

Homework 9 Solutions

$$\begin{aligned}
 8.4 \quad \begin{bmatrix} \bar{V}_{an} \\ \bar{V}_{bn} \\ \bar{V}_{cn} \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 50 \angle 80^\circ \\ 100 \angle 0^\circ \\ 50 \angle 90^\circ \end{bmatrix} = 50 \begin{bmatrix} 1 \angle 80^\circ + 2 \angle 0^\circ + 1 \angle 90^\circ \\ 1 \angle 80^\circ + 2 \angle 240^\circ + 1 \angle 210^\circ \\ 1 \angle 80^\circ + 2 \angle 120^\circ + 1 \angle 330^\circ \end{bmatrix} \\
 &= 50 \begin{bmatrix} 2.174 + j1.985 \\ -1.692 - j1.247 \\ 0.0397 + j2.217 \end{bmatrix} = \begin{bmatrix} 147.2 \angle 42.4^\circ \\ 105.1 \angle 216.4^\circ \\ 110.9 \angle 88.97^\circ \end{bmatrix} \text{ V}
 \end{aligned}$$

- 8.12 Note an error in printing: $\bar{V}_{ab}$ should be $1840 \angle 82.8^\circ$ selecting a base of 2300 V and 500 kVA, each resistor has an impedance of $1 \angle 0^\circ \text{ pu}$; $V_{ab} = 0.8$; $V_{bc} = 1.2$; $V_{ca} = 1.0$. The symmetrical components of the line voltages are:

$$\begin{aligned}
 \bar{V}_{ab1} &= \frac{1}{3} (0.8 \angle 82.8^\circ + 1.2 \angle 120^\circ - 41.4^\circ + 1.0 \angle 240^\circ + 180^\circ) = 0.2792 + j0.9453 \\
 &= 0.9857 \angle 73.6^\circ
 \end{aligned}$$

$$\begin{aligned}
 \bar{V}_{ab2} &= \frac{1}{3} (0.8 \angle 82.8^\circ + 1.2 \angle 240^\circ - 41.4^\circ + 1.0 \angle 120^\circ + 180^\circ) = -0.1790 - j0.1517 \\
 &= 0.2346 \angle 220.3^\circ
 \end{aligned}$$

(These are in pu on line-to-line voltage base.)

Phase voltages in pu on the base of voltage to neutral are given by

$$\begin{aligned}
 \bar{V}_{an1} &= 0.9857 \angle 73.6^\circ - 30^\circ = 0.9857 \angle 43.6^\circ \\
 \bar{V}_{an2} &= 0.2346 \angle 220.3^\circ + 30^\circ = 0.2346 \angle 250.3^\circ \quad [\text{Note: An angle of } 180^\circ \text{ is assigned to } \bar{V}_{ca}]
 \end{aligned}$$

Zero-sequence currents are not present due to the absence of a neutral connection.

$$\begin{aligned}
 \bar{I}_{a1} &= \bar{V}_{a1} / 1 \angle 0^\circ = 0.9857 \angle 43.6^\circ \text{ pu} \\
 \bar{I}_{a2} &= \bar{V}_{a2} / 1 \angle 0^\circ = 0.2346 \angle 250.3^\circ \text{ pu}
 \end{aligned}$$

The positive direction of current is from the supply toward the load.

9.1 Calculation of per unit reactances

Synchronous generators:

$$G1 \quad X_1 = X_d'' = 0.18 \quad X_2 = X_d'' = 0.18 \quad X_0 = 0.07$$

$$G2 \quad X_1 = X_d'' = 0.20 \quad X_2 = X_d'' = 0.20 \quad X_0 = 0.10$$

$$G3 \quad X_1 = X_d'' = 0.15 \left(\frac{13.8}{15} \right)^2 \left(\frac{1000}{500} \right) \quad X_0 = 0.05 \left(\frac{13.8}{15} \right)^2 \left(\frac{1000}{500} \right)$$

$$= 0.2539 \quad = 0.08464$$

$$X_2 = X_d'' = 0.2539 \quad 3X_n = 3X_0 = 0.2539$$

$$G4 \quad X_1 = X_d'' = 0.30 \left(\frac{13.8}{15} \right)^2 \left(\frac{1000}{750} \right)$$

