ILLINOIS
Electrical & Computer Engineering
GRAINGER COLLEGE OF ENGINEERING

Spectra of Digital Waveform

Time-domain and frequency-domain Spectrum

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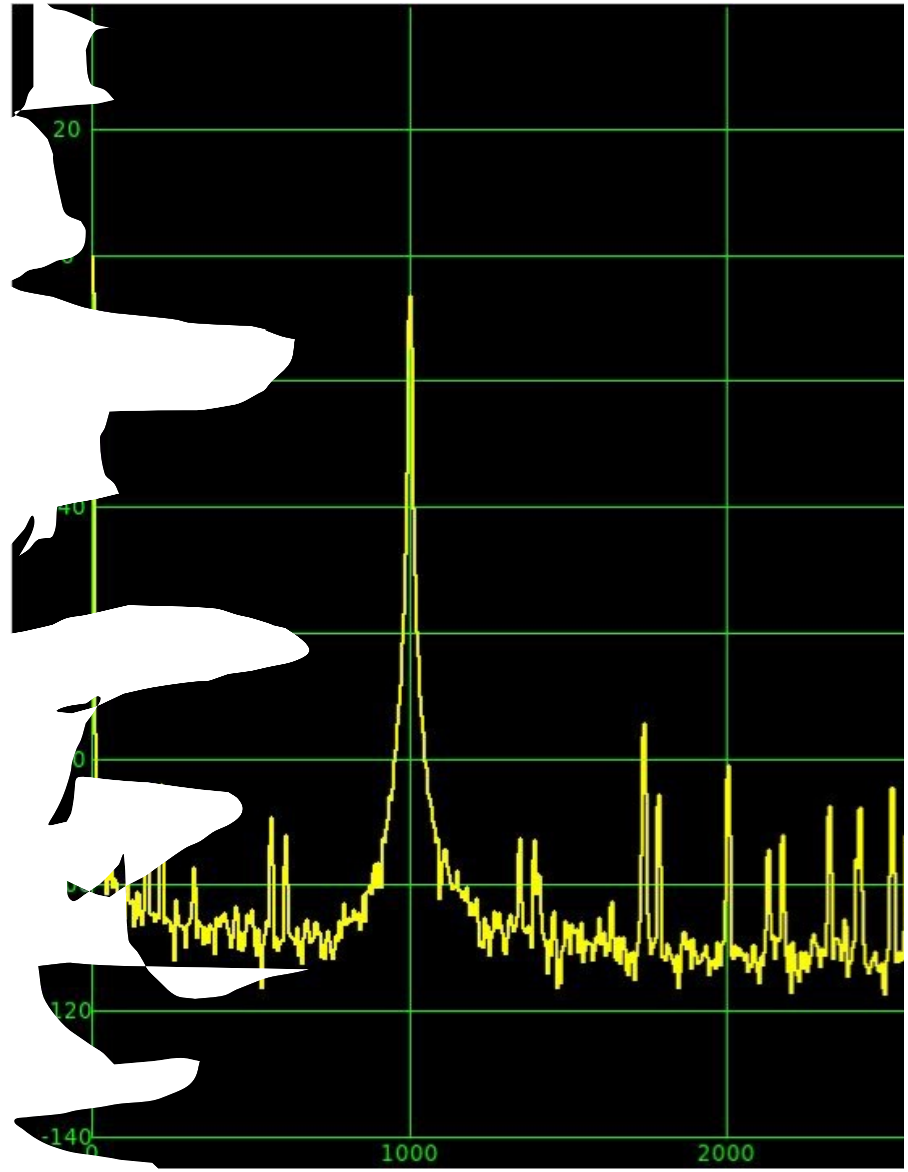
Outline

- Basic Properties of Fourier Transform
- Spectra of digital circuit waveforms
- Spectral bounds for trapezoidal waveforms
  - Effect of rise/fall time on Spectral Content
  - Effect of Repetition rate
  - Effect of Duty Cycle
  - Effect of Ringing (Undershoot/Overshoot)
- Spectrum Analyzer



2.1 Basic Properties of Fourier Transform

The more important signals that contribute directly to the radiated and conducted emissions of digital electronic systems are periodic signals (clock and data signals).

Periodic signal $x(t) = x(t + nT) \qquad n = 0, \pm 1, \pm 2, \cdots$

Periodic Signal = Time-domain signals or waveforms that occur repetitively

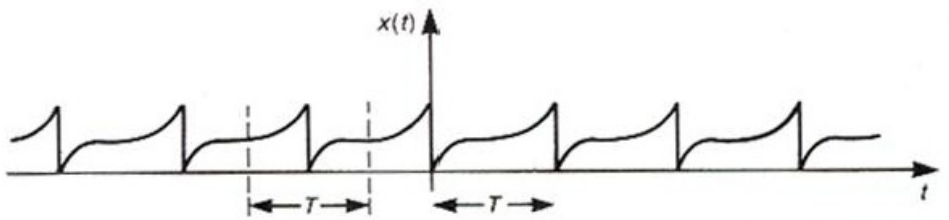


FIGURE 7.1 A periodic signal with period T.

T: repetition period

$f_0 = \dfrac{1}{T}$: fundamental frequency [Hz]

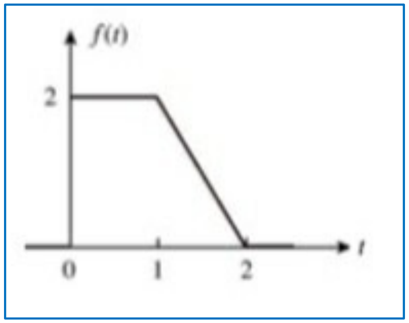
$\omega_o = 2\pi f_0 = \frac{2\pi}{T}$ [rad/s]

$P_{av} = \dfrac{1}{T}\displaystyle\int_{t_1}^{t_1+T} x^2(t)\,dt$ average power (periodic)

Periodic vs Nonperiodic Signals

Signals that are not periodic are referred to as nonperiodic

Energy (nonperiodic) $$E = \int_{-\infty}^{\infty} x^2(t)dt$$



Periodic signals have:
- **infinite energy**
- **finite average power**

Periodic signals are referred to as **power signals**

Nonperiodic signals have:
- **finite energy**
- **zero average power**

Nonperiodic signals are referred to as **energy signals**

Representation of Periodic Signals

Periodic signals can be represented as **linear combination** of **basis functions** (i.e., more basic signals) as

Table from ECE 210

$f(t)$, period $T = 2\pi/\omega_0$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$	Exponential	$F_n = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)e^{-jn\omega_0 t}dt$
$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n\rvert$ $\theta_n = \angle F_n$

Exponential form

$$x(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$F_n = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)e^{-jn\omega_0 t}dt$$

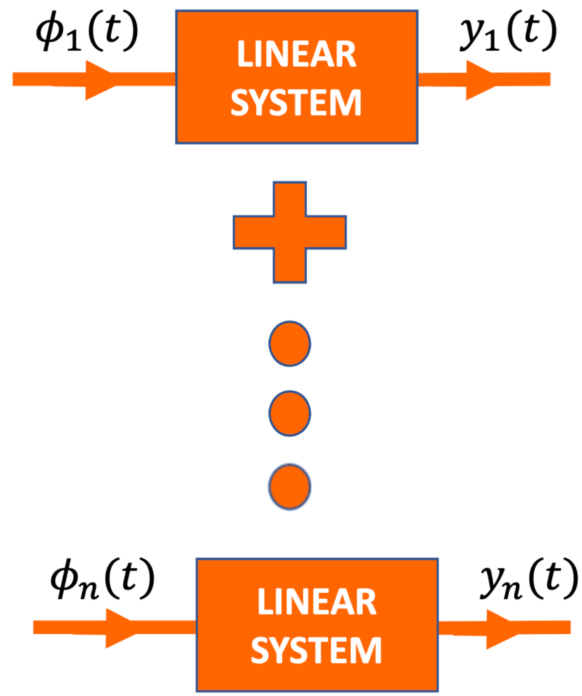
$\phi_n(t) = e^{jn\omega_0 t}$: basis functions, periodic with the same period as x(t)

c_n: expansion coefficients

Advantage

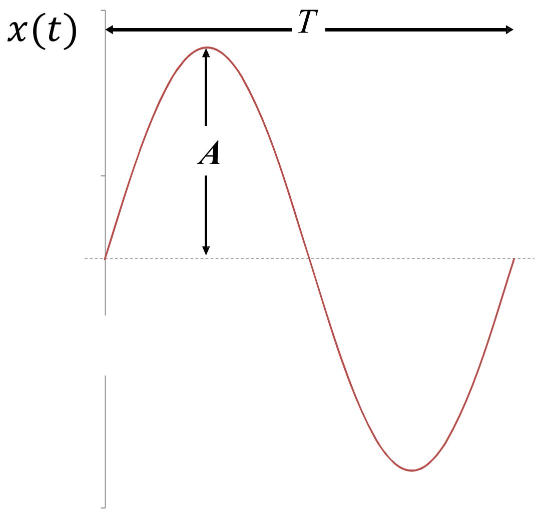
The response to the original input signal can be found, by **superposition**, as the **sum of the weighted responses** to the individual components or basis functions that are used to represent the original input signal.

$$y(t) = \sum_{n=0}^{\infty} c_n y_n(t)$$

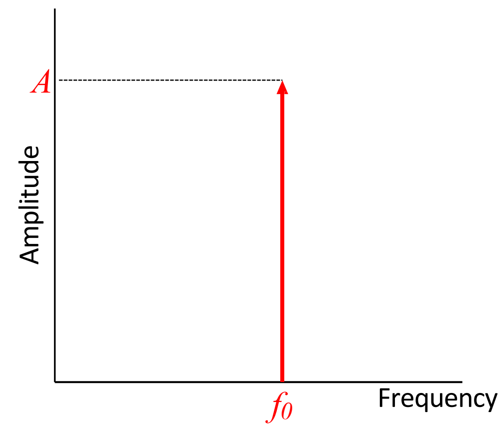


Sinusoidal Functions

TIME DOMAIN



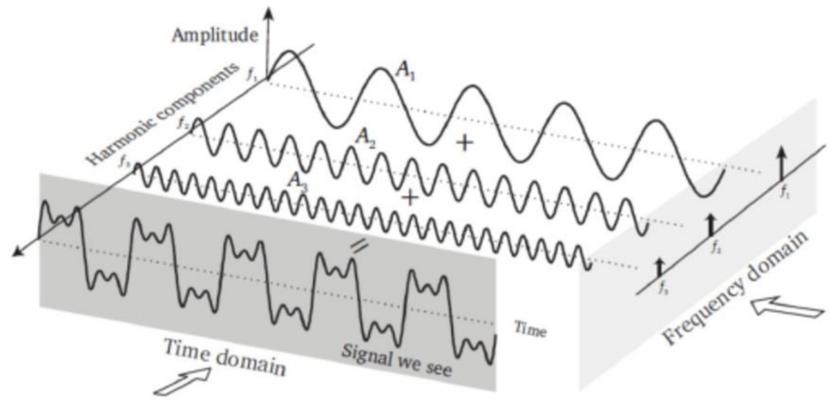
FREQUENCY DOMAIN



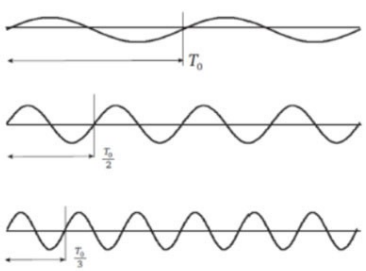
Sinusoidal functions are the basic blocks of the **Fourier series representation** of a periodic signal

Fourier Series Representation

Intuitive Visualization



Makes use of sinusoids as basis functions



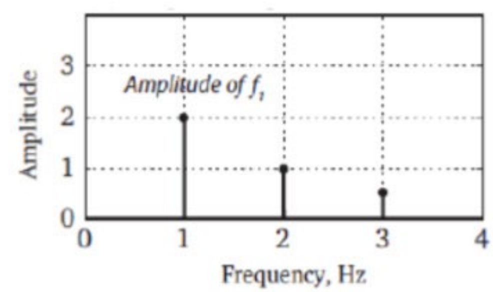
Fundamental $f_0 = \frac{1}{T}$

2nd harmonic $2f_0$

3rd harmonic $3f_0$

.
.
.

Signal Spectrum



One-sided spectrum

spectral lines separated by $\Delta f = 1/T$

Periodic Square Wave Pulse Train

$$c_n = \frac{1}{T}\int_{t_1}^{t_1+T} e^{-jn\omega_0 t} x(t)\,dt$$

$$= \frac{1}{T}\int_{0}^{\tau} e^{-jn\omega_0 t} A\,dt + \frac{1}{T}\int_{\tau}^{T} e^{-jn\omega_0 t} \times 0\,dt$$

$$= \frac{A}{jn\omega_0 T}(1 - e^{-jn\omega_0 \tau})$$

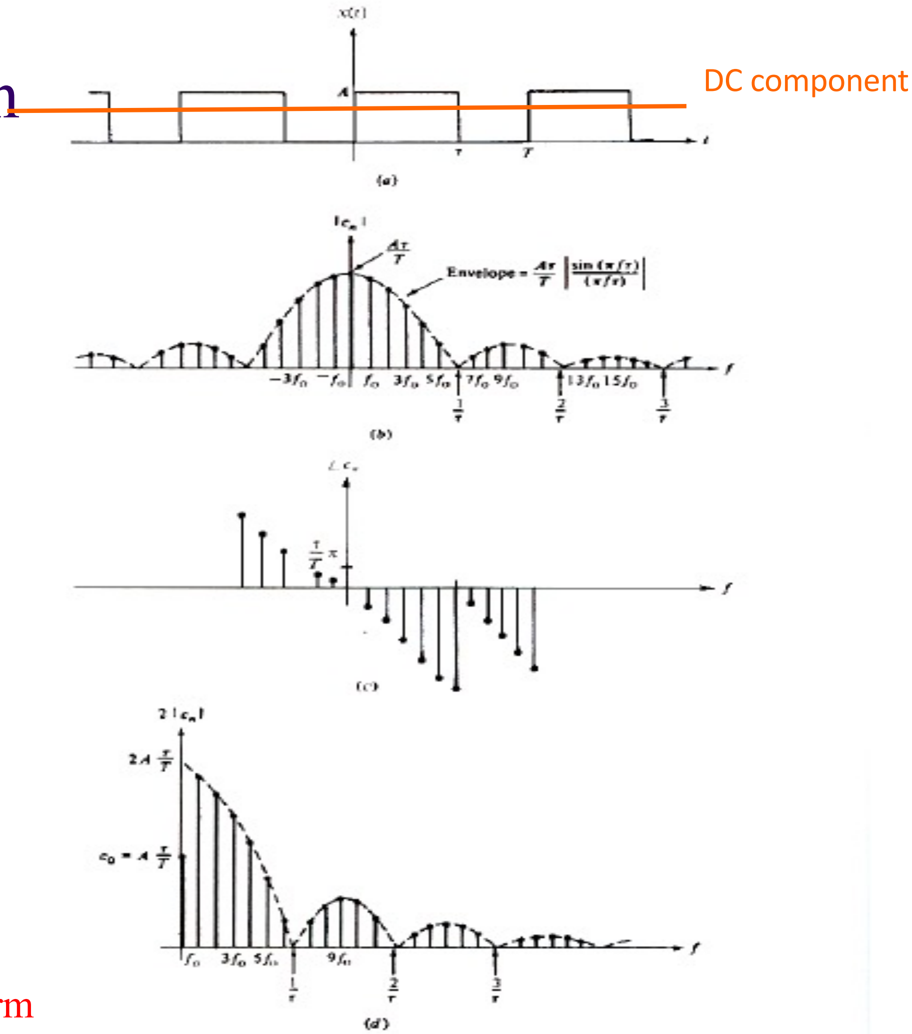
$$c_n = \frac{A}{jn\omega_0 T} e^{-jn\omega_0 \tau/2}(e^{jn\omega_0 \tau/2} - e^{-jn\omega_0 \tau/2})$$

$$= \frac{A}{jn\omega_0 T} e^{-jn\omega_0 \tau/2} 2j \sin\left(\tfrac{1}{2} n\omega_0 \tau\right)$$

$$\boxed{= \frac{A\tau}{T} e^{-jn\omega_0 \tau/2} \frac{\sin\left(\frac{1}{2} n\omega_0 \tau\right)}{\frac{1}{2} n\omega_0 \tau}}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

Complex exponential form
(double sided spectrum)



Frequency-domain representation of a square wave: (a) the signal; (b) the two-sided magnitude spectrum; (c) the phase spectrum; (d) the one-sided magnitude spectrum [1].

