points: (a) dimensions of the problem; (b) an equivalent but simpler problem

$$\begin{aligned}
V &= -\int_C \bar{E}_T \cdot d\bar{l} \\
&= -\int_{C_1} \bar{E}_T \cdot d\bar{l} - \underbrace{\int_{C_2} \bar{E}_T \cdot d\bar{l}}_{0} \\
&= -\int_{r=R_2}^{R_1} \frac{q}{2\pi\varepsilon_0 r} dr \\
&= \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right) \quad (\text{in V})
\end{aligned}$$

$$\boxed{c = \frac{q}{V} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)}}$$

$lc = \mu_0\varepsilon_0.$

Example: Determine the L.C.G. of the two-wire line.

(note: homogeneous medium)

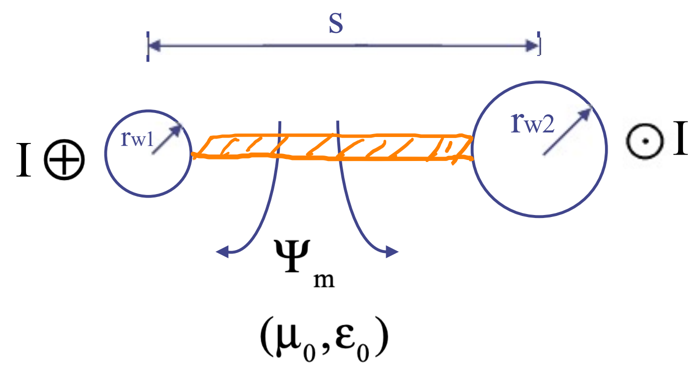
Inductance:

$$l = \frac{\Psi_m}{I}$$

Where $\frac{\mu_0 I}{2\pi}\ln\left(\frac{R_2}{R_1}\right)$

$$\Psi_m = \frac{\mu_0 I}{2\pi}\ln\left(\frac{s-r_{w2}}{r_{w1}}\right) + \frac{\mu_0 I}{2\pi}\ln\left(\frac{s-r_{w1}}{r_{w2}}\right)$$

$$= \frac{\mu_0 I}{2\pi}\ln\left(\frac{(s-r_{w2})(s-r_{w1})}{r_{w1}r_{w2}}\right)$$



$$\boxed{\psi_m = \frac{\mu_0 I}{2\pi}\ln\left(\frac{R_2}{R_1}\right) \quad (R_2 > R_1) \text{ for single wire}}$$

assume $s \gg r_{w1}, r_{w2}$

$$\Rightarrow \quad \boxed{l = \frac{\mu_0}{2\pi}\ln\left(\frac{s^2}{r_{w1}r_{w2}}\right)}$$

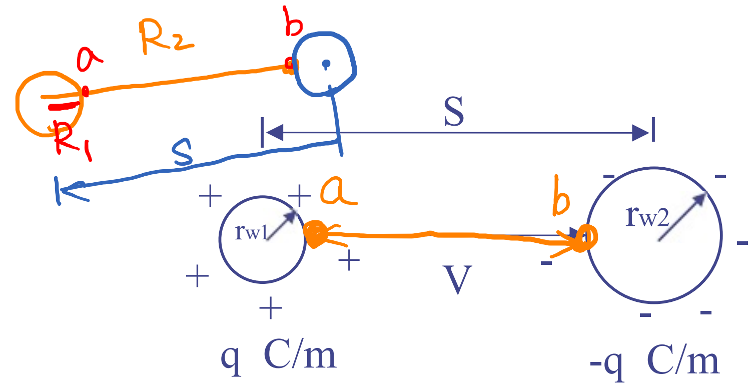
A special case, both wires have same radii

$$l = \frac{\mu_0}{\pi}\ln\left(\frac{s}{r_w}\right), \quad r_{w1} = r_{w2} = r_w$$

$$= 0.4\ln\left(\frac{s}{r_w}\right) \quad (\text{in } \mu\text{H/m})$$

$$= 10.16\ln\left(\frac{s}{r_w}\right) \quad (\text{in nH/in.})$$

Capacitance:



Method 1) $\boxed{lc = \varepsilon\mu}$ $\quad l = \dfrac{\mu_0}{2\pi}\ln\left(\dfrac{s^2}{r_{w1}r_{w2}}\right) \quad (s >> r_w)$

$$\Rightarrow c = \frac{2\pi\varepsilon_0}{\ln\left(\dfrac{s^2}{r_{w1}r_{w2}}\right)} \qquad (\text{in F/m})$$

Method 2) $V = \dfrac{q}{2\pi\varepsilon_0}\ln\left(\dfrac{s-r_{w2}}{r_{w1}}\right) + \dfrac{q}{2\pi\varepsilon_0}\ln\left(\dfrac{s-r_{w1}}{r_{w2}}\right)$

$$= \frac{q}{2\pi\varepsilon_0}\ln\left(\frac{(s-r_{w2})(s-r_{w1})}{r_{w1}r_{w2}}\right)$$

$$\cong \frac{q}{2\pi\varepsilon_0}\ln\left(\frac{s^2}{r_{w1}r_{w2}}\right) \qquad \text{if } s \gg r_{w1}, r_{w2}$$

$$\Rightarrow c = \frac{q}{V} = \frac{2\pi\varepsilon_0}{\ln\left(\dfrac{s^2}{r_{w1}r_{w2}}\right)} \qquad (\text{in F/m})$$

← the same with 1st method

$$V = \frac{q}{2\pi\epsilon_o}\ln\left(\frac{R_2}{R_1}\right) \quad (R_2 > R_1)$$

for single wire

A special case, both wires have same radii

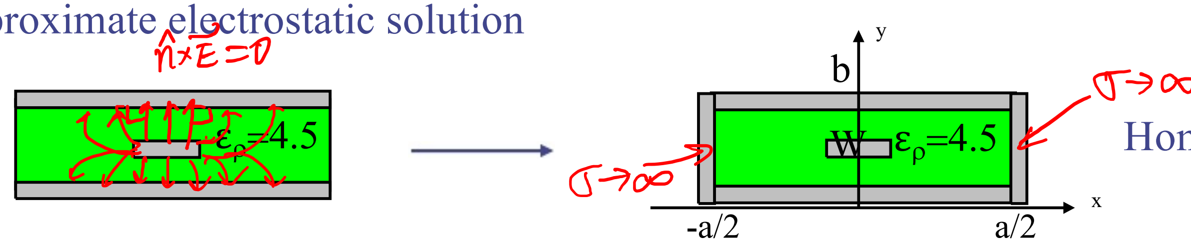
$$\boxed{c = \frac{\pi\epsilon_o}{\ln\left(\frac{S}{r_W}\right)}, \qquad r_{w1} = r_{w2} = r_W}$$

$Z_0 = \sqrt{\dfrac{l}{c}}$ $\quad V_p = \dfrac{1}{\sqrt{lc}}$ $\quad T_d = \dfrac{1\text{m}}{V_p} = \sqrt{lc}$.

per unit length

3.5 Analysis approach for Z_0 and Td (Stripline)

Approximate electrostatic solution

1. 

Homogeneous medium

2. The fields in TEM mode must satisfy Laplace equation

$$\nabla^2 \Phi_t \ (x, y) = \frac{\partial^2 \phi_x}{dx^2} + \frac{\partial^2 \phi_y}{dy^2} + \cancel{\frac{\partial^2 \phi_z}{dz^2}} = 0$$

where Φ is the electric potential

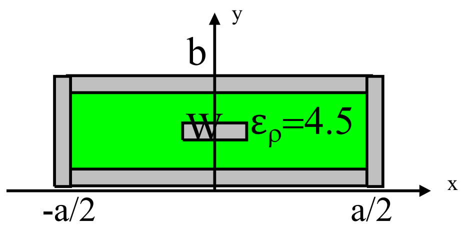
The boundary conditions are

$$\Phi(x, y) = 0 \quad at\ x = \pm \frac{a}{2}$$
$$\Phi(x, y) = 0 \quad at\ y = 0, b$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

