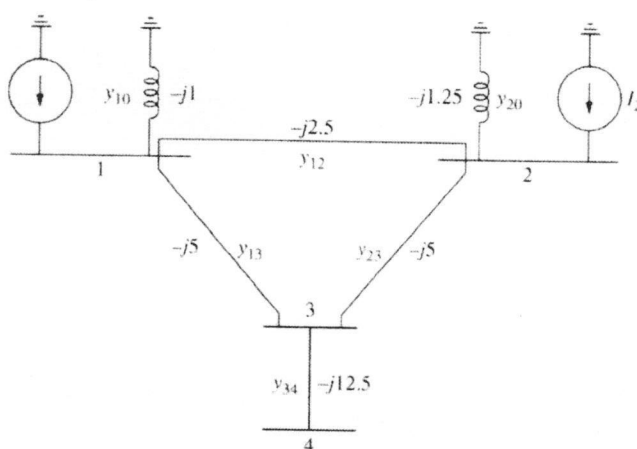


Homework 5 Solutions

2.38 The admittance diagram for the system is shown below:



$$\bar{Y}_{BUS} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \bar{Y}_{13} & \bar{Y}_{14} \\ \bar{Y}_{21} & \bar{Y}_{22} & \bar{Y}_{23} & \bar{Y}_{24} \\ \bar{Y}_{31} & \bar{Y}_{32} & \bar{Y}_{33} & \bar{Y}_{34} \\ \bar{Y}_{41} & \bar{Y}_{42} & \bar{Y}_{43} & \bar{Y}_{44} \end{bmatrix} = j \begin{bmatrix} -8.5 & 2.5 & 5.0 & 0 \\ 2.5 & -8.75 & 5.0 & 0 \\ 5.0 & 5.0 & -22.5 & 12.5 \\ 0 & 0 & 12.5 & -12.5 \end{bmatrix} S$$

where $\bar{Y}_{11} = \bar{y}_{10} + \bar{y}_{12} + \bar{y}_{13}$; $\bar{Y}_{22} = \bar{y}_{20} + \bar{y}_{12} + \bar{y}_{23}$; $\bar{Y}_{23} = \bar{y}_{13} + \bar{y}_{23} + \bar{y}_{34}$

$\bar{Y}_{44} = y_{34}$; $\bar{Y}_{12} = \bar{Y}_{21} = -\bar{y}_{12}$; $\bar{Y}_{13} = \bar{Y}_{31} = -\bar{y}_{13}$; $\bar{Y}_{23} = \bar{Y}_{32} = -\bar{y}_{23}$

and $\bar{Y}_{34} = \bar{Y}_{43} = -\bar{y}_{34}$

6.9 $x_2 - 3x_1 + 1.9 = 0$

$x_2 + x_1^2 - 3.0 = 0$

Rearrange to solve for x_1 and x_2 :

$x_1 = \frac{1}{3}x_2 + 0.6333$

$x_2 = 3.0 - x_1^2$

Starting with an initial guess of $x_1(0) = 1$ and $x_2(0) = 1$

$x_1(1) = \frac{1}{3}x_2(0) + 0.6333 = 0.9667$

$x_2(1) = 3 - [x_1(0)]^2 = 2$

We repeat this procedure using the general equations

$x_1(n+1) = \frac{1}{3}x_2(n) + 0.6333$

$x_2(n+1) = 3 - [x_1(n)]^2$

n	0	1	2	3	...	47	48	49	50
x_1	1	0.9667	1.3	1.3219	...	1.1746	1.1735	1.1734	1.1743
x_2	1	2.0	2.0656	1.31	...	1.6205	1.6202	1.6229	1.6231

6.18 $x^3 + 8x^2 + 2x - 50 = 0$

$x(0) = 1$

$J(i) = \left. \frac{df}{dx} \right|_{x=x(i)} = [3x^2 + 16x + 2]_{x=x(i)}$

In the general form:

$x(i+1) = x(i) + \frac{1}{3x(i)^2 + 16x(i) + 2} \times (-x(i)^3 - 8x(i)^2 - 2x(i) + 50)$

Using $x(0) = 1$, we arrive at

i	0	1	2	3	4	5
x	1	2.857	2.243	2.129	2.126	2.126
ϵ	1.857	0.215	0.051	0.0018	2.0E-6	

After 5 iterations, $\epsilon < 0.001$.