Square Wave Compact Form

$$x(t) = c_0 + \sum_{n=1}^{\infty} 2|c_n|\cos(n\omega_0 t + \angle c_n)$$

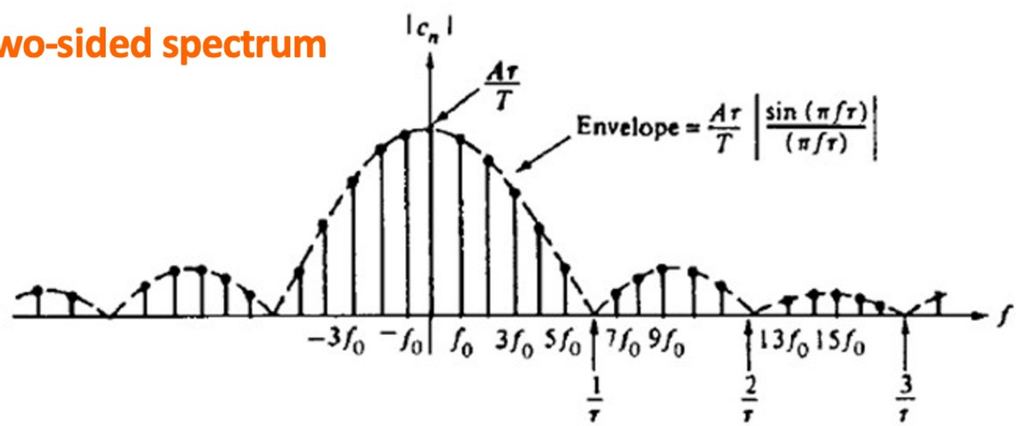
$$|c_n| = \frac{A\tau}{T}\left|\frac{\sin\left(\frac{1}{2}n\omega_0\tau\right)}{\frac{1}{2}n\omega_0\tau}\right|$$

$$\angle c_n = \pm\frac{1}{2}n\omega_0\tau$$

➢The expansion coefficients for the one-sided spectrum (positive frequencies only) = $2|c_n|$

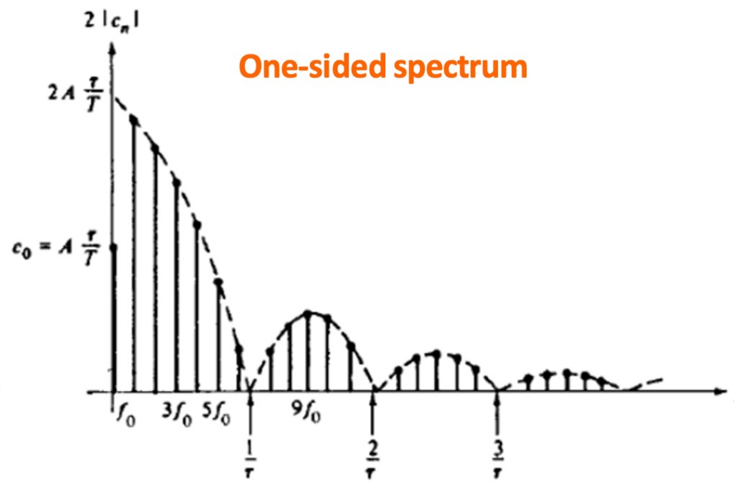
➢The dc component c_0 remains unchanged

Two-sided spectrum



Complex Exponential Form

One-sided spectrum



Trigonometric Form

Square Wave: Spectrum Envelope

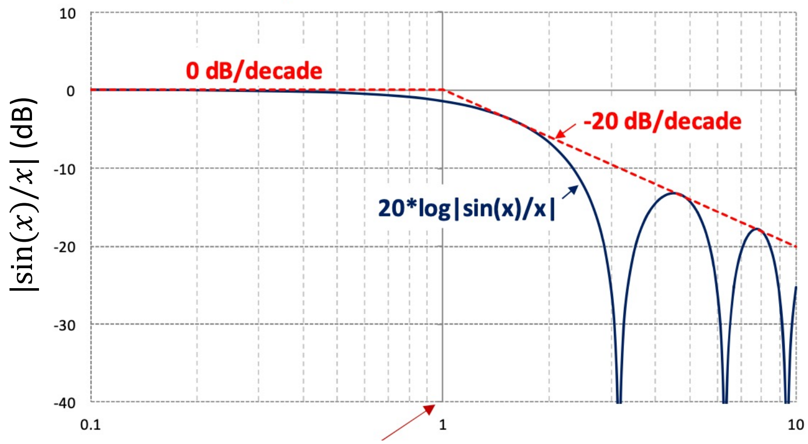
$$\omega_0 = \frac{2\pi}{T} = 2\pi f \Rightarrow \textit{Spectrum Envelope} = \frac{A\tau}{T}\left|\frac{\sin(\pi f\tau)}{\pi f\tau}\right|$$

Spectral Bound

$$\left|\frac{\sin(x)}{x}\right| = \begin{cases} 1 & for\ x \to 0 \\ 0 & for\ x = n\pi\ and\ n = 1,2,\ldots \\ \frac{1}{|x|} & for\ x = \frac{(2n+1)\pi}{2} \end{cases}$$

$$x = \pi f\tau$$

Spectral bound - Bode Plot

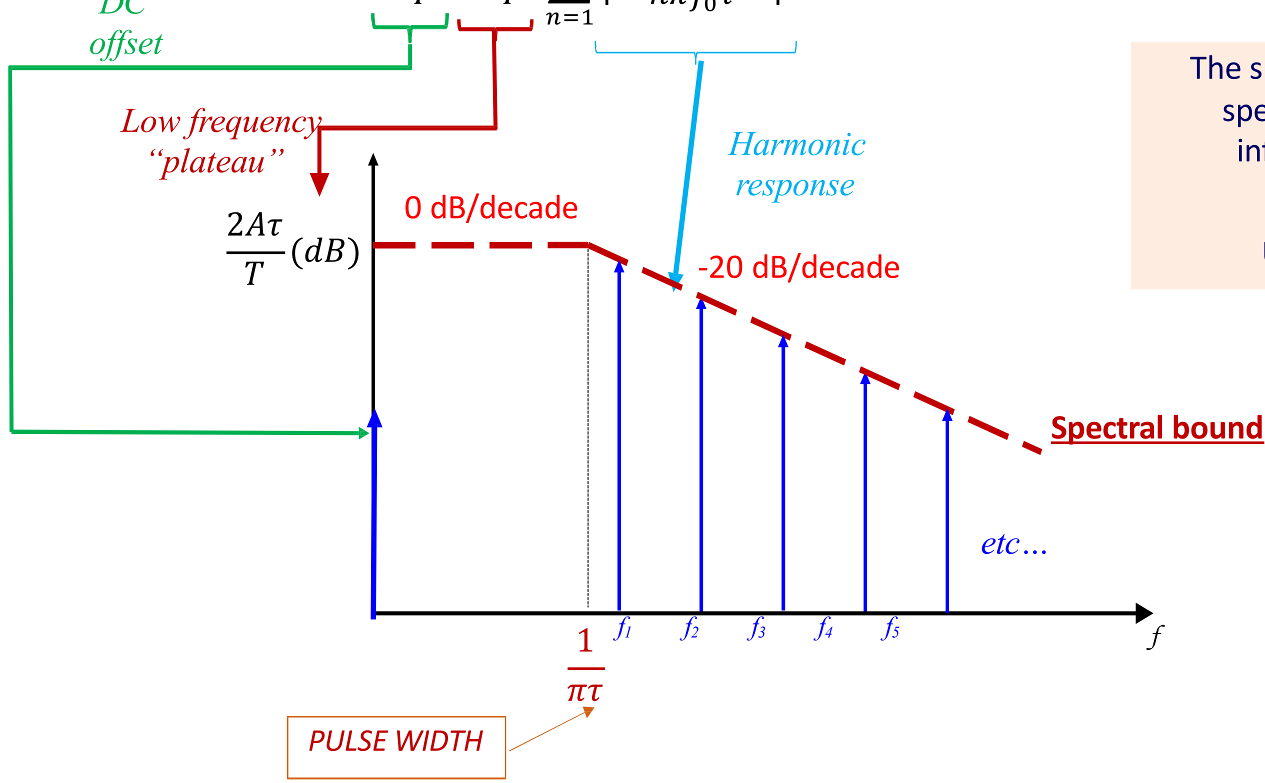


$x = 1 \Rightarrow f = 1/(\pi\tau)$

(break frequency is determined by pulse width)

Spectral Bound of a Square Wave Pulse Train

$$x(t) = \frac{A\tau}{T} + \frac{2A\tau}{T}\sum_{n=1}^{\infty}\left|\frac{\sin(n\pi f_0 \tau)}{n\pi f_0 \tau}\right|\cos(2\pi n f_0 t - \pi n f_0 \tau)$$



The spectral bound on the magnitude spectrum provides more intuitive information of spectral content

Upper Bound (Worst case)

Basic Properties of Fourier Transform

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)e^{-jn\omega_o t}dt$$

1. Linear decomposition

$$F(x_1(t) + x_2(t)) = F(x_1(t)) + F(x_2(t))$$

2. Hermitian for real $x(t)$ $\quad C_n = C_{-n}^*$

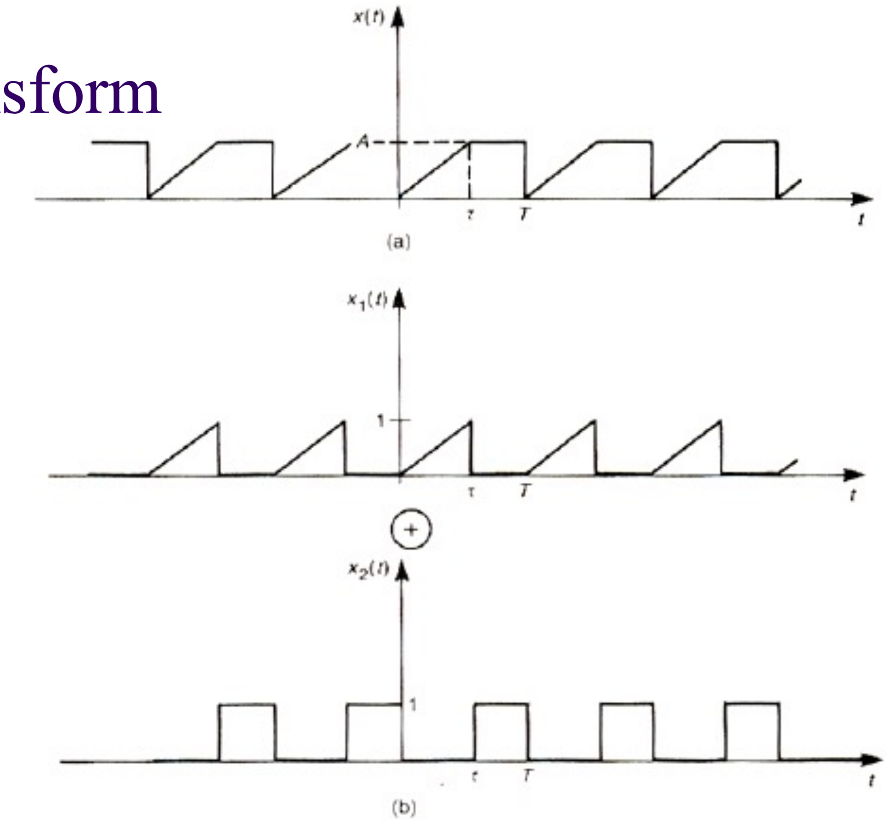


Illustration of the principle of linear decomposition of a signal. (a) x(t); (b) $x(t)=Ax_1(t)+Ax_2(t)$

Basic Properties of Fourier Transform

3. Time shift

$$x(t - t_o) \leftrightarrow C_n e^{-jn\omega_o t_o}$$

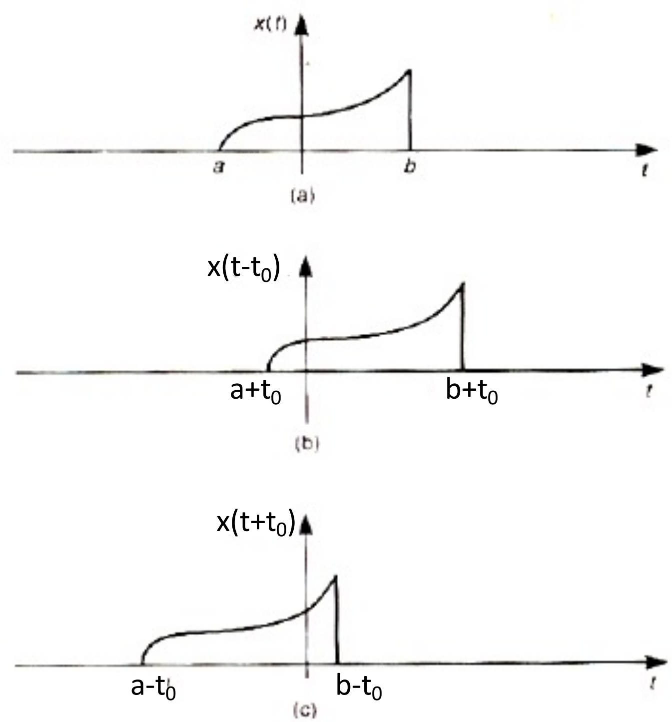


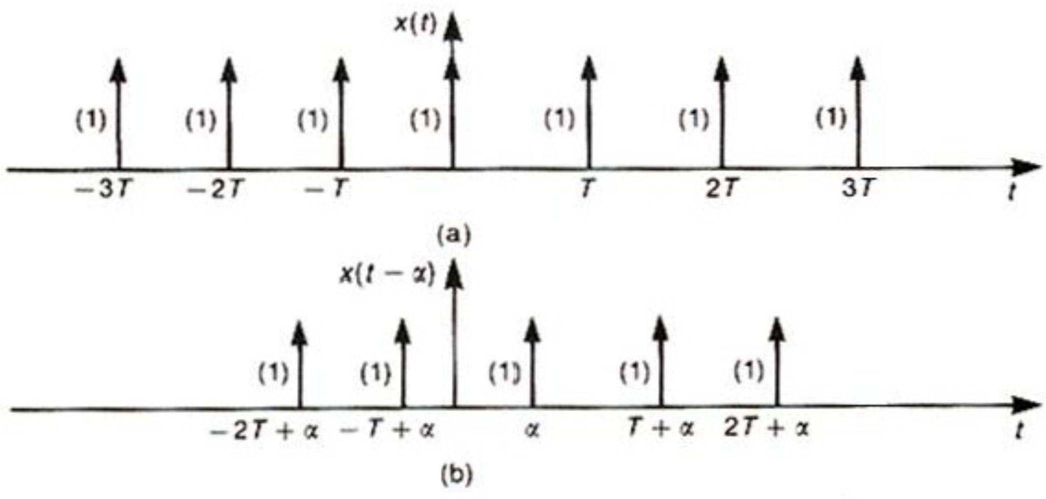
Illustration of the time-shift principle. (a) $x(t)$; (b) $x(t-t_0)$; (c) $x(t+t_0)$

Basic Properties of Fourier Transform

4. Impulse response

$$F\left(\delta(t \pm kT)\right) \Rightarrow C_n = \frac{1}{T} \qquad n = 0, \pm 1, \pm 2, \cdots$$

$$\delta(t) = \begin{cases} 0 & \text{for } t < 0 \\ 0 & \text{for } t > 0 \\ \int_{0^-}^{0^+} \delta(t)\, dt = 1 \end{cases}$$



A periodic train of unit impulses (a) and (b) shifted in time.