Analysis approach for Z_0 and Td

3. Since the center conductor will contain the surface charge, so



$$\Phi(x,y) = \begin{cases} \sum\limits_{\substack{n=1 \\ odd}}^{\infty} A_n \cos\frac{n\pi x}{a} \sinh\frac{n\pi y}{a} & \text{for } 0 \le y \le b/2 \quad \text{(bottom half)} \\ \sum\limits_{\substack{n=1 \\ odd}}^{\infty} B_n \cos\frac{n\pi x}{a} \sinh\frac{n\pi}{a}(b-y) & \text{for } b/2 \le y \le b \quad \text{(top half)} \end{cases}$$

Why?

4. The unknowns A_n and B_n can be solved by two known conditions:

The potential at $y = b/2$ must continuous

The surface charge distribution for the strip: $\rho_s = \begin{cases} 1 & \text{for } |x| \le W/2 \\ 0 & \text{for } |x| \ge W/2 \end{cases}$

Analysis approach for Z_0 and Td

5. $$\begin{cases} V = -\int_0^b E_y(x=0,y)dy = -\int_0^b -\partial\Phi(x,y)/\partial y(x=0,y)dy \\ Q = \int_{-w/2}^{w/2} \rho_s(x)dx = W(C/m) \end{cases}$$

6. $$C = \frac{Q}{V} = \frac{W}{\sum_{\substack{n=1 \\ odd}}^{\infty} \frac{2a\sin(n\pi W/2a)\sinh(n\pi b/2a)}{(n\pi)^2 \varepsilon_0 \varepsilon_r \cosh(n\pi b/2a)}}$$

$$\boxed{Z_0 = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_r}}{cC}}$$ Characteristic impedance

Answers!!

7. $$\boxed{T_d = \frac{1}{v_p} = \frac{\sqrt{\epsilon_r}}{c}}$$ Time delay

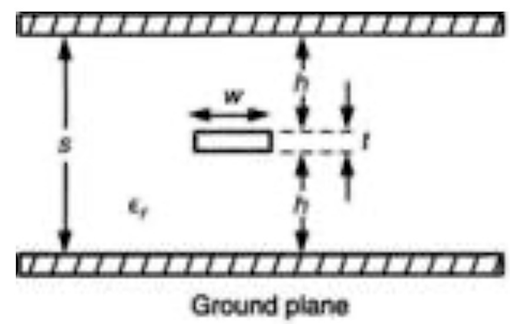
Approximate Solution for Z_0 and Td

Assume a zero thickness strip, $t = 0$

$$Z_c = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{1}{\left[\frac{w_e}{s} + 0.441\right]}$$

Where the effective width of the center conductor is

$$\frac{w_e}{s} = \begin{cases} \frac{w}{s} & \frac{w}{s} \geq 0.35 \\ \frac{w}{s} - \left(0.35 - \frac{w}{s}\right)^2 & \frac{w}{s} \leq 0.35 \end{cases}$$



Stripline

$$Z_o = \frac{1}{V_p C} = \frac{\sqrt{\epsilon_r}}{cC}$$

$$\boxed{C = \frac{\sqrt{\epsilon_r}}{cZ_o}}$$

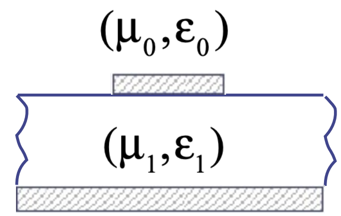
Per-unit-length capacitor

$$\boxed{T_d = \frac{1}{v_p} = \frac{\sqrt{\epsilon_r}}{c}}$$

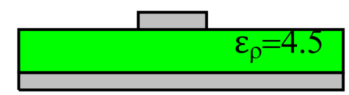
Time delay

c: speed of light

d. How to determine L,C for microstrip-line.