$$= 0.3386$$

$$X_2 = 0.40 \left(\frac{13.8}{15} \right)^2 \left(\frac{1000}{750} \right) \quad X_0 = 0.10 \left(\frac{13.8}{15} \right)^2 \left(\frac{1000}{750} \right)$$

$$= 0.4514 \quad = 0.1129$$

Transformers:

$$X_{T1} = 0.10 \quad X_{T2} = 0.10 \quad X_{T3} = 0.12 \left(\frac{1000}{500} \right)$$

$$X_{T4} = 0.11 \left(\frac{1000}{750} \right) = 0.1467 \quad = 0.24$$

Transmission Lines:

$$Z_{base H} = \frac{(765)^2}{1000} = 585.23 \Omega$$

Positive/Negative Sequence

$$X_{12} = \frac{50}{585.23} = 0.08544$$

$$X_{13} = X_{23} = \frac{40}{585.23}$$

$$= 0.06835$$

Zero Sequence

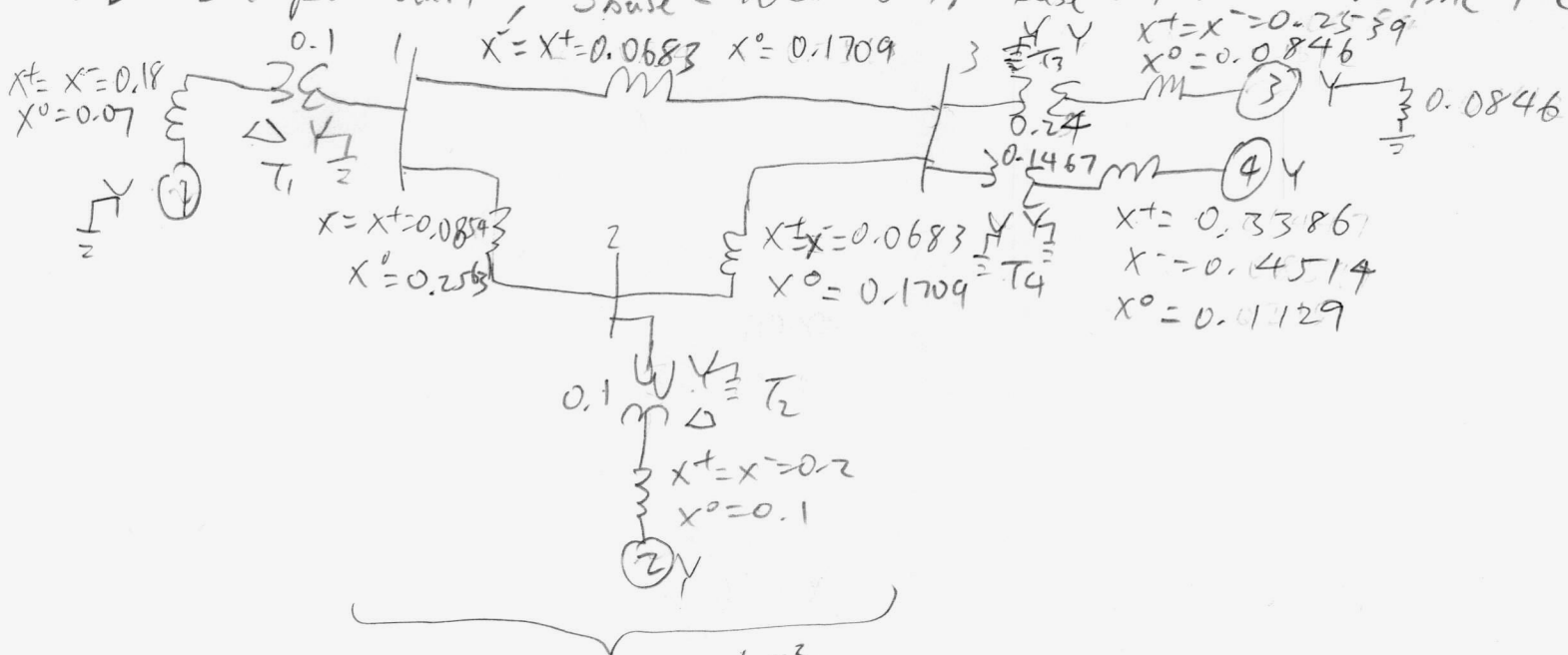
$$X_{12} = \frac{150}{585.23}$$

$$= 0.2563$$

$$X_{13} = X_{23} = \frac{100}{585.23}$$

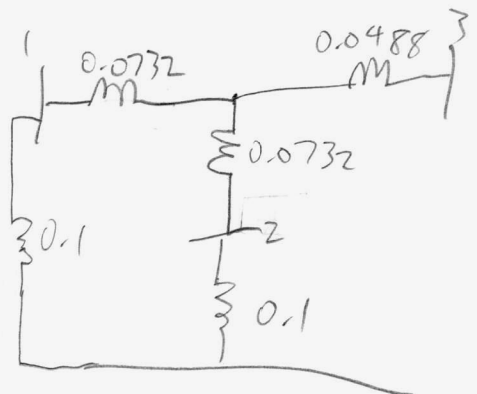
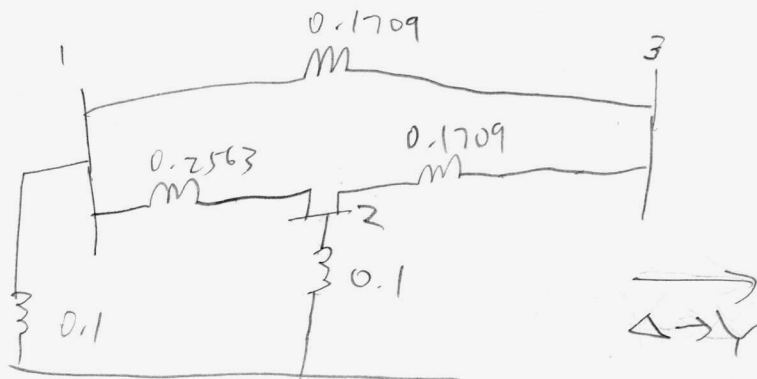
$$= 0.1709$$

9.2 In per unit, $S_{base} = 1000 \text{ MVA}$, $V_{base} = 765 \text{ kV}$ for line 1-2

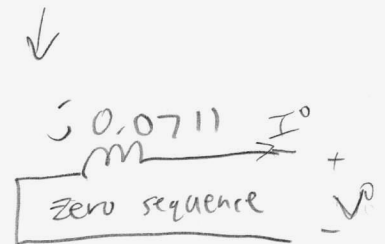


$$Z_{base} = \frac{765^2}{1000} = 585.225 \Omega$$

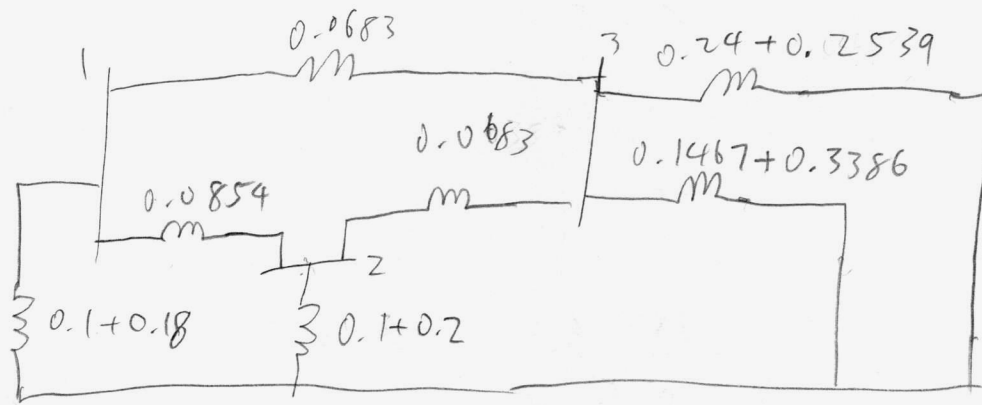
Zero sequence



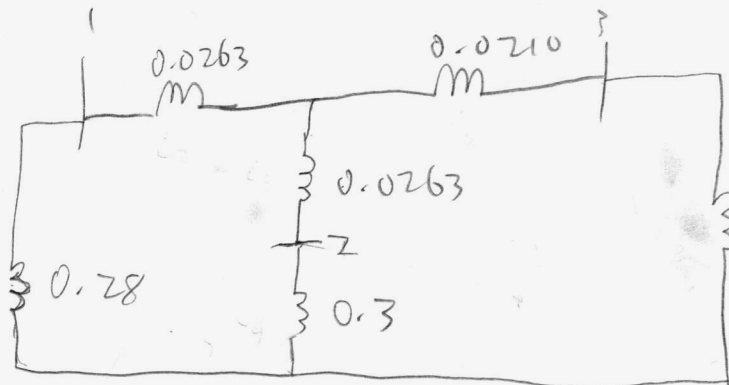
$$\begin{aligned} X_{TH}^0 &= 0.1 // 0.0732 + 0.0732 + 0.1 \\ &= 0.1 // 0.2464 \\ &= 0.0711 \end{aligned}$$