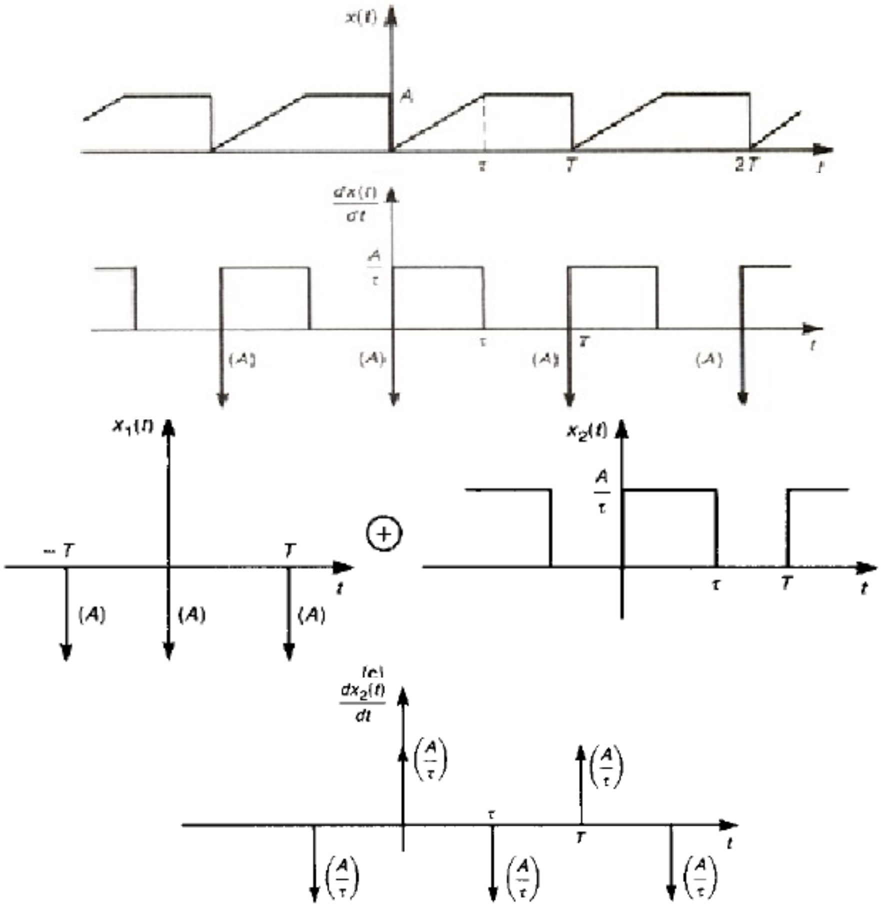
Basic Properties of Fourier Transform

5. Derivative $F\left(d^k x(t) / dt^k\right) \Rightarrow C_n^{(k)} = (jn\omega_0)^k C_n$

Basic Properties of Fourier Transform

$$F\left(d^k x(t)/dt^k\right) \Rightarrow C_n^{(k)} = (jn\omega_0)^k C_n$$

Example



$$c_{1n}^{(1)} = -\frac{A}{T}e^{-jn\omega_0 T} = -\frac{A}{T}$$

$$c_{2n}^{(2)} = \frac{A}{\tau}\frac{1}{T} - \frac{A}{\tau}\frac{1}{T}e^{-jn\omega_0\tau}$$

$$c_n = \frac{1}{jn\omega_0}c_{1n}^{(1)} + \frac{1}{(jn\omega_0)^2}c_{2n}^{(2)} \quad n \neq 0$$

$$= -\frac{1}{jn\omega_0}\frac{A}{T}e^{-jn\omega_0 T} + \frac{1}{(jn\omega_0)^2}\left(\frac{A}{\tau}\frac{1}{T} - \frac{A}{\tau}\frac{1}{T}e^{-jn\omega_0\tau}\right)$$

$$= j\frac{A}{n\omega_0 T}\left[e^{-jn\omega_0 T} + j\frac{1}{n\omega_0\tau}(1 - e^{-jn\omega_0\tau})\right]$$

$$= j\frac{A}{n\omega_0 T}e^{-jn\omega_0 T} - j\frac{A}{n\omega_0 T}\frac{\sin\left(\frac{1}{2}n\omega_0\tau\right)}{\frac{1}{2}n\omega_0\tau}e^{-jn\omega_0\tau/2}$$

$$= j\frac{A}{2\pi n}\left[1 - e^{-j\frac{n\pi\tau}{T}}\frac{\sin(n\pi\tau/T)}{n\pi\tau/T}\right]$$

Spectra of Digital Clock Signals

- It should be mentioned that, in general, data signals are nondeterministic/random signals, otherwise no information would be conveyed
- Anyway, we will analyze the frequency-domain description of **periodic, deterministic waveforms** (i.e., digital clock signals) to give insight into the spectral composition of data signals (at least, at some degree)

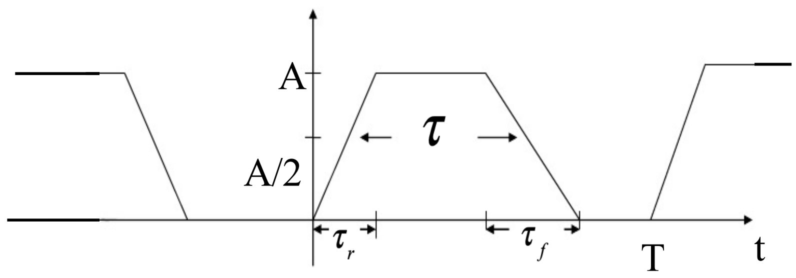
2.2 Spectra of digital circuit waveforms

The periodic, trapezoidal pulse train representing clock and data signals of digital systems.

Key Parameters

- period T (fundamental frequency $f0=1/T$)
- amplitude A
- rise time τ_r
- fall time τ_f
- pulse width τ

•NOTE: τ_r and τ_f are generally measured between 10% and 90% of the minimum and maximum values of the waveform



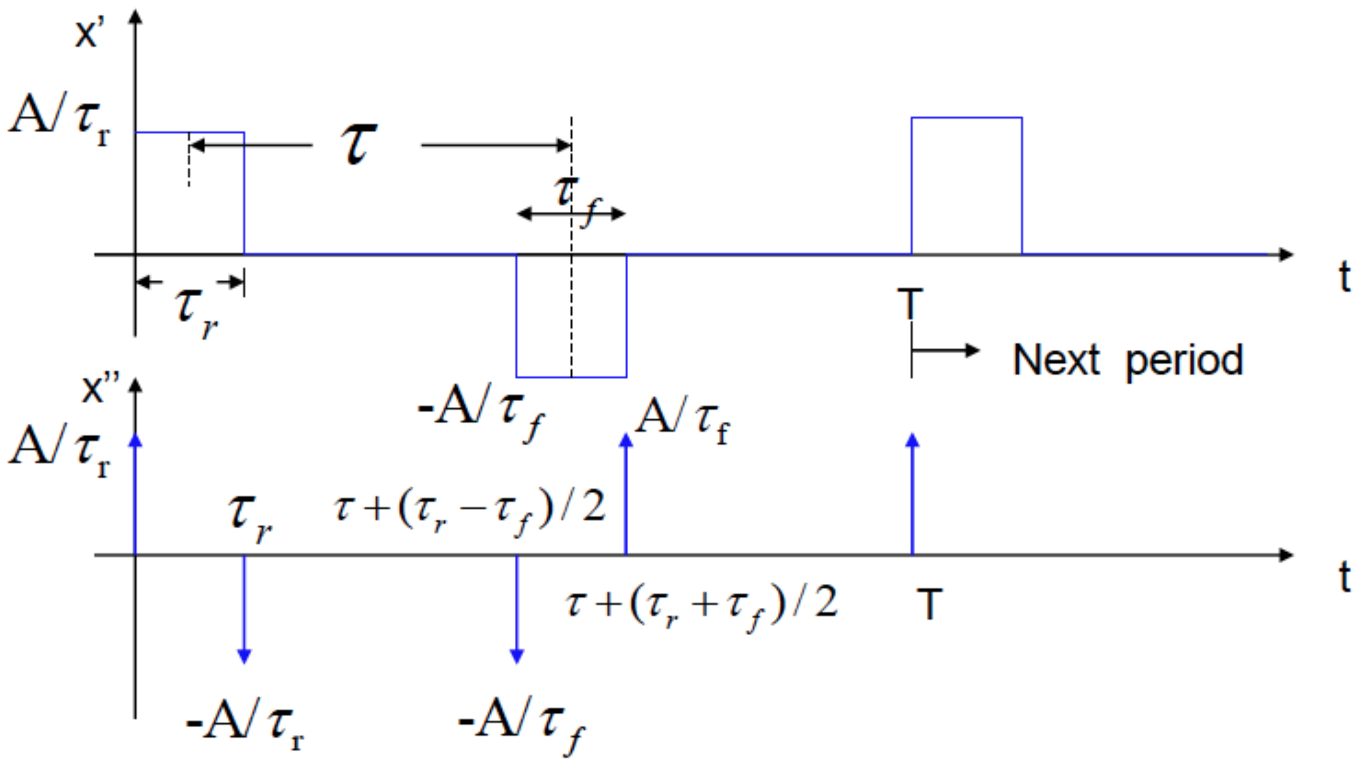
GOAL

Investigate the effect of these parameters on the spectrum of clock waveforms

Spectra of digital circuit waveforms

To obtain the complex exponential Fourier series of this waveform

(1)



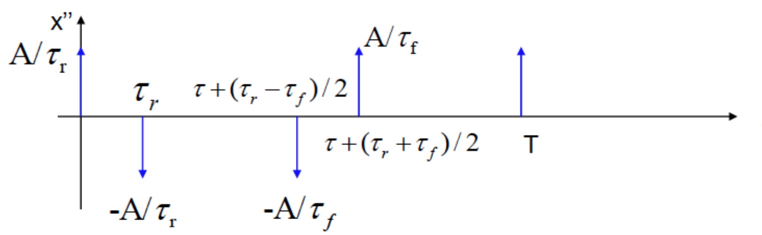
(2)

$$c_n^{(2)} = \frac{1}{T}\frac{A}{\tau_r} - \frac{1}{T}\frac{A}{\tau_r}e^{-jn\omega_0\tau_r} - \frac{1}{T}\frac{A}{\tau_f}e^{-jn\omega_0[\tau+(\tau_r-\tau_f)/2]} + \frac{1}{T}\frac{A}{\tau_f}e^{-jn\omega_0[\tau+(\tau_r+\tau_f)/2]}$$

$$= \frac{A}{T}\left[\frac{1}{\tau_r}e^{-jn\omega_0\tau_r/2}(e^{jn\omega_0\tau_r/2} - e^{-jn\omega_0\tau_r/2}) - \frac{1}{\tau_f}e^{-jn\omega_0\tau_r/2}e^{-jn\omega_0\tau}(e^{jn\omega_0\tau_f/2} - e^{-jn\omega_0\tau_f/2})\right]$$

$$= j\frac{A}{2\pi n}(n\omega_0)^2 e^{-jn\omega_0(\tau+\tau_r)/2}\left[\frac{\sin(\frac{1}{2}n\omega_0\tau_r)}{\frac{1}{2}n\omega_0\tau_r}e^{jn\omega_0\tau/2} - \frac{\sin(\frac{1}{2}n\omega_0\tau_f)}{\frac{1}{2}n\omega_0\tau_f}e^{-jn\omega_0\tau/2}\right]$$

$$F(x) = F(x'')/(jn\omega_0)^2 \quad , \quad n \neq 0$$



$$c_n = \frac{1}{(jn\omega_0)^2}c_n^{(2)} \quad n \neq 0 \quad = -\frac{c_n^{(2)}}{(n\omega_0)^2}$$

$$= -j\frac{A}{2\pi n}e^{-jn\omega_0(\tau+\tau_r)/2}\left(\frac{\sin(\frac{1}{2}n\omega_0\tau_r)}{\frac{1}{2}n\omega_0\tau_r}e^{jn\omega_0\tau/2} - \frac{\sin(\frac{1}{2}n\omega_0\tau_f)}{\frac{1}{2}n\omega_0\tau_f}e^{-jn\omega_0\tau/2}\right)$$

(3)

$$if \quad \boxed{\tau_r = \tau_f}$$

$$C_n = A\frac{\tau}{T}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}\frac{\sin(n\omega_0\tau_r/2)}{(n\omega_0\tau_r/2)}e^{-jn\omega_0(\tau+\tau_r)/2}$$

Complex Exponential Form (one-sided spectrum)

$$x(t) = \frac{A\tau}{T} + \frac{2A\tau}{T}\sum_{n=1}^{\infty}\left|\frac{\sin\left(\frac{n\omega_o\tau}{2}\right)}{\frac{n\omega_o\tau}{2}}\right|\left|\frac{\sin\left(\frac{n\omega_o\tau_r}{2}\right)}{\frac{n\omega_o\tau_r}{2}}\right| e_0^{-jn\omega(\tau+\tau_r)/2}e^{-jn\omega_0 t}$$

One-side Spectrum (Compact Form)

$$x(t) = C_0 + \sum_{n=1}^{\infty} \left|C_n^+\right| \cos(n\omega_0 t + \angle C_n)$$

When $|C_n^+| = 2|C_n|$ for $n \neq 0$ with $\tau_r = \tau_f, c_0 = \frac{A\tau}{T}, and\ \omega_0 = 2\pi f_0$

Compact Form

$$x(t) = \frac{A\tau}{T} + \frac{2A\tau}{T} \sum_{n=1}^{\infty} \underbrace{\left|\frac{\sin(\pi n f_o \tau)}{\pi n f_o \tau}\right| \left|\frac{\sin(\pi n f_o \tau_r)}{\pi n f_o \tau_r}\right|} \cos(2\pi n f_0 t - \pi n f_o(\tau - \tau_r))$$

Position: $nf_o \to f$ Envelope= $\frac{2A\tau}{T} \left|\frac{\sin(\pi n f_o \tau)}{\pi n f_o \tau}\right| \left|\frac{\sin(\pi n f_o \tau_r)}{\pi n f_o \tau_r}\right|$

Note : (1) $\boxed{\text{50\% duty cycle}} \Leftrightarrow \tau / T = 1/2 \Leftrightarrow$ no even harmonics

$$|c_n| = \frac{A}{2}\left|\frac{\sin(n\pi/2)}{n\pi/2}\right| \qquad \frac{\tau}{T} = \frac{1}{2}$$
$$= \frac{A}{n\pi} \qquad n = 1, 3, 5, \ldots$$
$$= 0 \qquad n = 2, 4, 6, \ldots$$

$$\angle c_n = \angle \frac{-n\omega_0\tau}{2} + \angle \sin\left(\tfrac{1}{2}n\omega_0\tau\right)$$
$$= \angle \frac{-n\pi}{2} + \angle \sin\left(\frac{n\pi}{2}\right)$$
$$= -90^\circ \qquad n = 1, 3, 5, \ldots$$

Compact Fourier series

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}\cos(\omega_0 t - 90^\circ) + \frac{2A}{3\pi}\cos(3\omega_0 t - 90^\circ) + \cdots$$
$$= \frac{A}{2} + \frac{2A}{\pi}\sin(\omega_0 t) + \frac{2A}{3\pi}\sin(3\omega_0 t) + \cdots$$

Even harmonics, n=2, 4, 6, ..., are zero

(2) slight variations from an 50% $\Rightarrow$ even harmonics varywidely

2.3 Spectral bounds for trapezoidal waveforms

Assumption: $\tau_r = \tau_f$

$$\text{Envelope} = \frac{2A\tau}{T}\left|\frac{\sin\left(\frac{n\pi\tau}{T}\right)}{\frac{n\pi\tau}{T}}\right|\left|\frac{\sin\left(\frac{n\pi\tau_r}{T}\right)}{\frac{n\pi\tau_r}{T}}\right|$$

Bode Plot

Spectral Bound

$$20\log_{10}(Envelope) = 20\log_{10}\left(\frac{2A\tau}{T}\right) + 20\log_{10}\left(\left|\frac{\sin(\pi n f_o \tau)}{\pi n f_o \tau}\right|\right) + 20\log_{10}\left(\left|\frac{\sin(\pi n f_o \tau_r)}{\pi n f_o \tau_r}\right|\right)$$