1) This is inhomogeneous medium.
2) Numerical method should be used to solve the capacitance C of this structure, such as Finite element, Finite Difference...
3) But L can be obtained by

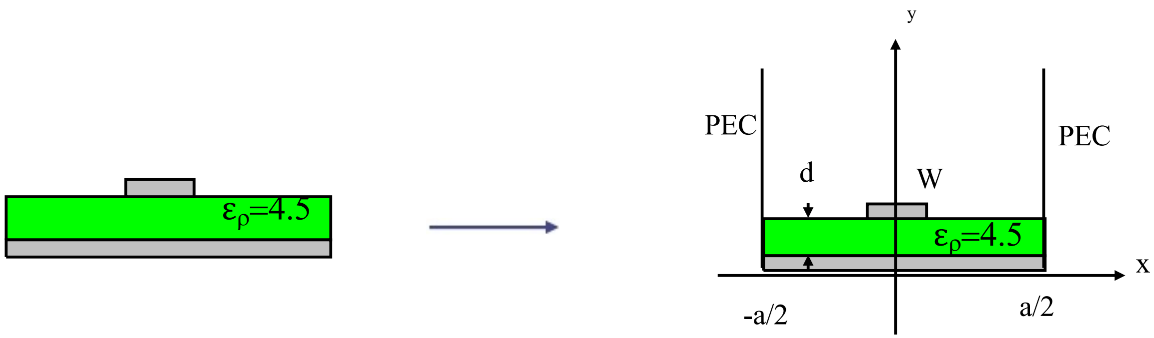


$$LC = \mu\epsilon_e \qquad \Longrightarrow \qquad L = \frac{\mu\epsilon_e}{C}$$

where ϵ_e is effective permittivity of medium 0 and medium 1.

Analysis approach for Z_0 and Td (Microstrip Line)

1.



2.

The fields in Quasi-TEM mode must satisfy Laplace equation

$$\nabla_t^2 \Phi(x, y) = 0$$

where Φ is the electric potential

The boundary conditions are

$$\Phi(x, y) = 0 \quad \text{at } x = \pm a/2$$

$$\Phi(x, y) = 0 \quad \text{at } y = 0, \infty$$

Analysis approach for Z_0 and Td (Microstrip Line)

3. Since the center conductor will contain the surface charge, so

$$\Phi(x,y) = \begin{cases} \sum_{\substack{n=1 \\ odd}}^{\infty} A_n \cos\frac{n\pi x}{a} \sinh\frac{n\pi y}{a} & \text{for } 0 \le y \le d \quad \text{(in dielectric)} \\ \sum_{\substack{n=1 \\ odd}}^{\infty} B_n \cos\frac{n\pi x}{a} e^{-n\pi y/a} & \text{for } d \le y \le \infty \quad \text{(in air)} \end{cases}$$

4. The unknowns A_n and B_n can be solved by two known conditions and the orthogonality of *cos* function :

The potential at $y = d$ must continuous

The surface charge distribution for the strip: $\rho_s = \begin{cases} 1 & \text{for } |x| \le W/2 \\ 0 & \text{for } |x| \ge W/2 \end{cases}$

Analysis approach for Z_0 and Td (Microstrip Line)

5.
$$\begin{cases} V = -\int_0^{b/2} E_y(x=0,y)dy = -\int_0^{b/2} -\partial\Phi(x,y)/\partial y(x=0,y)dy = \sum_{\substack{n=1 \\ odd}}^{\infty} A_n \sinh\frac{n\pi d}{a} \\ Q = \int_{-w/2}^{w/2} \rho_s(x)dx = W(C/m) \end{cases}$$

6.
$$C = \frac{Q}{V} = \frac{W}{\sum_{\substack{n=1 \\ odd}}^{\infty} \frac{4a\sin(n\pi W/2a)\sinh(n\pi d/2a)}{(n\pi)^2 W\varepsilon_0[\sinh(n\pi d/a) + \varepsilon_r \cosh(n\pi d/a)]}}$$

Analysis approach for Z_0 and Td (Microstrip Line)

7.

To find the effective dielectric constant ε_e, we consider two cases of capacitance

1. C = capacitance per unit length of the microstrip line with the dielectric substrate $\varepsilon_r \neq 1$
2. C_0 = capacitance per unit length of the microstrip line with air, $\varepsilon_r = 1$

$$\varepsilon_{eff} = \frac{C}{C_0}$$

8.