Positive Sequence



↓
 $\Delta \rightarrow Y$



$$0.4939 // 0.4853 = 0.2448$$

↓

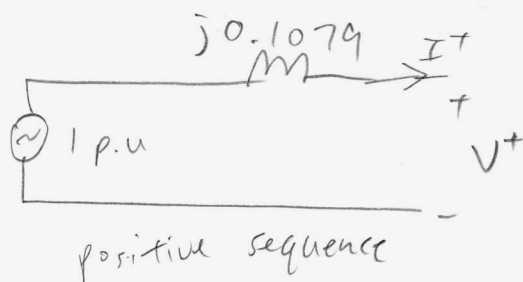
$$X_{Th}^+ = 0.3 // (0.0263 + (0.0210 + 0.2448) // (0.0263 + 0.28))$$

$$= 0.3 // (0.0263 + 0.2658 // 0.3063)$$

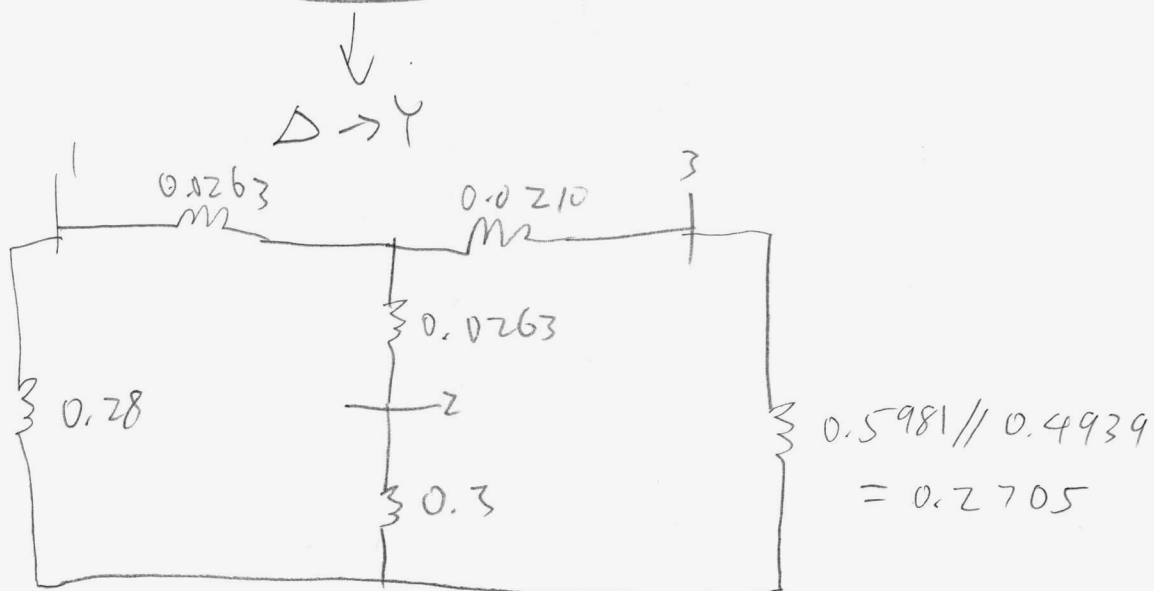
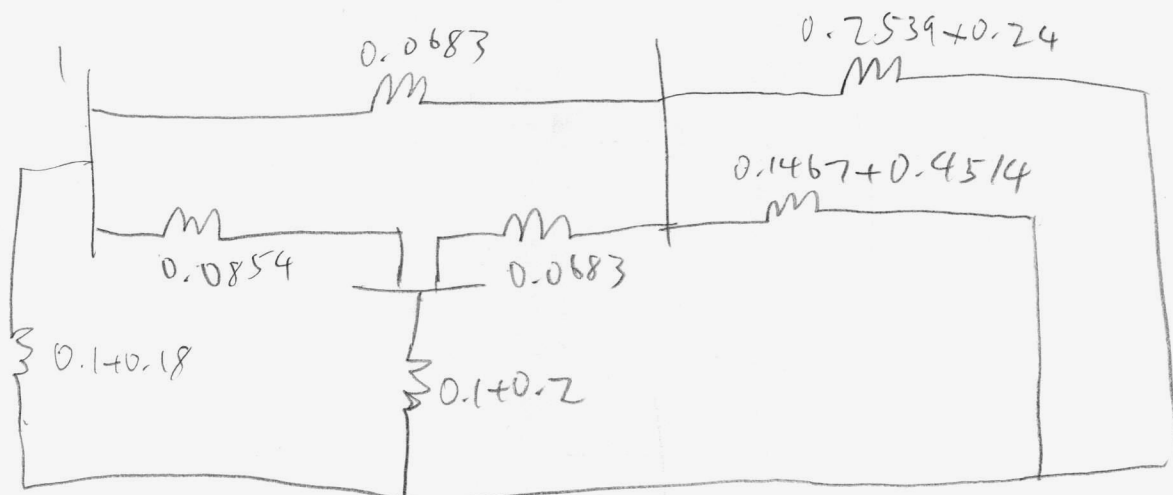
$$= 0.3 // (0.0263 + 0.1423)$$