Plot 1

- 0 dB/decade slope
- $2A\tau/T$ level

Plot 2

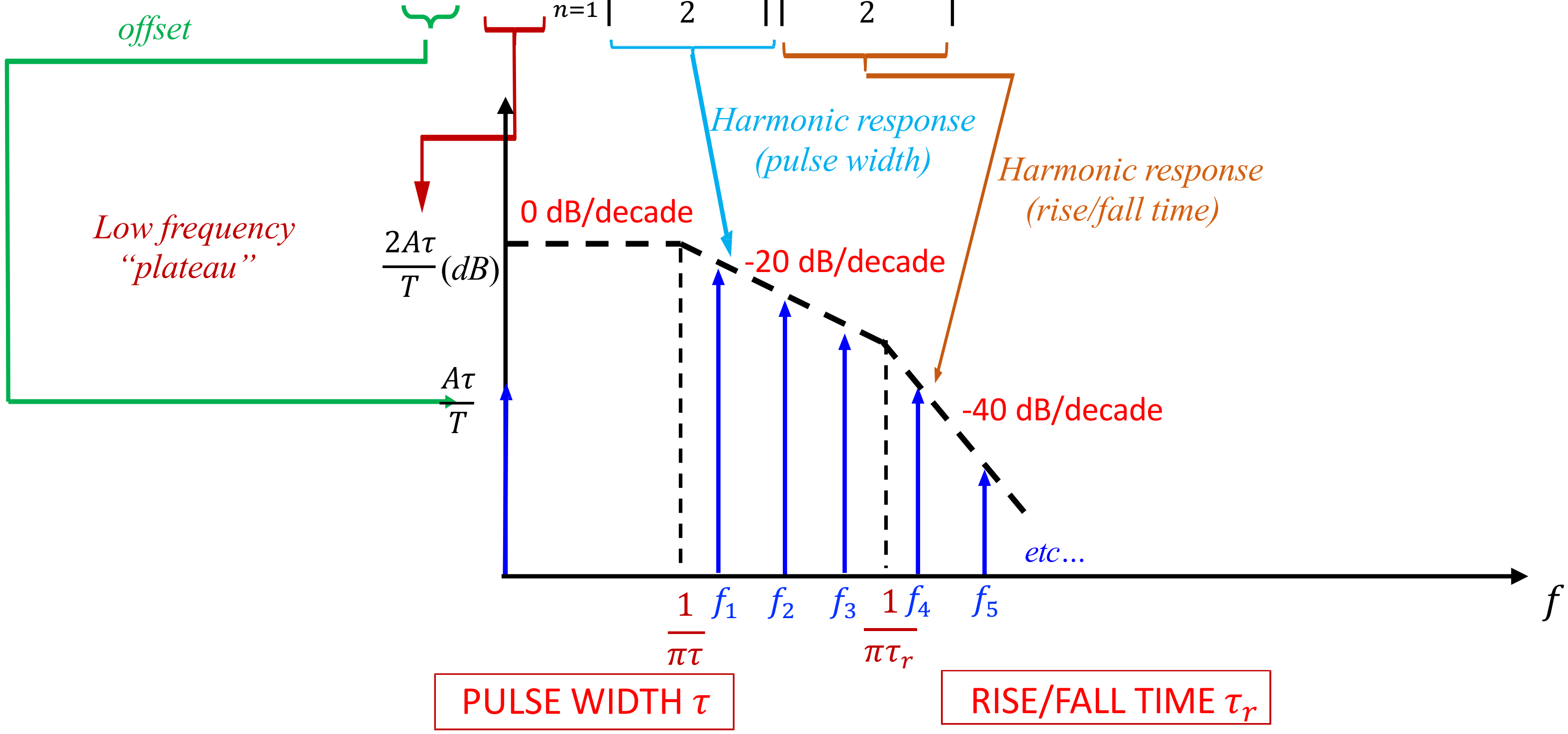
	Slope
Asymptote 1	0 dB/decade
Asymptote 2	-20 dB/decade
Break frequency	$f = 1/(\pi\tau)$

Plot 3

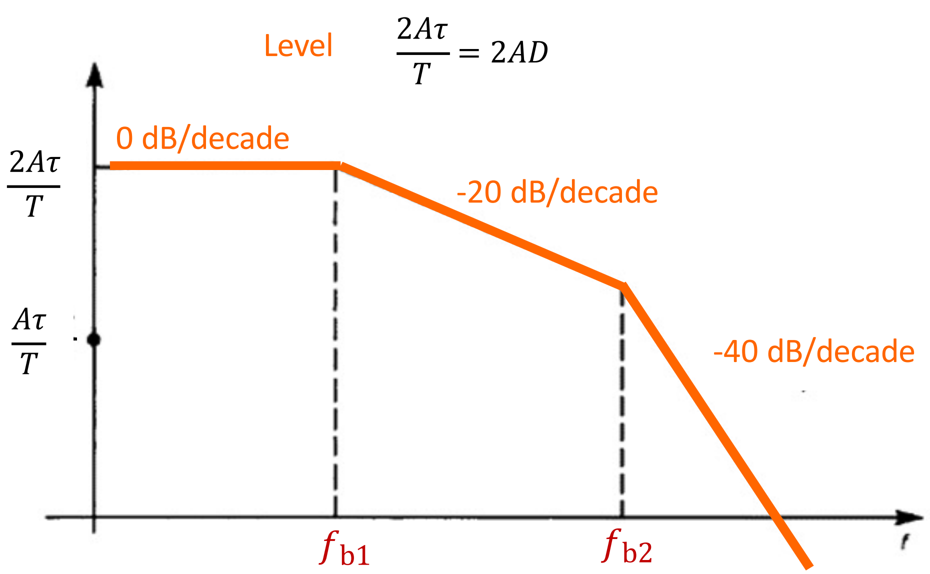
	Slope
Asymptote 1	0 dB/decade
Asymptote 2	-20 dB/decade
Break frequency	$f = 1/(\pi\tau_r)$

Spectral Bound of Trapezoidal Wave Spectrum

$$x(t) = \underbrace{\frac{A\tau}{T}}_{\text{DC offset}} + \frac{2A\tau}{T}\sum_{n=1}^{\infty}\left|\frac{\sin\left(\frac{n\omega_o\tau}{2}\right)}{\frac{n\omega_o\tau}{2}}\right|\left|\frac{\sin\left(\frac{n\omega_o\tau_r}{2}\right)}{\frac{n\omega_o\tau_r}{2}}\right| e_0^{-jn\omega(\tau+\tau_r)/2} e^{-jn\omega_0 t}$$



Spectral Bound of Trapezoidal Wave Spectrum



- The high-frequency content is due primarily to the rise/fall time of the pulse
- Pulses having small rise/fall times will have larger high-frequency spectral content than will pulses having larger rise/fall times.
- "Fast" (short) rise/fall times are the primary contributors to the high-frequency spectral content and therefore the product's inability to meet the governmental regulatory requirements on **radiated** and **conducted emissions**.

First break frequency $f_{b1} = \frac{1}{\pi\tau} = \frac{f_0}{\pi D}$ → For smaller duty cycle *D*, the breaking frequency f_{b1} moves towards higher frequencies, while reducing the signal level

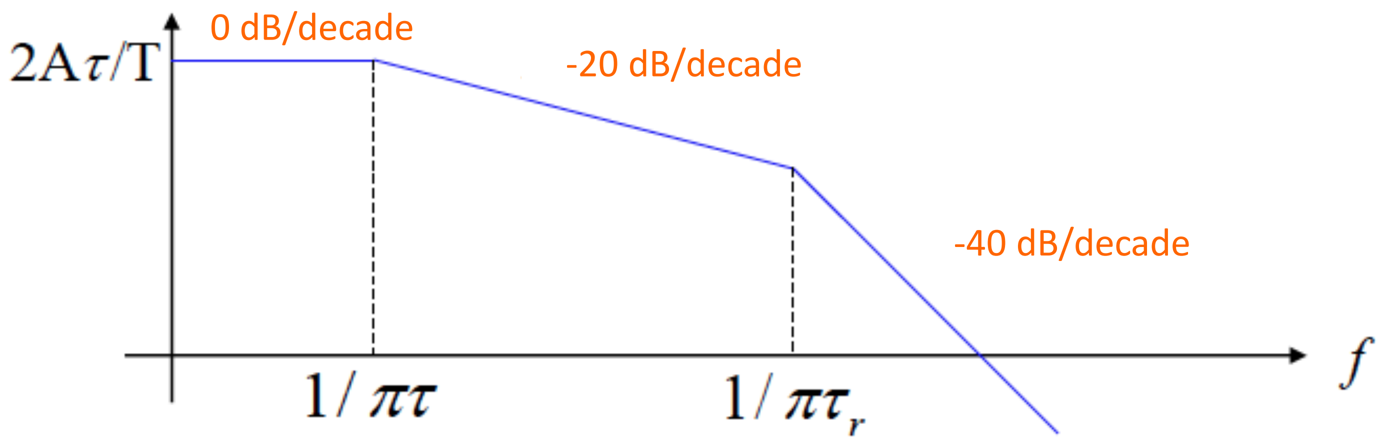
Second break frequency $f_{b2} = \frac{1}{\pi\tau_r}$ → For fast (short) rise/fall times, the breaking frequency f_{b2} moves towards higher frequencies

2.3.1 Effect of rise/fall time on Spectral Content

Trapezoidal pulse train, assume $\tau_r = \tau_f$

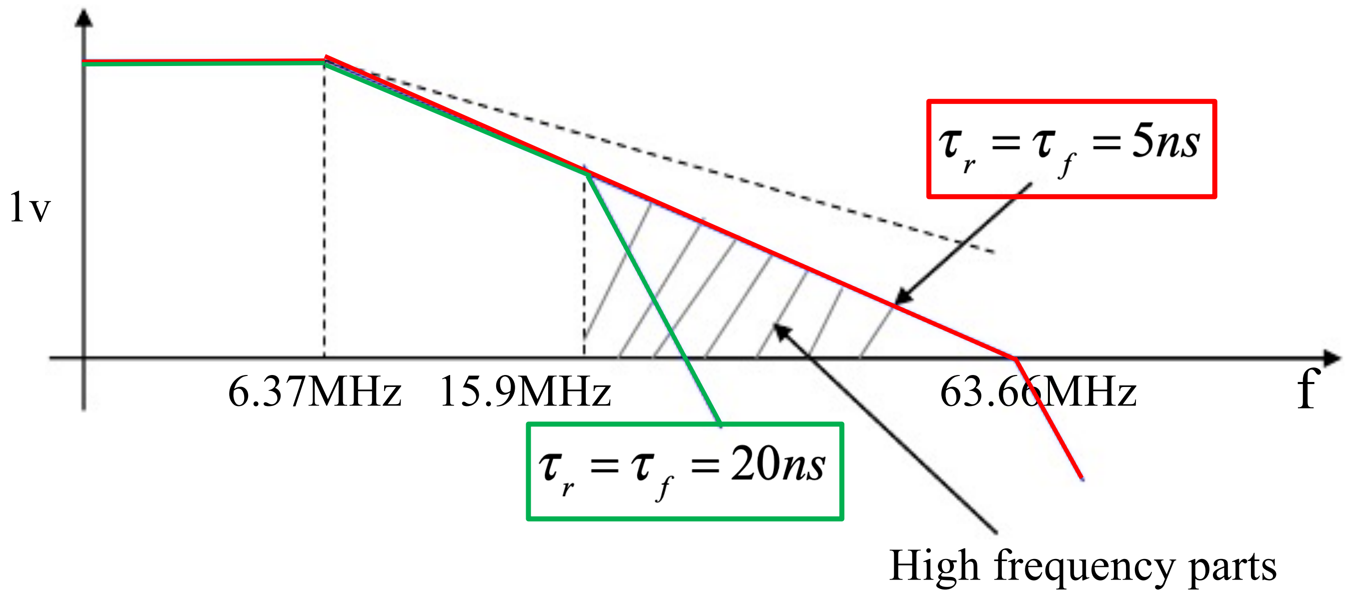
Spectral Bound

$$20\log_{10}(Envelope) = 20\log_{10}\left(\frac{2A\tau}{T}\right) + 20\log_{10}\left(\left|\frac{\sin(\pi n f_o \tau)}{\pi n f_o \tau}\right|\right) + 20\log_{10}\left(\left|\frac{\sin(\pi n f_o \tau_r)}{\pi n f_o \tau_r}\right|\right)$$



trapezoidal pulse train with equal rise- and fall times

2.3.1 Effect of rise/fall time on Spectral Content



Poll: Which pulse has more high frequency components?

A. Green curve
B. Red curve

Same T, different τ_r/τ_f

2.3.1 Effect of rise/fall time on Spectral Content

Spectral bound prediction

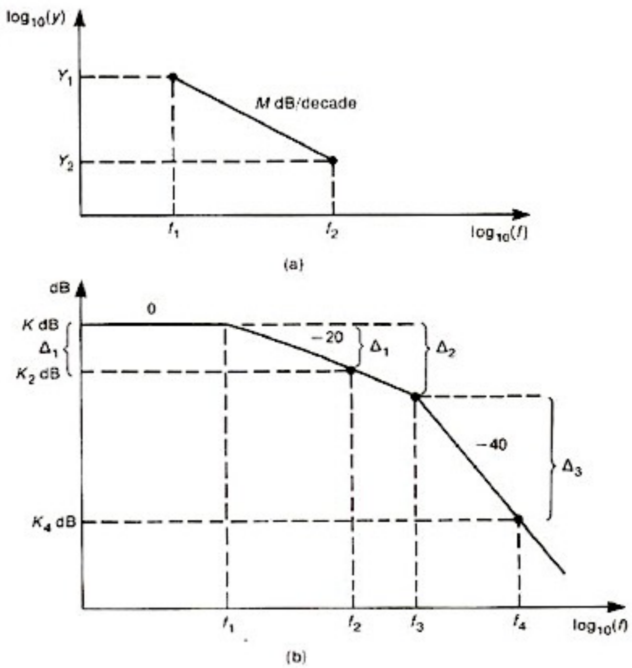


Illustration of interpolation on log-log (Bode) plots.

a) the general characterization of a linear segment

b) application to the spectral bound for a trapezoidal pulse train

$$\Delta_1 = -20\log_{10}\left(\frac{f_2}{f_1}\right)$$

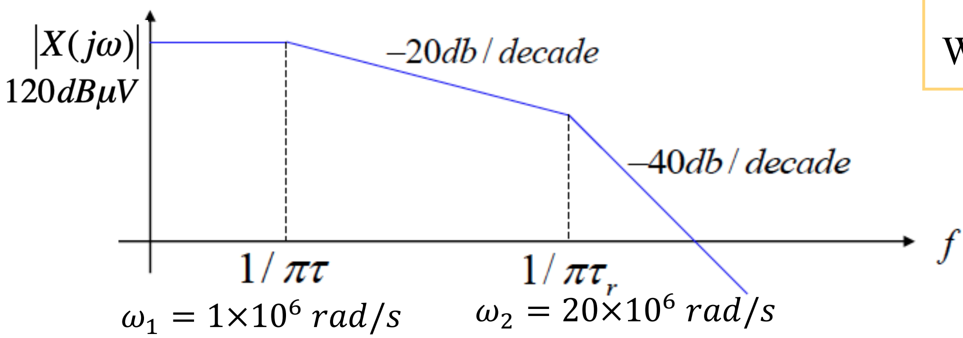
$$\Delta_2 = -20\log_{10}\left(\frac{f_3}{f_1}\right)$$

$$\Delta_3 = -40\log_{10}\left(\frac{f_4}{f_3}\right)$$

$$K_2\,\text{dB} = K\,\text{dB} + \Delta_1$$

$$K_4\,\text{dB} = K\,\text{dB} + \Delta_2 + \Delta_3$$

Spectral of Digital Waveforms



What is $|X(j\omega)|$ at $\omega = 50\times10^6 rad/s$?

trapezoidal pulse train with equal rise- and fall times

at $\omega_2 = 20\times10^6 rad/s$

$$|X(j\omega_2)| = 120 + (-20)\log_{10}\left(\frac{20\times10^6}{10^6}\right) = 120 - 20\times1.3 = 94dB\mu V$$

at $\omega = 50\times10^6 rad/s$

$$|X(j\omega)| = 94 + (-40)\log_{10}\left(\frac{50\times10^6}{20\times10^6}\right) = 78dB\mu V$$

2.3.1 Effect of rise/fall time on Spectral Content

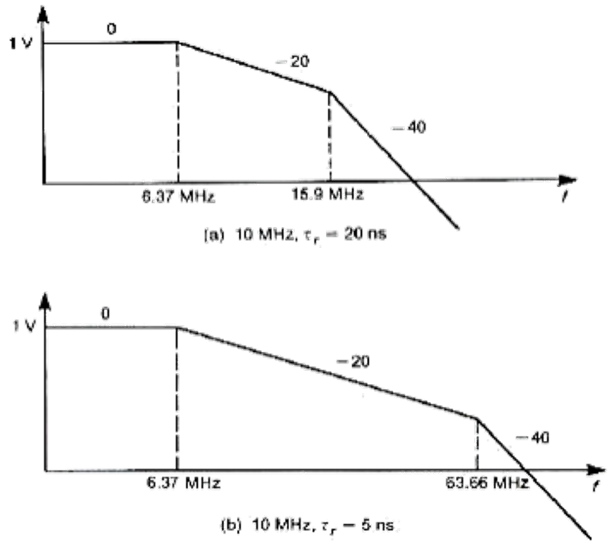
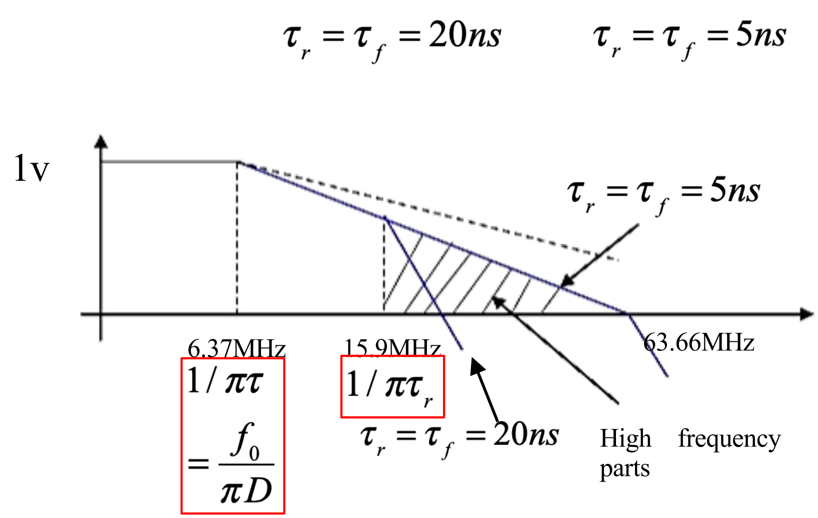