$\therefore x = 2.126$

6.30

6.25 - j18.695	-5.00 + j15.00	-1.25 + j3.75	0	0
-5.00 + j15.00	12.9167 - j38.665	-1.6667 + j5.00	-1.25 + j3.75	-5.00 + j15.00
-1.25 + j3.75	-1.6667 + j5.00	8.7990 - j32.2294	-5.8824 + j23.5294	0
0	-1.25 + j3.75	-5.8824 + j23.5294	9.8846 - j36.4037	-2.7523 + j9.1743
0	-5.00 + j15.00	0	-2.7523 + j9.1743	7.7523 - j24.1443

6.34 $V_1 = 1.0 \angle 0$

$$V_2(i+1) = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^*(i)} - Y_{21}V_1 - Y_{23}V_3(i) \right]$$

$$= \frac{1}{-j10} \left[\frac{1 - j0.5}{V_2^*(i)} - j5V_1 - j5V_3(i) \right]$$

$$V_3(i+1) = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^*(i)} - Y_{31}V_1 - Y_{32}V_2(i+1) \right]$$

$$= \frac{1}{-j10} \left[\frac{1.5 - j0.75}{V_3^*(i)} - j5V_1 - j5V_2(i+1) \right]$$

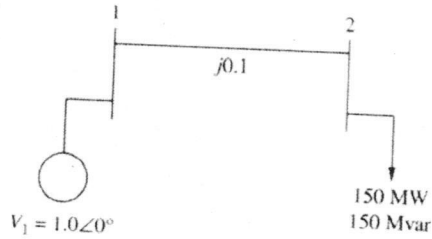
Solving using MATLAB with $V_2(0) = 1.0 \angle 0$ and $V_3(0) = 1.0 \angle 0$:

i	0	1	2
V_2	$1.0 \angle 0$	$0.9552 \angle -6.0090^\circ$	$0.9090 \angle -12.6225^\circ$
V_3	$1.0 \angle 0$	$0.9220 \angle -12.5288^\circ$	$0.8630 \angle -16.1813^\circ$

After 2 iterations,

$$V_2 = 0.9090 \angle -12.6225^\circ$$

$$V_3 = 0.8630 \angle -16.1813^\circ$$



First, convert all values to per-unit.

$$P_{pu} = \frac{P}{S_{base}} = \frac{-150 \text{ MW}}{100 \text{ MVA}} = -1.5 \text{ p.u.}$$

$$Q_{pu} = \frac{Q}{S_{base}} = \frac{-50 \text{ MW}}{100 \text{ MVA}} = -0.5 \text{ p.u.}$$

$$\Delta y_{max,pu} = \frac{\Delta y_{max}}{S_{base}} = \frac{0.1 \text{ MVA}}{100 \text{ MVA}} = 1\text{E-}4 \text{ p.u.}$$

Since there are 2 buses, we need to solve $2(n-1) = 2$ equations.

Therefore, J has dimension 2×2 .

Using Table 6.5

$$J1_{22} = -V_2 V_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21})$$

$$J2_{22} = V_2 Y_{22} \cos(\theta_{22}) + Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \cos(-\theta_{22})$$

$$J3_{22} = V_2 Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21})$$

$$J4_{22} = -V_2 Y_{22} \sin \theta_{22} + Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} + V_2 \sin(-\theta_{22})$$

Also,

$$Y_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix} = \begin{bmatrix} 10 \angle -90 & 10 \angle 90 \\ 10 \angle 90 & 10 \angle -90 \end{bmatrix}$$

Finally, using Eqs. (6.6.2) and (6.6.3),

$$P_2(\delta_2, V_2) = V_2 [Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \cos(-\theta_{22})]$$

$$Q_2(\delta_2, V_2) = V_2 [Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \sin(-\theta_{22})]$$

Solving using MATLAB with $V_2(0) = 1.0 \angle 0$,

i	0	1	2	3
V_2	$1.0 \angle 0$	$0.9500 \angle -8.5944^\circ$	$0.9338 \angle -9.2319^\circ$	$0.9334 \angle -9.2473^\circ$

After 3 iterations

$$V_2 = 0.9334 \angle -9.2473^\circ$$