Characteristic impedance

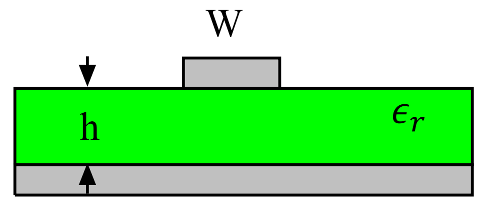
$$Z_0 = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_e}}{cC}$$

Time delay

$$T_d = \frac{1}{v_p} = \frac{\sqrt{\epsilon_e}}{c}$$

Approximate approach for Z_0 and Td (Microstrip Line)

The characteristic impedance is given as*



$$Z_C = \begin{cases} \dfrac{60}{\sqrt{\epsilon_r'}} \ln\left[\dfrac{8h}{w} + \dfrac{w}{4h}\right] & \dfrac{w}{h} \le 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_r'}} \left[\dfrac{w}{h} + 1.393 + 0.667 \ln\left(\dfrac{w}{h} + 1.444\right)\right]^{-1} & \dfrac{w}{h} \ge 1 \end{cases}$$

The effective relative permittivity is

$$\epsilon_r' = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 10h/w}}$$

$h/w \ll 1$. $\varepsilon_r' = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2} = \varepsilon_r$. $\rightarrow$ parallel plate.

$h/w \gg 1$. $\varepsilon_r' = \frac{\varepsilon_r+1}{2}$.

A simpler (less accurate) approximation

$$\epsilon_r' \approx \begin{cases} \epsilon_r & h \ll w \\ \dfrac{\epsilon_r + 1}{2} & h \gg w \end{cases}$$

$$Z_c = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left(\frac{5.98h}{0.8w + t}\right)$$

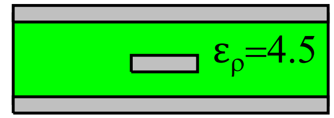
Then, Velocity of propagation $v = c/\sqrt{\epsilon_r'}$

per-unit-length parameters $l = \frac{Z_c}{v}, c = \frac{1}{vZ_c}$

Time delay per unit length $T_d = \frac{1}{v} = \sqrt{\epsilon_r'}/c$

*Cohn, Problems in strip TL, 1955

Tables for Z_0 and Td (Stripline)



W $\downarrow\downarrow \longrightarrow$

$Z_0(\Omega)$	20	28	40	50	75	90	100
$\varepsilon_{eff} = \varepsilon_r$	4.5	4.5	4.5	4.5	4.5	4.5	4.5
L (nH/mm)	0.141	0.198	0.282	0.353	0.53	0.636	0.707
C (pF/mm)	0.354	0.252	0.171	0.141	0.094	0.078	0.071
$\frac{\sqrt{\varepsilon_r}}{c} =$ Td (ps/mm)	7.09	7.09	7.09	7.09	7.09	7.09	7.09

const. (ε_{eff})

Const. (Td)

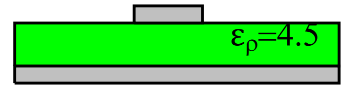
$Z_c = \sqrt{\frac{L}{C}} \uparrow\uparrow$

$L \uparrow\uparrow$

$C \downarrow\downarrow$

Fr4 : dielectric constant = 4.5
Frequency: 1GHz

Tables for Z_0 and Td (Microstrip Line)



$Z_0(\Omega)$	20	28	40	50	75	90	100
ε_{eff} $\Downarrow$.	3.8	3.68	3.51	3.39	3.21	3.13	3.09
L (nH/mm)	0.119	0.183	0.246	0.320	0.468	0.538	0.591
C (pF/mm)	0.299	0.233	0.154	0.128	0.083	0.067	0.059
T_d (ps/mm) $\Downarrow$.	6.54	6.41	6.25	6.17	5.99	5.92	5.88

$Z_0 \Uparrow = \sqrt{\frac{L}{C}}.$

$LC = \mu \varepsilon_{eff}$

$L \Uparrow$

$C \Downarrow$

$W \Downarrow$ or $h \Uparrow$.