Illustration of the effect of rise/fall times on the spectral content of 1V, 50% duty cycle trapezoidal pulse trains: (a) 10 MHz, $t_r = t_f = 20ns$;
$(b) 10MHz, t_r = t_f = 5ns$



20 ns rise time, 10MHz

$$\text{Level}_{110\,\text{MHz}} = 20\log_{10}(10^6\,\mu\text{V}) - 20\log_{10}\left(\frac{15.9\,\text{MHz}}{6.37\,\text{MHz}}\right) - 40\log_{10}\left(\frac{110\,\text{MHz}}{15.9\,\text{MHz}}\right)$$

$$= 120\,\text{dB}\mu\text{V} - 7.95\,\text{dB} - 33.6\,\text{dB}$$

$$= 78.45\,\text{dB}\mu\text{V}$$

5 ns rise time, 10MHz

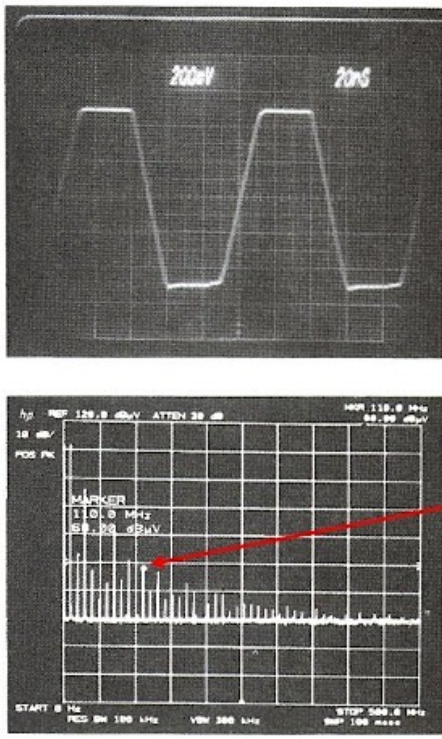
$$\text{Level}_{110\,\text{MHz}} = 20\log_{10}(10^6\,\mu\text{V}) - 20\log_{10}\left(\frac{63.7\,\text{MHz}}{6.37\,\text{MHz}}\right) - 40\log_{10}\left(\frac{110\,\text{MHz}}{63.7\,\text{MHz}}\right)$$

$$= 120\,\text{dB}\mu\text{V} - 20\,\text{dB} - 9.49\,\text{dB}$$

$$= 90.5\,\text{dB}\mu\text{V}$$

Q: calculate the radiation level at 110 MHz

2.3.1 Effect of rise/fall time on Spectral Content



t_r = 20ns

rise/fall time = 20 ns

(a)

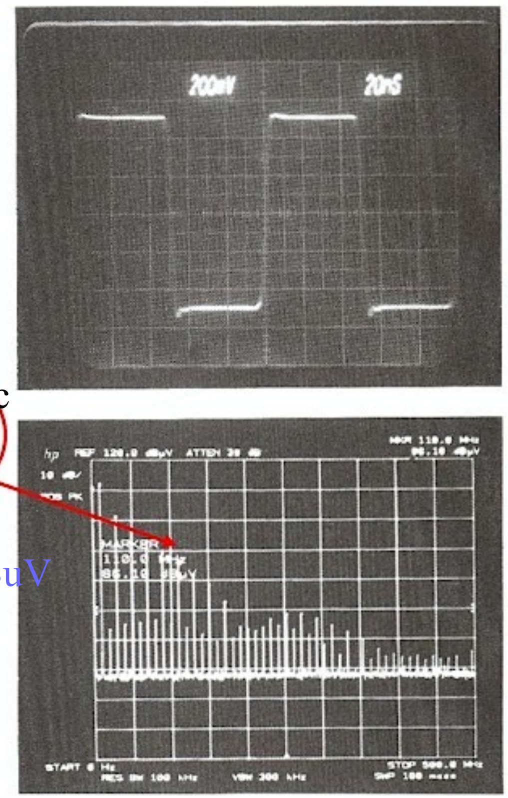
FIGURE 7.19 Experimentally measured spectra of 1 V, 10 MHz, 50% duty cycle trapezoidal pulse trains for rise/falltimes of (a) 20 ns.

11th harmonic

110MHz

68dBuV

86.1dBuV



t_r = 5ns

rise/fall time = 5 ns

(b)

FIGURE 7.19 continued (b) 5 ns.

1-V, 10-MHz trapezoidal pulse train

2.3.1 Effect of rise/fall time on Spectral Content

The exact value calculated by *sinc* function

20ns	73.8dBμV
5ns	90.4dBμV

Very close to the spectral bound prediction

The measured values are

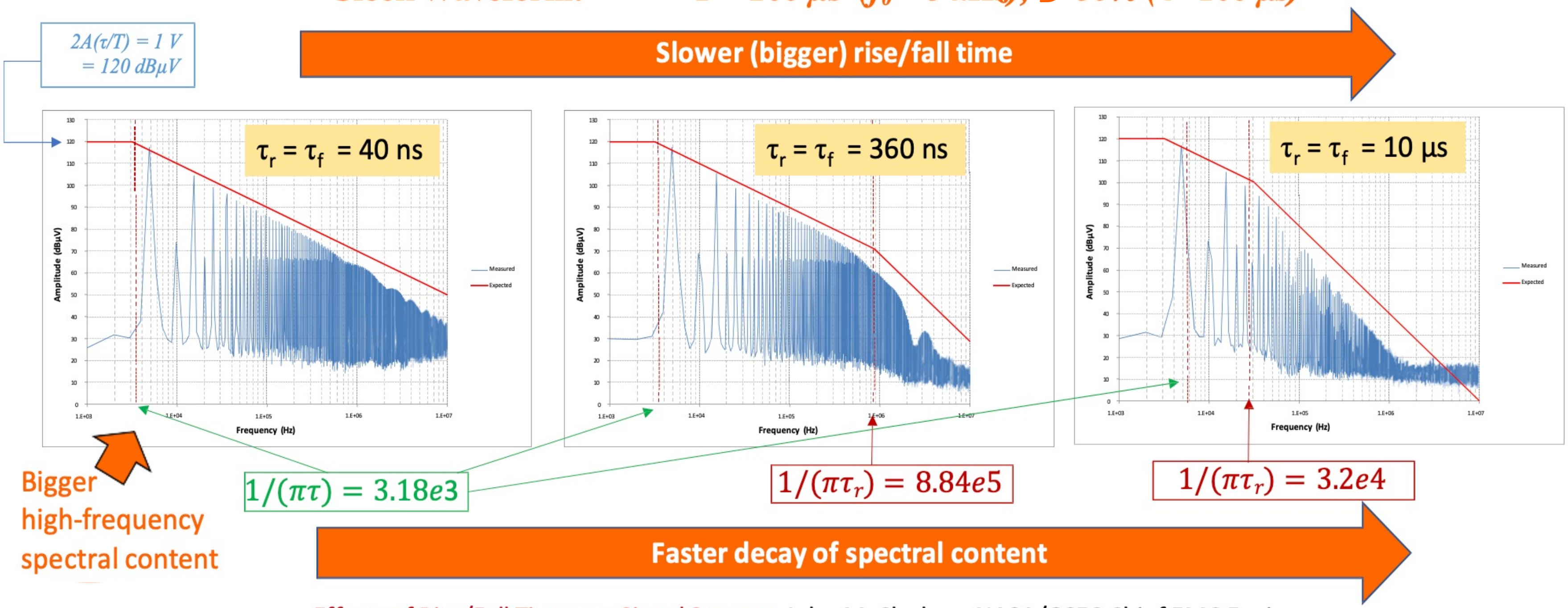
20ns	68.0dBμV
5ns	86.1dBμV

Why are the measured values lower than the values by the exact calculations?

2.3.1 Measurements vs Spectral Bound Approach

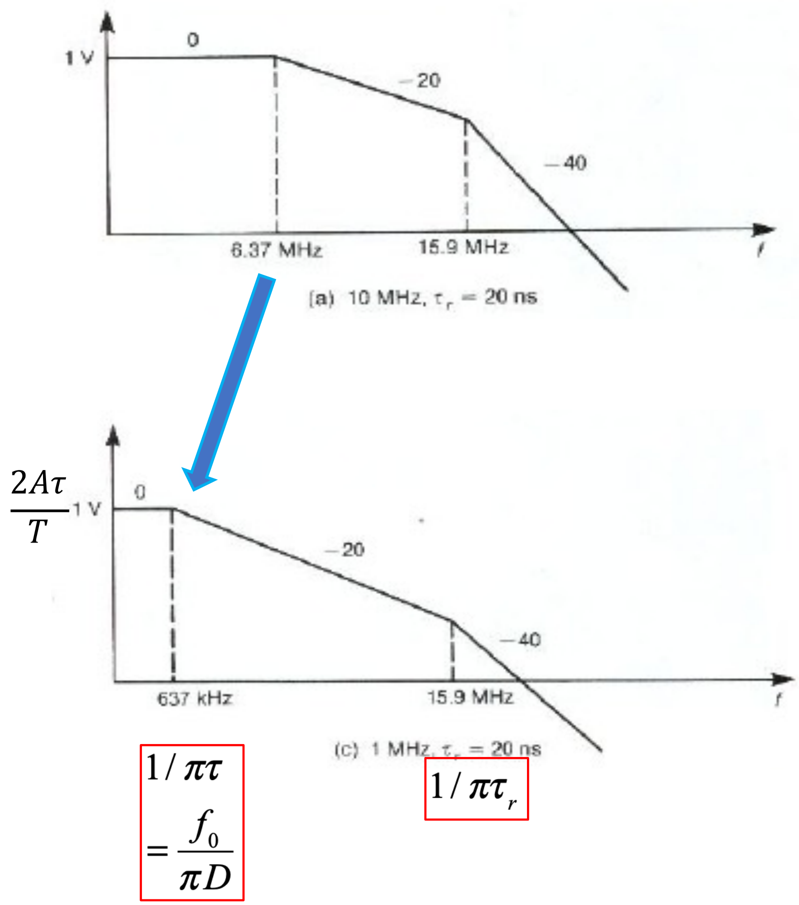
SQUARE to TRAPEZOIDAL

Clock Waveform: $T = 200\ \mu s\ \ (f_0 = 5\ kHz);\ D{=}50\%\ (\tau = 100\ \mu s)$



Effects of Rise/Fall Times on Signal Spectra, John McCloskey, NASA/GSFC Chief EMC Engineer

2.3.2 Effect of Repetition rate



20ns rise time, 10MHz, original signal

$$\mathrm{Level}_{110\,\mathrm{MHz}} = 20\log_{10}(10^{6}\,\mu\mathrm{V}) - 20\log_{10}\left(\frac{15.9\,\mathrm{MHz}}{6.37\,\mathrm{MHz}}\right) - 40\log_{10}\left(\frac{110\,\mathrm{MHz}}{15.9\,\mathrm{MHz}}\right)$$

$$= 120\,\mathrm{dB}\mu\mathrm{V} - 7.95\,\mathrm{dB} - 33.6\,\mathrm{dB}$$

$$= 78.45\,\mathrm{dB}\mu\mathrm{V}$$

20ns rise time, frequency f_o reduced to 1MHz

$$\mathrm{Level}_{110\,\mathrm{MHz}} = 20\log_{10}(10^{6}\,\mu\mathrm{V}) - 20\log_{10}\left(\frac{15.9\,\mathrm{MHz}}{637\,\mathrm{KHz}}\right) - 40\log_{10}\left(\frac{110\,\mathrm{MHz}}{15.9\,\mathrm{MHz}}\right)$$

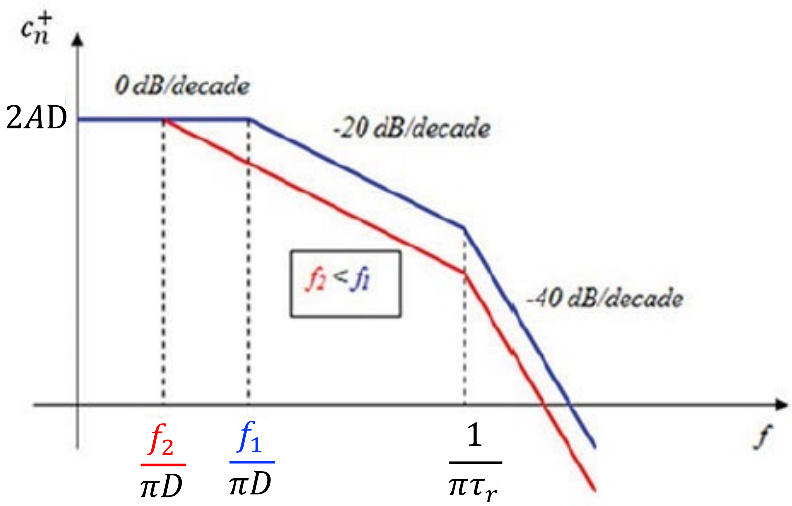
$$= 120\,\mathrm{dB}\mu\mathrm{V} - 28\,\mathrm{dB} - 33.6\,\mathrm{dB}$$

$$= 58.4\,\mathrm{dB}\mu\mathrm{V}$$

2.3.2 Effect of Repetition rate

change repetition rate while keep duty cycle the same

Changing the repetition rate for a **given duty cycle** …



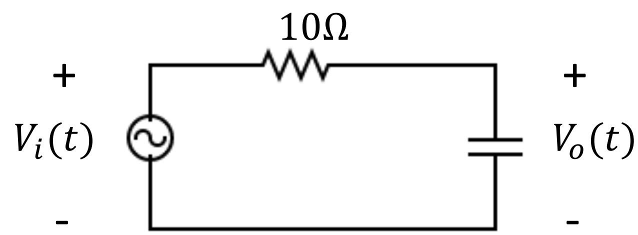
- The effect of a change in the repetition rate (frequency f_0) of the pulse train is to change the spacing between the discrete harmonics
- Reducing the frequency of the pulse train (i.e., increasing the period T) does not usually affect the starting level.
- When the repetition rate f_1 is reduced to f_2:
  - the pulsewidth ($\tau = D/f$) is increased
  - the first breakpoint in the spectral bound, $f_{b1} = \frac{1}{\pi\tau} = \frac{f_0}{\pi D}$ moves down in frequency
  - part of the spectral content in the region of 0 dB/decade now rolls of at a rate of -20 dB/decade.

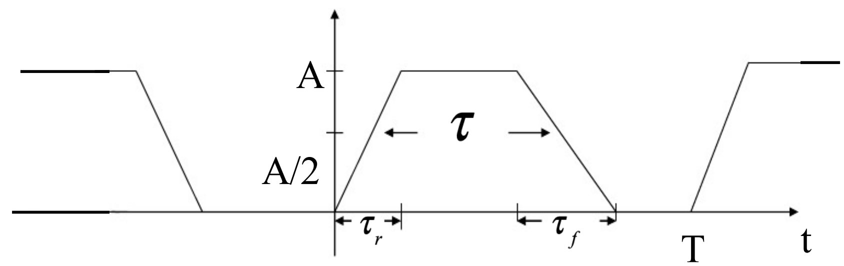
Example

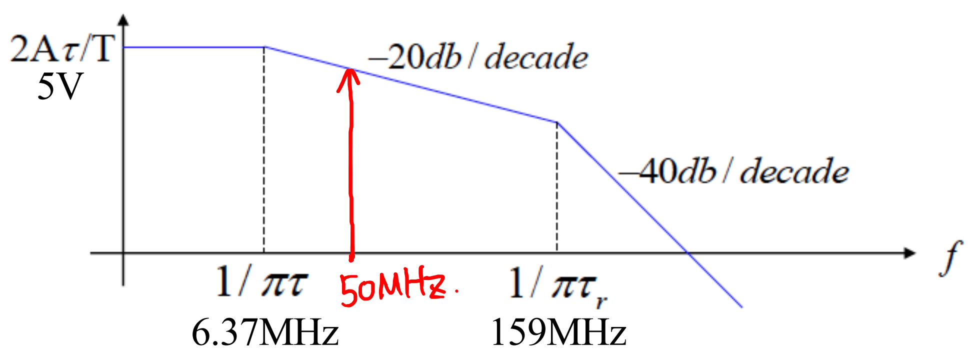
A 10-MHz clock oscillator having a 10 Ω internal resistance has an open circuit voltage waveform as a trapezoidal pulse train with a 50% duty cycle and rise- and fall times of 2 ns.

a) Determine the value of a capacitor (an ideal one) such that when placed the capacitor across the oscillator, the output voltage of the fifth harmonic will reduce by 10 dB.

b) Given that the open-circuit voltage transitions from 0 to 5 V, estimate the level of this fifth harmonic.