$$= 0.3 // 0.1686$$

$$= 0.1079$$



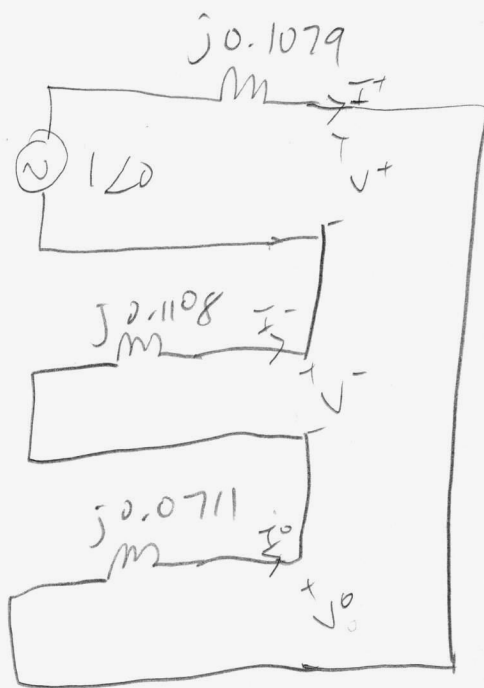
Negative Sequence



$$\begin{aligned}
 \downarrow \quad X_{Th} &= 0.3 // (0.0263 + (0.0210 + 0.2705) // (0.0263 + 0.28)) \\
 &= 0.3 // (0.0263 + 0.2915 // 0.3063) \\
 &= 0.3 // (0.0263 + 0.1494) \\
 &= 0.3 // 0.1757
 \end{aligned}$$

$$\begin{aligned}
 0.1108 \quad I^- &= 0.1108 \\
 \text{Negative sequence} \quad V^- &
 \end{aligned}$$

9.14



$$I^+ = I^- = I^0 = \frac{1\angle 0}{j(0.1079 + 0.1108 + 0.0711)}$$

$$= -j3.45 \text{ p.u.}$$

$$\begin{bmatrix} I_a'' \\ I_b'' \\ I_c'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} -j3.45 \\ -j3.45 \\ -j3.45 \end{bmatrix}$$

$$= \begin{bmatrix} -j10.35 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j7.81 \\ 0 \\ 0 \end{bmatrix} \text{ kA}$$

$$I_{base} = \frac{1000 \text{ E6}}{\sqrt{3} 765 \text{ E3}} = 0.755 \text{ kA}$$

$$\begin{bmatrix} V^0 \\ V^+ \\ V^- \end{bmatrix} = \begin{bmatrix} 0 \\ 1\angle 0 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.0711 & 0 & 0 \\ 0 & j0.1079 & 0 \\ 0 & 0 & j0.1108 \end{bmatrix} \begin{bmatrix} -j3.45 \\ -j3.45 \\ -j3.45 \end{bmatrix} = \begin{bmatrix} -0.245 \\ 0.628 \\ -0.382 \end{bmatrix}$$

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} -0.245 \\ 0.628 \\ -0.382 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.949\angle -113^\circ \\ 0.949\angle 113^\circ \end{bmatrix} \text{ p.u.}$$