

Homework 3 Solutions

$$4.32 \quad C_n = \frac{2\pi\epsilon_0}{\ln\left(\frac{D}{r}\right)} = \frac{2\pi(8.854 \times 10^{-12})}{\ln\left(\frac{0.5}{0.015/2}\right)} = \underline{1.3246 \times 10^{-11} \frac{\text{F}}{\text{m}}} \text{ to neutral}$$

$$\bar{Y}_n = j\omega C_n = j(2\pi 60)(1.3246 \times 10^{-11}) \frac{\text{S}}{\text{m}} \times 1000 \frac{\text{m}}{\text{km}}$$

$$\bar{Y}_n = \underline{j4.994 \times 10^{-6} \frac{\text{S}}{\text{km}}} \text{ to neutral}$$

$$4.41 \quad D_{eq} = \sqrt[3]{10 \times 10 \times 20} = 12.6 \text{ m}$$

$$\text{From Table A.4, } r = \frac{1.293}{2} \ln\left(\frac{0.0254 \text{ m}}{1 \text{ in}}\right) = 0.01642 \text{ m}$$

$$D_{sc} = \sqrt[3]{rd^2} = \sqrt[3]{0.01642(0.5)^2} = 0.16 \text{ m}$$

$$C_1 = \frac{2\pi\epsilon_0}{\ln\left(\frac{D_{eq}}{D_{sc}}\right)} = \frac{2\pi(8.854 \times 10^{-12})}{\ln\left(\frac{12.6}{0.16}\right)} = 1.275 \times 10^{-11} \text{ F/m}$$

$$\bar{Y}_1 = j\omega C_1 = j2\pi(60)1.275 \times 10^{-11}(1000) = j4.807 \times 10^{-6} \text{ S/km}$$

$$Q_1 = V_{LL}^2 Y_1 = (500)^2 4.807 \times 10^{-6} = 1.2 \text{ MVAR/km}$$

$$5.1 \quad (a) \quad \bar{A} = \bar{D} = 1.0 \angle 0^\circ \text{ pu}; \bar{C} = 0. \text{ S}$$

$$\bar{B} = \bar{Z} = (0.19 + j0.34) 25 = 9.737 \angle 60.8^\circ \Omega$$

$$(b) \quad \bar{V}_R = \frac{33}{\sqrt{3}} \angle 0^\circ = 19.05 \angle 0^\circ \text{ kV}_{L-N}$$

$$\bar{I}_R = \frac{S_R \angle -\cos^{-1}(pf)}{\sqrt{3} V_{R-L-L}} = \frac{10}{\sqrt{3}(33)} \angle -\cos^{-1}(0.9)$$

$$= 0.175 \angle -25.84^\circ \text{ kA}$$

$$\bar{V}_S = \bar{A}\bar{V}_R + \bar{B}\bar{I}_R = (1.0)(19.05) + (9.737 \angle 60.8^\circ)(0.175 \angle -25.84^\circ)$$

$$= 20.45 + j0.976 = 20.47 \angle 2.732^\circ \text{ kV}_{L-N}$$

$$V_{S-L-L} = 20.47\sqrt{3} = 35.45 \text{ kV}$$

$$(c) \quad \bar{I}_R = 0.175 \angle 25.84^\circ$$

$$\bar{V}_S = \bar{A}\bar{V}_R + \bar{B}\bar{I}_R = (1.0)(19.05) + (9.737 \angle 60.8^\circ)(0.175 \angle 25.84^\circ)$$

$$= 19.15 + j1.701 = 19.23 \angle 5.076^\circ \text{ kV}_{L-N}$$

$$V_{S-L-L} = 19.23\sqrt{3} = 33.3 \text{ kV}$$

$$5.14 \quad (a) \quad \bar{Z}_C = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{0.03 + j0.35}{4.4 \times 10^{-6} \angle 90^\circ}} = 282.6 \angle -2.45^\circ \Omega$$

$$(b) \quad \bar{\gamma}l = \sqrt{\bar{z}\bar{y}}(l) = \sqrt{(0.35128 \angle 85.101^\circ)(4.4 \times 10^{-6} \angle 90^\circ)}(400) \\ = 0.4973 \angle 87.55^\circ = 0.02126 + j0.4968 \text{ pu}$$

$$(c) \quad \bar{A} = \bar{D} = \cosh \bar{\gamma}l = \cosh(0.02126 + j0.4968) \\ = (\cosh 0.02126)(\cos 0.4968 \text{ radians}) + j(\sinh 0.02126)(\sin 0.4968 \text{ radians}) \\ = (1.00023)(0.87911) + j(0.02126)(0.47661) \\ = 0.87931 + j0.01013 = 0.8794 \angle 0.66^\circ \text{ pu}$$

$$\sinh \bar{\gamma}l = \sinh(0.02126 + j0.4968) \\ = \sinh(0.02126)\cos(0.4968 \text{ radians}) + j(\cosh 0.02126)(\sin 0.4968 \text{ radians}) \\ = (0.02126)(0.87911) + j(1.00023)(0.47661) \\ = 0.01869 + j0.4767 = 0.4771 \angle 87.75^\circ$$

$$\bar{B} = \bar{Z}_C \sinh(\bar{\gamma}l) = (282.6 \angle -2.45^\circ)(0.4771 \angle 87.75^\circ) \\ = 134.8 \angle 85.3^\circ \Omega$$

$$\bar{C} = \frac{1}{\bar{Z}_C} \sinh(\bar{\gamma}l) = \frac{0.4771 \angle 87.75^\circ}{282.6 \angle -2.45^\circ} = 1.688 \times 10^{-3} \angle 90.2^\circ \text{ S}$$