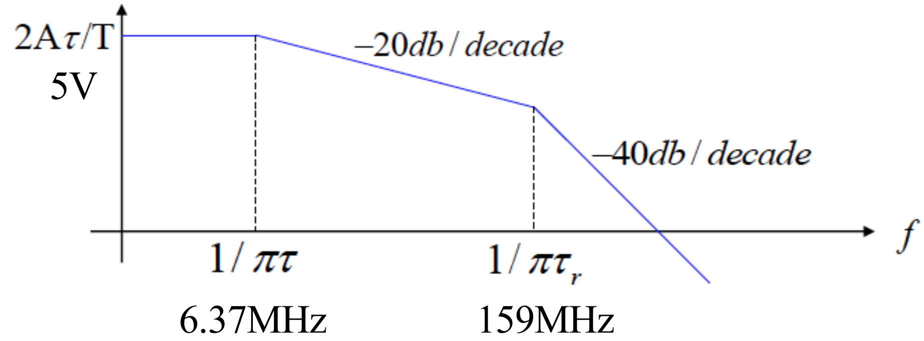


$$T = \frac{1}{10MHz} \qquad A = 5V$$

$$\tau_r = \tau_f = 2ns$$

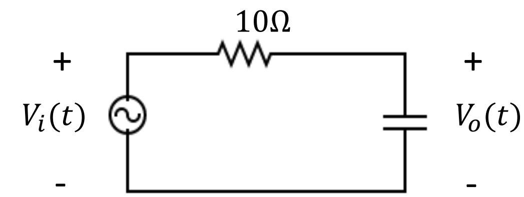
fifth harmonic $f_5 = 50MHz$

a)



$$T = \frac{1}{f_0} = 100ns$$

$$\tau = TD = 50ns$$



$$\boxed{20\log_{10}\left(\frac{V_o(f_5)}{V_i(f_5)}\right) = -10dB}$$

$$\left|\frac{\frac{1}{j\omega_5 C}}{10 + \frac{1}{j\omega_5 C}}\right| = \left|\frac{V_o(f_5)}{V_i(f_5)}\right| \Rightarrow$$

$$X = -\frac{1}{\omega_5 C} \qquad \left|\frac{jX}{10 + jX}\right| = 10^{-1/2} \qquad \frac{X^2}{100 + X^2} = 0.1$$

$$0.9X^2 = 10 \qquad X = -3.333$$

$$\frac{1}{\omega_5 C} = 3.333 \qquad C = \frac{1}{3.333 \times \omega_5} = \frac{1}{3.333 \times 2 \times \pi \times 50 \times 10^6} = 955pF$$

b) 2ns rise time → only -20dB/decade for 5th harmonic

$$\text{Level}_{50MHz} = 20\log_{10}(5V) - 20\log_{10}\left(\frac{50MHz}{6.37MHz}\right) = 133.98 - 17.90 = 116.08dB\mu V$$

1μV.

(3) Calculate actual level at 5th harmonic for one-sided Fourier series

$$F(x) = F(x'') / (jn\omega_0)^2 \quad , \; n \neq 0$$

$$if \quad \tau_r = \tau_f$$

$$C_n = 2A\frac{\tau}{T}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}\frac{\sin(n\omega_0\tau_r/2)}{(n\omega_0\tau_r/2)}e^{-jn\omega_0(\tau+\tau_r)/2}$$

Pulse width $\tau = DT = 0.5\times10^{-7}ns$

$$|C_5| = 2\times5\frac{0.5\times10^{-9}}{1\times10^{-9}}\frac{\sin(5\times2\times\pi\times10\times10^6\times0.5\times10^{-7}/2)}{5\times2\times\pi\times10\times10^6\times0.5\times10^{-7}/2}\frac{\sin(5\times2\times\pi\times10\times10^6\times2\times10^{-9}/2)}{5\times2\times\pi\times10\times10^6\times2\times10^{-9}/2}$$

$$= 5\times\frac{\sin(2.5\times\pi)}{2.5\times\pi}\frac{\sin(0.1\times\pi)}{0.1\times\pi}$$

$$= 5\times\frac{1}{7.854}\times\frac{0.309}{0.31416}$$

$$= 0.626\ V$$

$$= 115.93\ dB\mu V$$

2.3.3 Effect of Duty Cycle

What is the effect of the duty cycle on the spectral bound?

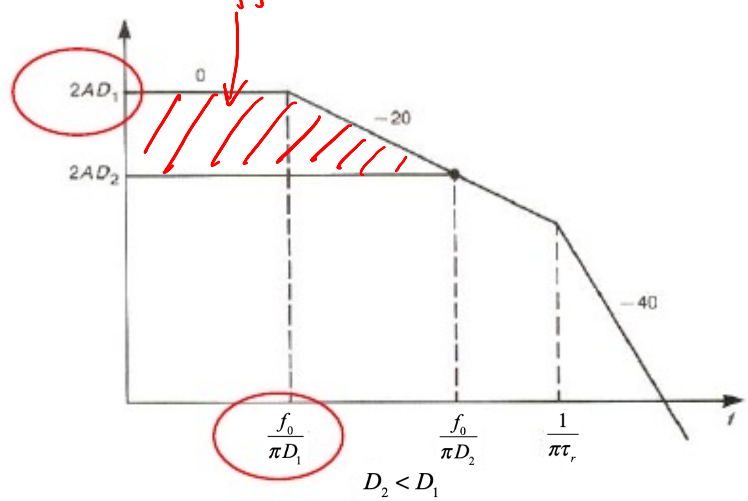
Duty cycle: $D = \dfrac{\tau}{T}$ Or, pulse width $\tau = DT$

One-sided spectrum:

$$\left|C_n^+\right| = 2AD\left|\frac{\sin(n\pi D)}{n\pi D}\right|\left|\frac{\sin(n\pi\tau_r f_0)}{n\pi\tau_r f_0}\right|$$

- DC level
- 1st breakpoint

diff in low freq.



Reducing the duty cycle (the pulsewidth) reduces the low-frequency spectral content of the waveform, but does not affect the high- frequency content

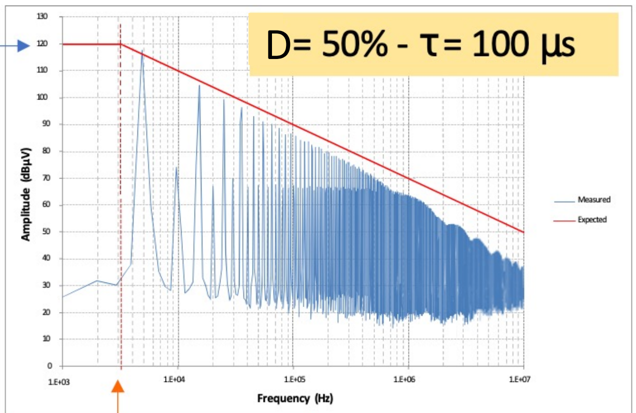
Illustration of the effect of duty cycle on the spectral bounds of trapezoidal pulse trains

Measurements vs Spectral Bound Approach

Clock Waveform: $T = 200\ \mu s$ $(f_0 = 5\ kHz)$; $\tau_r = \tau_f = 40\ ns$

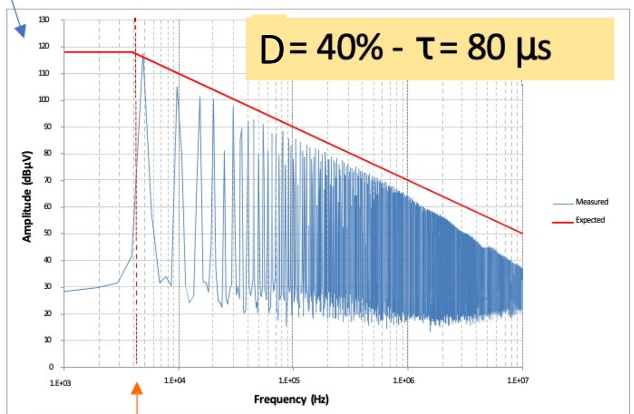
Lower duty cycle δ

$2A(\tau/T) = 1\ V = 120\ dB\mu V$



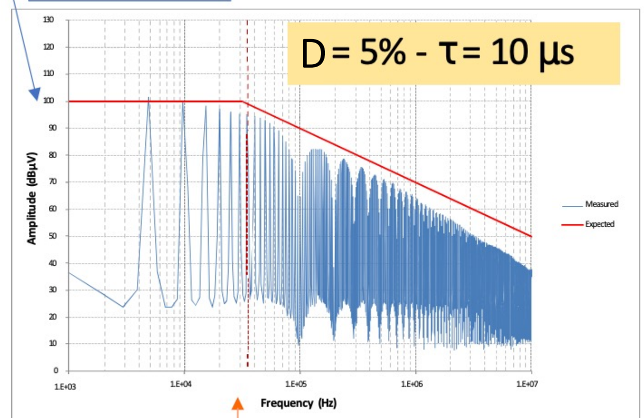
$1/(\pi\tau) = 3.18e3$

$2A(\tau/T) = 0.8\ V = 118\ dB\mu V$



$1/(\pi\tau) = 4.14e3$

$2A(\tau/T) = 0.1\ V = 100\ dB\mu V$



$1/(\pi\tau) = 3.2e4$

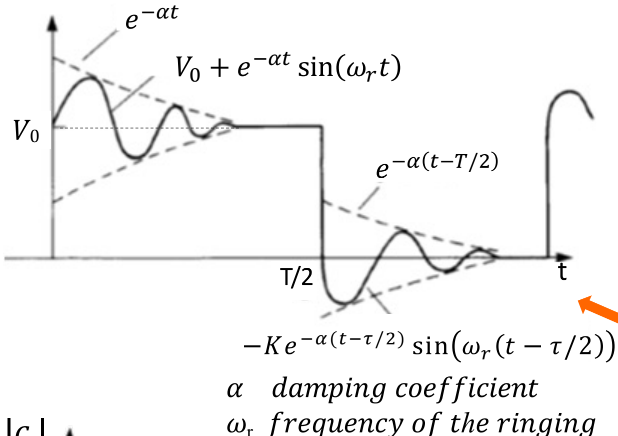
Wider low frequency plateau with a lower starting value

Effects of Rise/Fall Times on Signal Spectra, John McCloskey, NASA/GSFC Chief EMC Engineer

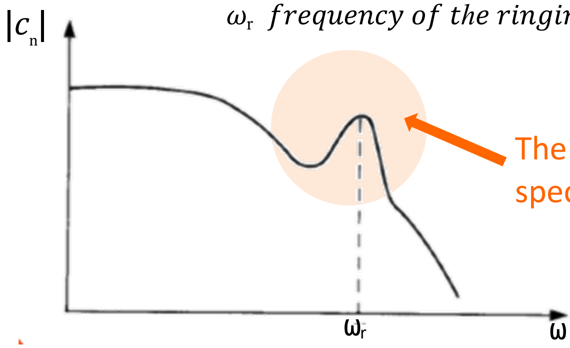
2.3.4 Effect of Ringing (Undershoot/Overshoot)

Inductance and capacitance of PCB lands and wires in a digital system can cause a phenomenon referred to as ringing.

Time domain waveform



Spectral content



The ringing tends to accentuate or enhance certain regions of the spectrum of the original waveform

It is necessary to prevent this ringing by the use of series resistors or ferrite beads or by matching the transmission line

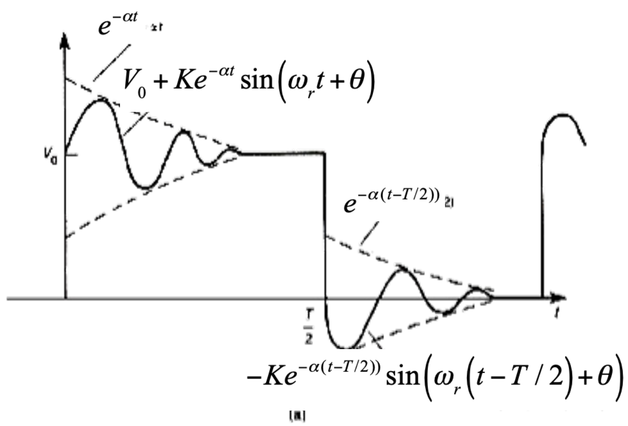
The spectrum is the superposition of three periodic functions:

- A square wave (original square waveform)
- A damped sinusoid $V_0 + e^{-\alpha t}\sin(\omega_r t)$
- The same damped sinusoid shifted ahead in t by T/2

The spectrum of a square wave with undershoot/overshoot has a part of its spectrum enhanced or increased about the ringing frequency ω_r

Undershoot/overshoot will have the effect of increasing the emissions about the ringing frequency

2.3.4 Effect of Ringing (Undershoot/Overshoot)



α: damping coefficient

$f_r = \omega_r / 2\pi$: frequency of the ringing

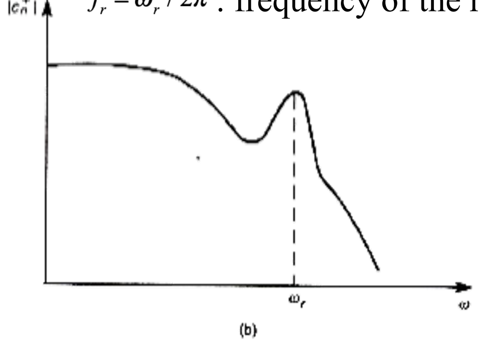


Illustration of ringing (undershoot/overshoot).
(a) time domain waveform; (b) spectral content.

Assume $\theta = 0$

$$c_n = c_{n(squarewave)} + \frac{1}{T}\int_0^{T/2} Ke^{-\alpha t}\sin(\omega_r t+\theta)e^{-jn\omega_0 t}dt - e^{-jn\omega_0 T/2}\left[\frac{1}{T}\int_0^{T/2} Ke^{-\alpha t}\sin(\omega_r t+\theta)e^{-jn\omega_0 t}dt\right]$$

$$= c_{n(squarewave)} + (1-e^{-jn\omega_0 T/2})\frac{1}{T}\int_0^{T/2} Ke^{-\alpha t}\sin(\omega_r t+\theta)e^{-jn\omega_0 t}dt$$

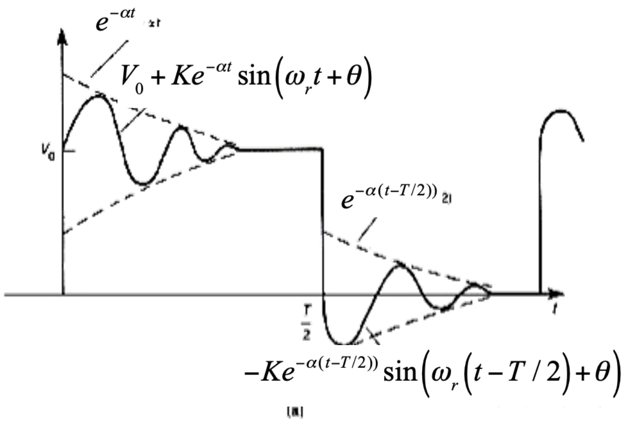
$$= \underbrace{\frac{V_0}{2}\frac{\sin(\frac{1}{4}n\omega_0 T)}{\frac{1}{4}n\omega_0 T}e^{-jn\omega_0 T/4}}_{\text{Square wave components}} + \underbrace{\frac{K}{2}\frac{\sin(\frac{1}{4}n\omega_0 T)}{\frac{1}{4}n\omega_0 T}e^{-jn\omega_0 T/4}\frac{p\omega_r}{p^2+2\alpha p+\alpha^2+\omega_r^2}}_{\text{Bandpass}}$$

Square wave components

Bandpass

$\boxed{\text{where } p = jn\omega_0}$

2.3.4 Effect of Ringing (Undershoot/Overshoot)



α: damping coefficient

$f_r = \omega_r / 2\pi$: frequency of the ringing

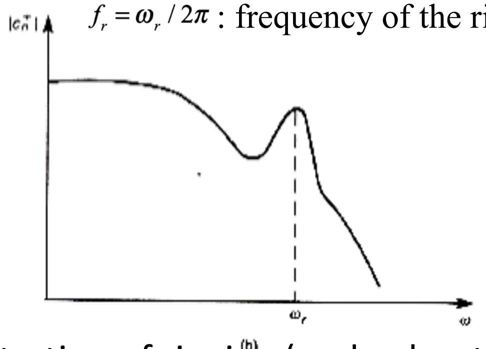


Illustration of ringing (undershoot/overshoot).
(a) time domain waveform; (b) spectral content.