$\frac{\sqrt{\varepsilon_{eff}}}{c} = T_d$.

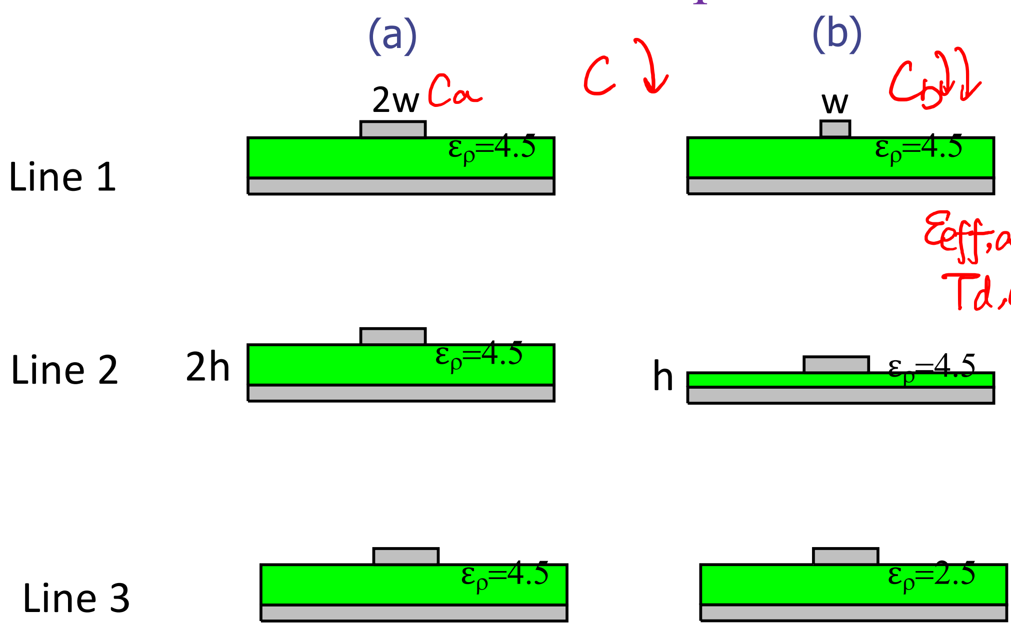
Fr4 : dielectric constant = 4.5
Frequency: 1GHz

Analysis approach for Z_o and T_d (EDA/Simulation Tool)

1. Cadence AWR Microwave Office
   - Free academic access available to students
   - RF/microwave circuit, transmission-line, and layout design
2. Ansys Electronics Desktop (HFSS)
   - Free Student version; EWS access may also be available
   - 3D full-wave electromagnetic field solver
3. SIMULIA CST Studio Suite
   - Free Learning Edition available to students
   - 3D electromagnetic simulation with multiple solver types
4. Other commercial EDA / EM simulation tools

Concept Test for Planar Transmission Lines

- Poll: Please compare their Z_0 and T_d



C_a $C \downarrow$ $C_b \downarrow < C_a$.

$Z_a < Z_b$.

$\varepsilon_{eff,a} > \varepsilon_{eff,b}$

$T_{d,a} > T_{d,b}$.

$\varepsilon_{eff,a} < \varepsilon_{eff,b}$.

$T_{d,a} < T_{d,b}$.

$C_a < C_b$. $C_{o,a} < C_{o,b}$.

$Z_{oa} > Z_{o,b}$.

$\varepsilon_{eff,a} > \varepsilon_{eff,b}$. $T_{d,a} > T_{d,b}$.

$C_a > C_b$. C_o same. $Z_{oa} < Z_{o,b}$.

$\varepsilon_{eff} = \frac{C}{C_o}$.

$$C = \epsilon \frac{A}{d}$$

$$Z_0 = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_e}}{cC} = \frac{1}{c\sqrt{CC_o}}.$$

$$T_d = \frac{\sqrt{\epsilon_e}}{c}$$

c: speed of light
C: Capacitance