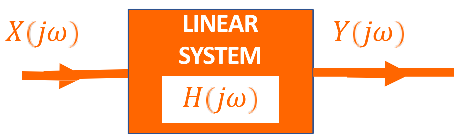
The spectrum of a square wave with undershoot/overshoot has a part of its spectrum enhanced or increased about the ringing frequency

$$\omega = \sqrt{\alpha^2 + \omega_r^2} \cong \omega_r$$

Quite often we see in the radiated emission profile a seemingly Resonant region of enhanced emissions in a narrow frequency band.
One possible explanation for this is the undershoot/overshoot present on the digital waveforms.

Spectral Bound of the Spectrum of Linear Systems

Even though a periodic signal will have frequencies that occur only at multiples of the basic repetition rate, the use of **smooth piecewise linear segments** on a log–log format have considerable advantage in estimating the spectral content of the output of a linear system when the signal is the input to the system.



$$Y(jn\omega_0) = H(jn\omega_0)X(jn\omega_0)$$ **Output spectrum**

$$|Y(jn\omega_0)| = |H(jn\omega_0)||X(jn\omega_0)|$$ *magnitude spectrum*

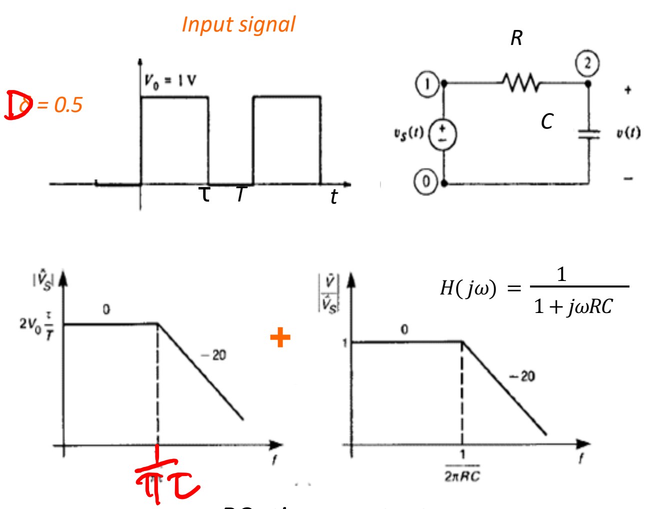
$$\angle Y(jn\omega_0) = \angle H(jn\omega_0)\angle X(jn\omega_0)$$ *phase spectrum*

Bode Plot

$$20\log_{10} Y(jn\omega_0) = 20\log_{10} H(jn\omega_0) + 20\log_{10} X(jn\omega_0)$$

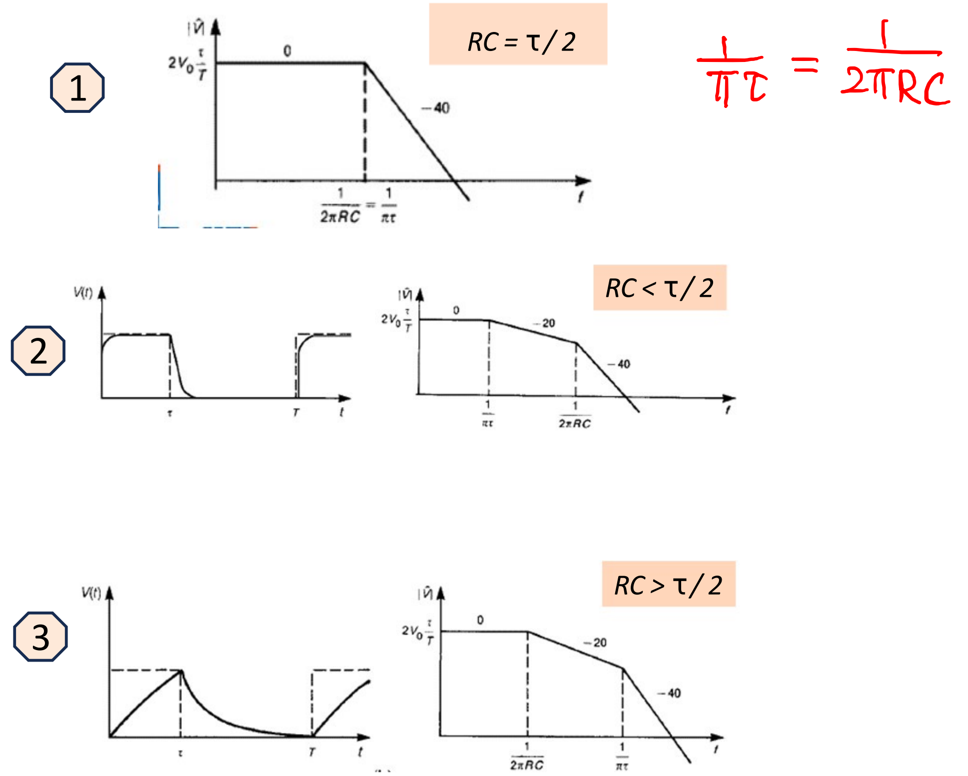
Example

Assume same input signal frequency f and duty cycle D, design the transfer function with diff RC circuit, which is better choice?



RC time constant
τ pulse width

Poll: When pulse width τ is fixed, in order to reduce high-frequency spectrum content, which of the three plots are most suitable?



The sawtooth waveform has reduced high frequency content

Summary

- **Measured spectra show good agreement with expected values**

    Spectral bound provides simple, accurate, and powerful analytical tool

- **Signal spectra significantly more determined by rise/fall times than by fundamental frequency of waveform**

    Uncontrolled rise/fall times can produce significant frequency content up to 1000th harmonic and beyond

    - Common cause of radiated emissions
    - 5 MHz clock can easily produce harmonics out to 5 GHz and beyond
    - Potential interference to S-band receiver (~2 GHz), GPS (~1.5 GHz), etc.

    Controlling rise/fall times can significantly limit high frequency content at the source

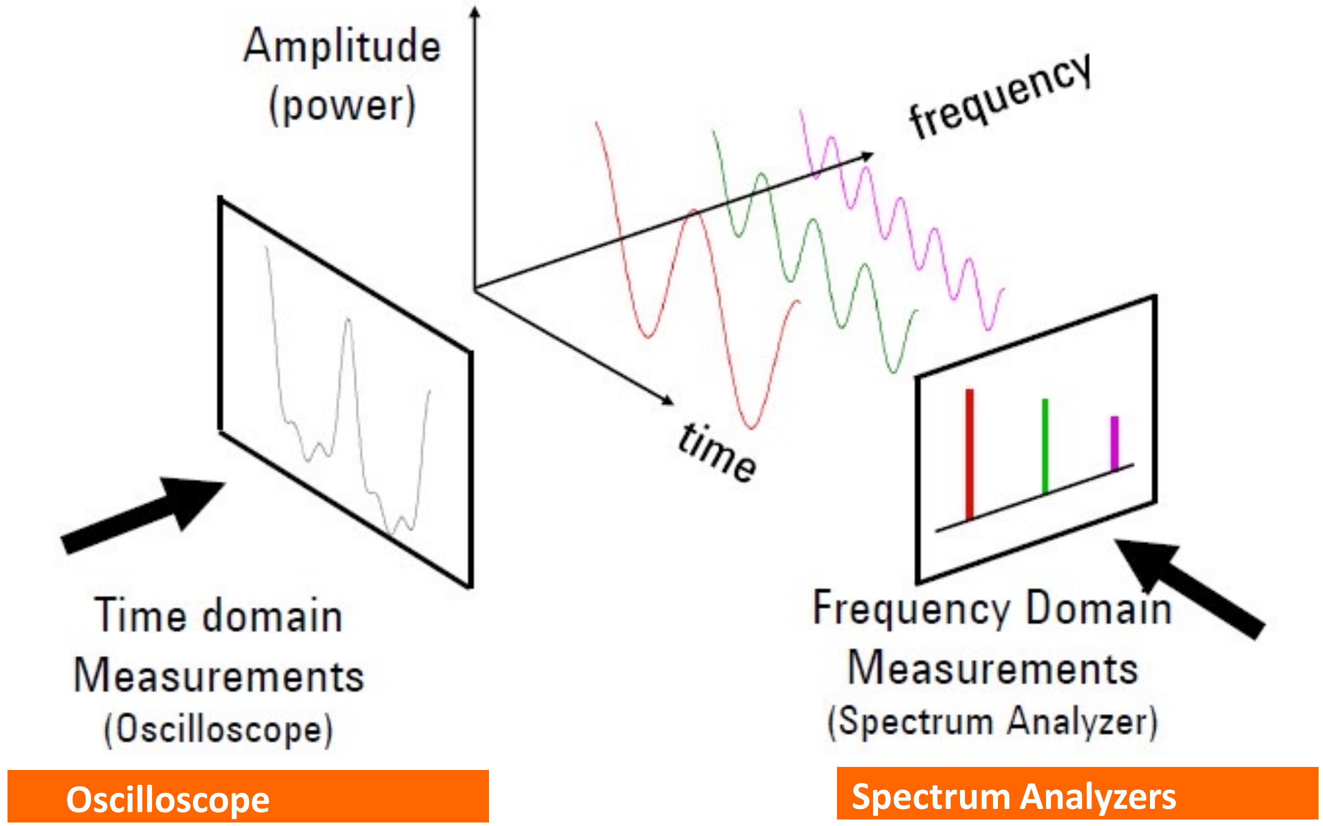
- **Low frequency plateau scales with duty cycle**

    Spectrum for **low duty cycle D** waveforms gives indication of

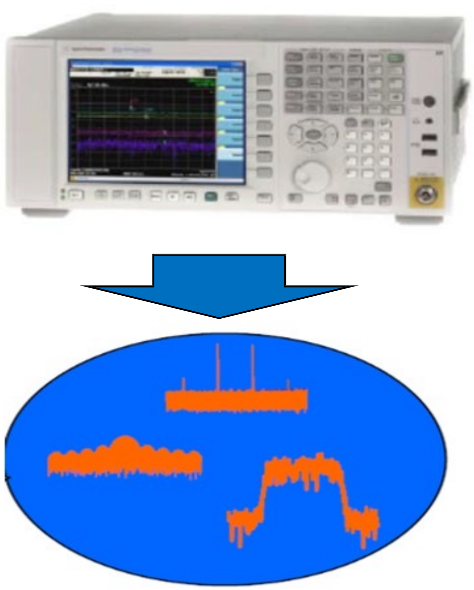
    - Low amplitude (Level=2*AD*)

    -First break frequency $f_{\mathrm{b1}} = 1/(\pi\tau) = f_0/(\pi \mathrm{D})$ may be quite high, producing relatively flat spectrum for many harmonics (100s or 1000s in these examples)

2.4 Spectrum Analyzer

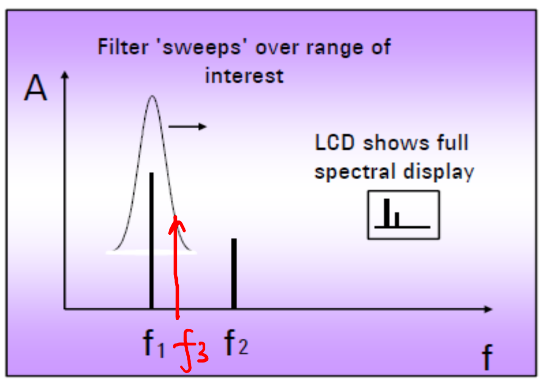


2.4 Spectrum Analyzer



Spectrum analyzers display and measure the magnitude spectrum for periodic signals

They are basically radio receivers having a bandpass filter that is swept in time

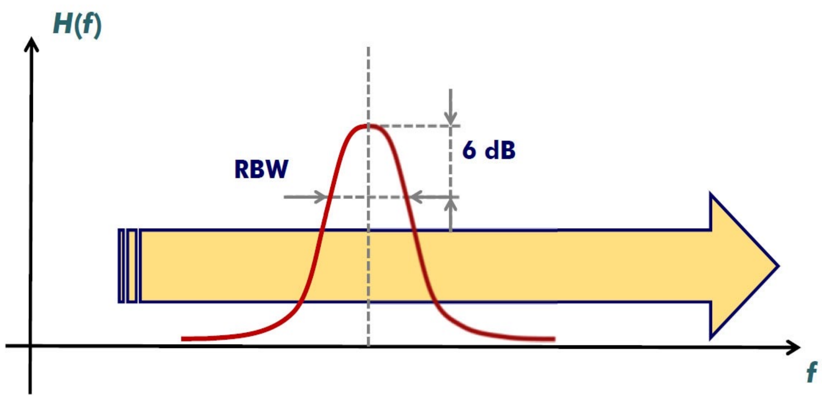


A bandpass filter whose center frequency is swept in time from the start frequency to the end frequency (chosen by the operator) selects and displays the spectral components of the input signal that are present within the bandwidth of the instrument at the point in the time of the sweep.

Resolution Bandwidth (RBW)

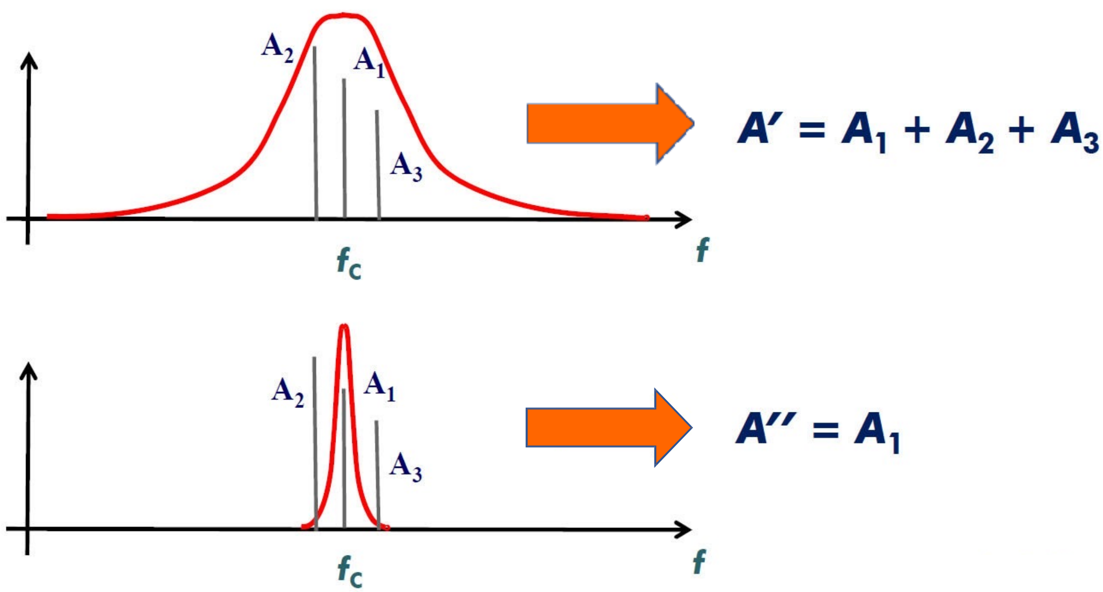
A key factor in determining the level that is displayed by the SA at that frequency is the resolution bandwidth

The bandwidth is the 6 dB bandwidth, where the response is reduced by 6 dB from its maximum level at the center frequency



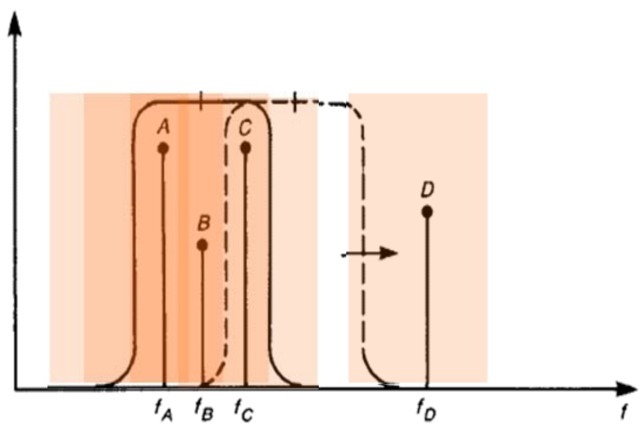
Resolution Bandwidth (RBW)

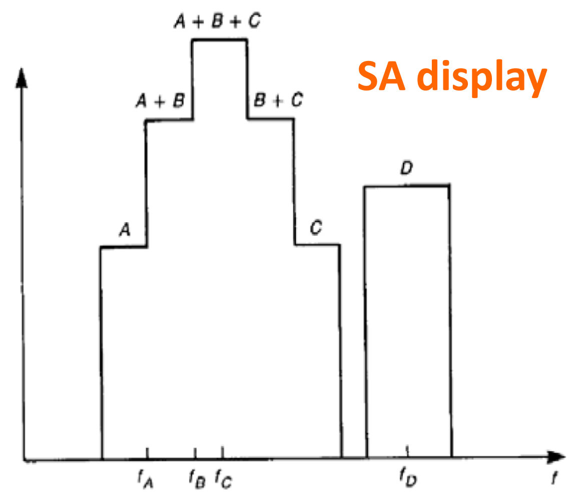
Let us "freeze" the sweep of the SA at some point in its cycle, and compare two different situations…



SA Basic Principles

RBW 1
RBW 2





SA display

The **displayed level at the center frequency of the bandwidth** is the sum of the spectral lines falling within the bandwidth of the filter at that time, i.e. **(A + B + C)**.

To obtain the lowest possible level on the SA display, we should choose as small a bandwidth as possible!

Resolution Bandwidth (RBW)

The regulatory agencies set a **minimum bandwidth** to be used for the measurement

FCC Minimum SA Bandwidths (6 dB)

TABLE 3.1 FCC Minimum Spectrum Analyzer Bandwidths (6 dB)

Radiated emissions:	30 MHz–1 GHz	120 kHz
Radiated emissions:	>1 GHz	1 MHz
Conducted emissions:	150 kHz–30 MHz	9 kHz

CISPR 22 Minimum SA Bandwidths (6 dB)

TABLE 3.2 CISPR 22 Minimum Spectrum Analyzer Bandwidths (6 dB)

Radiated emissions:	30 MHz–1 GHz	120 kHz
Conducted emissions:	150 kHz–30 MHz	9 kHz

To avoid possible summation effect occurring in possible non compliance with regulatory limits, **clock and data repetition rates should be chosen such that none of the harmonics of any signal in the system will be closer than the measurement bandwidth of the SA!**

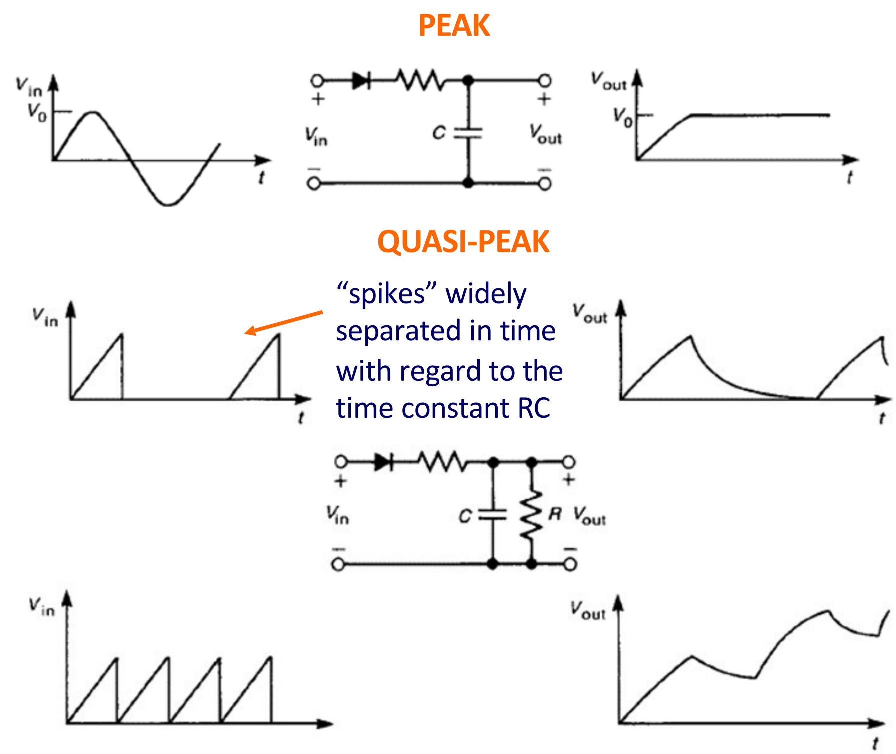
Peak vs Quasi Peak Measurement

<u>Peak detector</u> retains the peak value of each harmonic in an emitted signal, indicating the worst-case scenario.

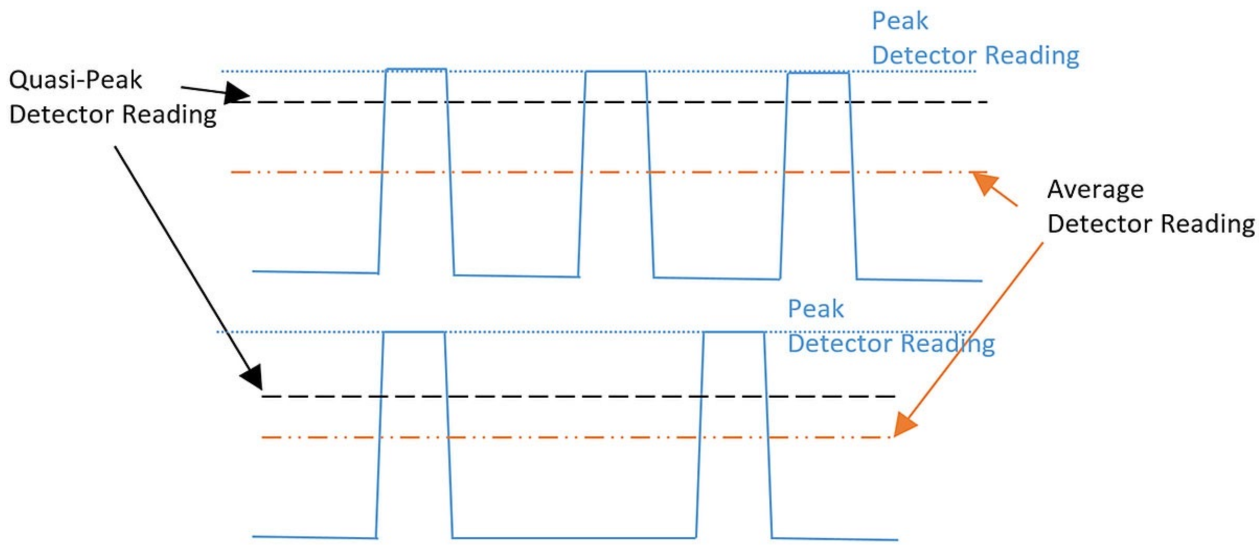
<u>Quasi-peak detector</u> weighs each component based on its repetition rate: The faster repetition rate, the higher the weight given to that component.

To determine compliance quasi-peak detection is needed as dictated by regulatory requirements.

Infrequently occurring signals will result in a measured quasi-peak level that is considerably smaller than a peak detector would give !

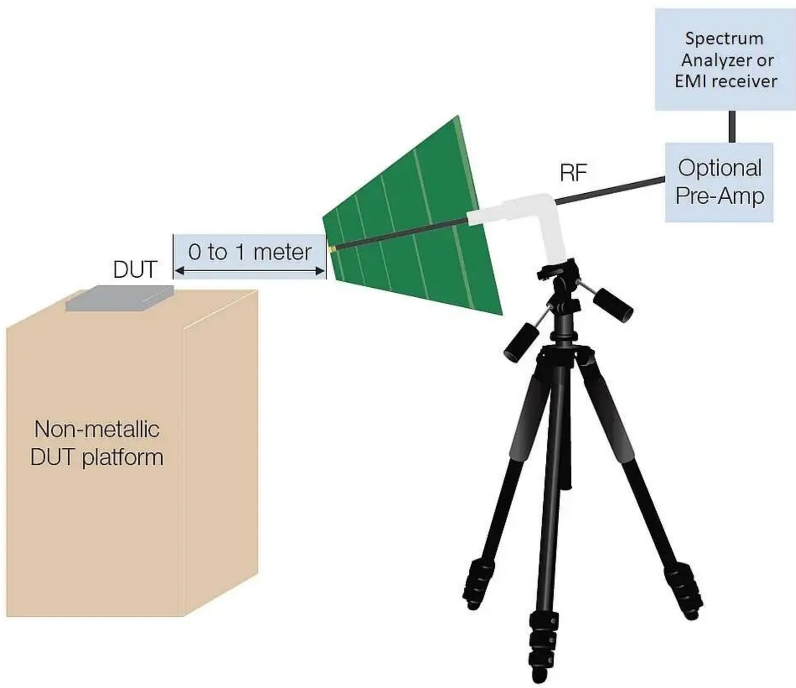


Peak vs Quasi Peak Measurement



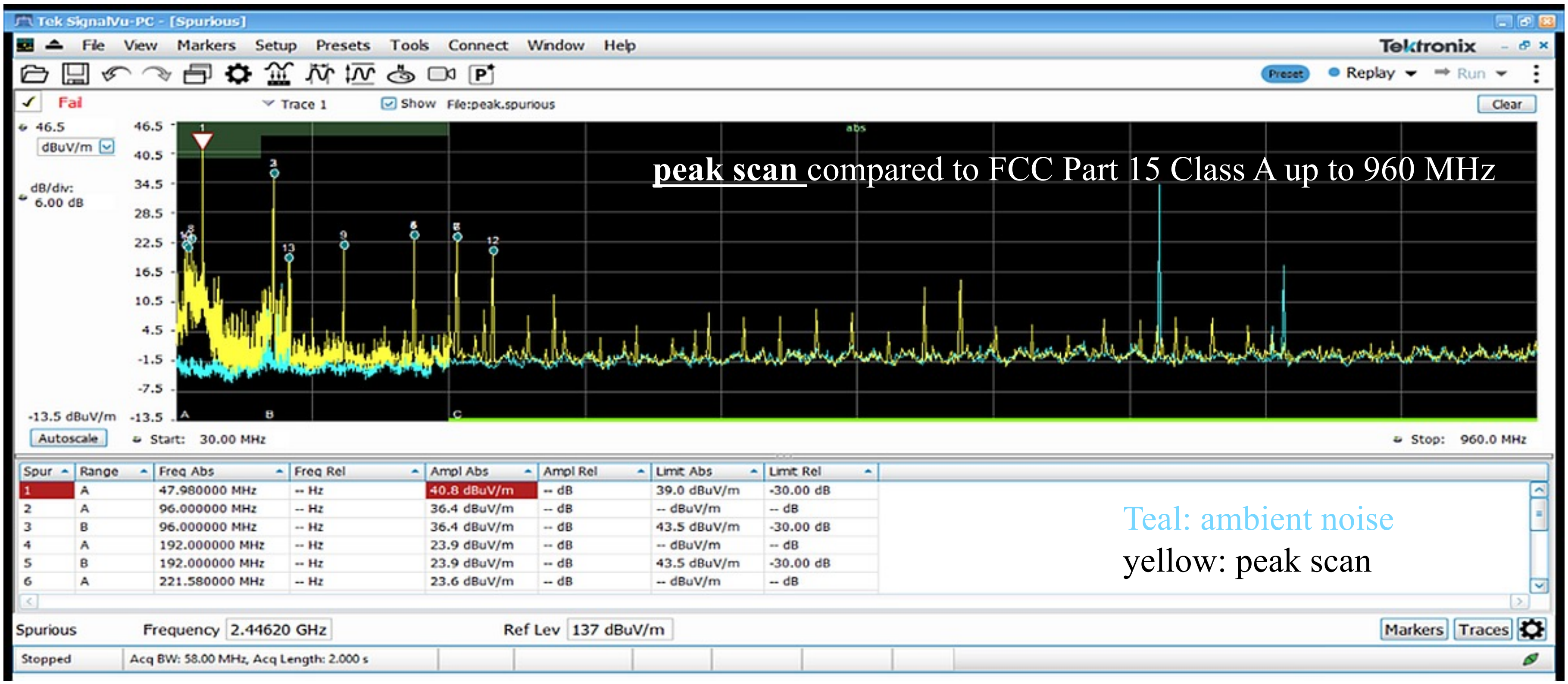
The quasi-peak detector generates a higher voltage output when the repetition rate is higher

Radiated Emission Measurement Example



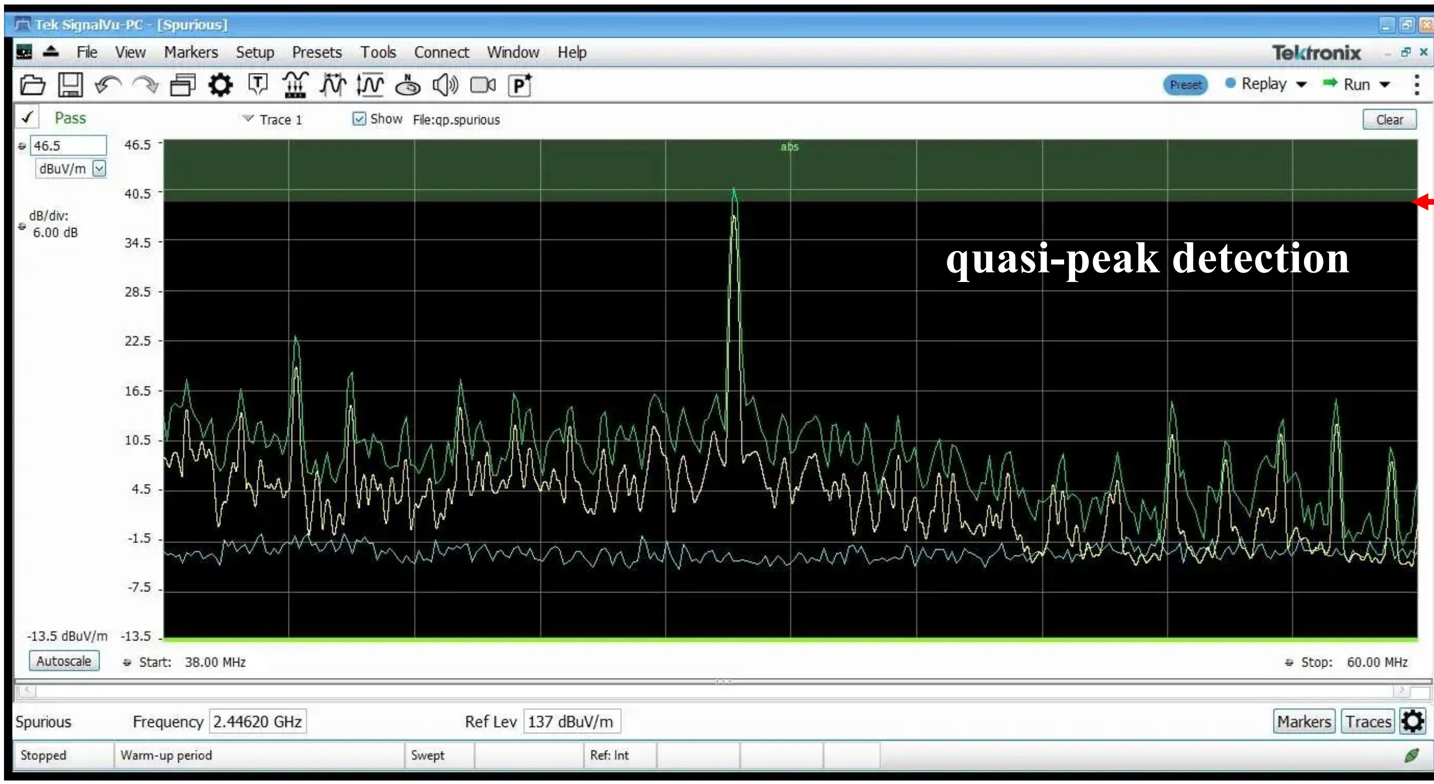
Test setup

Radiated Emission Measurement Example



These results show a large number of spurs and a failure .

Radiated Emission Measurement Example



FCC Part 15 Class A

Green: peak
yellow: quasi peak

The quasi-peak test shows that the devices barely passes and most likely will need to be modified.

https://www.edn.com/emi-emissions-testing-peak-quasi-peak-and-average-measurements/