

Homework 10 Solutions

11.4 Per unit swing equations

$$2H \frac{\omega_{pu}(t)}{\omega_{syn}} \frac{d^2 \delta(t)}{dt^2} = P_{mpu}(t) - P_{epu}(t) = P_{apu}(t)$$

$$\text{Assuming } \omega_{pu}(t) \approx 1 \rightarrow \frac{2H}{\omega_{syn}} \frac{d^2 \delta(t)}{dt^2} = P_{apu}(t)$$

$$\frac{2(6)}{2\pi 60} \frac{d^2 \delta(t)}{dt^2} = 0.7 - (0.40)(0.70) = 0.42 \quad (\text{i.e. a 60\% reduction to 0.4})$$

$$\text{Initial Conditions: } \delta(0) = 12^\circ = 0.2094 \text{ rad; } \frac{d\delta(0)}{dt} = 0$$

Integrating twice and using the above initial conditions:

$$\frac{d\delta(t)}{dt} = 13.14t + 0$$

$$\delta(t) = 6.57t^2 + 0.2094$$

At $t = 5 \text{ cycles} = 0.08333 \text{ seconds}$

$$\delta(5 \text{ cycles}) = 6.57(0.08333)^2 + 0.2094$$

$$\delta(5 \text{ cycles}) = 0.255 \text{ rad} = 14.62^\circ$$

11.7 $KE_{gen} = H S_{rated} = (3)(100 \times 10^6) = 3 \times 10^8 \text{ joules}$

$$\text{For a moving mass } W_{kinetic} = \frac{1}{2} MV^2$$

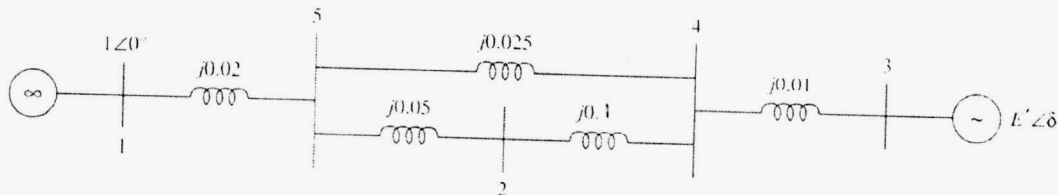
To equal the energy stored in the generator

$$KE_{gen} = W_{kinetic}$$

$$V = \sqrt{\frac{2(KE_{gen})}{M}}$$

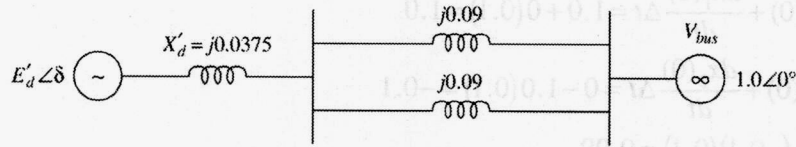
$$V = \sqrt{\frac{2(3 \times 10^8 \text{ joules})}{80,000 \text{ kg}}} = 86.60 \frac{\text{m}}{\text{s}} = 193.72 \frac{\text{miles}}{\text{hour}}$$

11.10



$$\begin{aligned} X_{Th} &= X_{34} + [X_{45} \parallel (X_{42} + X_{25})] + X_{15} \\ &= 0.01 + [0.025 \parallel (0.1 + 0.05)] + 0.02 \\ &= 0.0514 \Omega \end{aligned}$$

11.19



$$H = 20 \text{ sec p.u.}$$

$$D = 0.1 \text{ p.u.}$$

$$100 \text{ MVA base}$$

$$(a) X_{eq} = 0.0375 + (0.09 \parallel 0.09) = 0.0825$$

$$I = \frac{P}{V_{bus}(pf)} \angle -\cos^{-1}(pf) = \frac{\left(\frac{400}{100}\right)}{(1.0)(1.0)} \angle 0^\circ = 4.0 \angle 0^\circ$$

$$\begin{aligned} \bar{E}' &= E' \angle \delta = V_{bus} + jX_{eq}I \\ &= 1.0 \angle 0^\circ + j0.0825(4 \angle 0^\circ) \\ &= 1 + j0.33 \\ &= 1.053 \angle 18.2629^\circ \end{aligned}$$

$$(b) \frac{d\delta_t}{dt} = \omega_t - \omega_{syn}$$

$$\frac{d\omega_t}{dt} = \frac{P_{apu} \omega_{syn} - 4\omega_{syn} - D \frac{d\delta_t}{dt}}{2H\omega_{pu}} = \frac{4\omega_{syn} - D \frac{d\delta_t}{dt}}{2H\omega_{pu}}$$

$$P_{apu} = 4.0 - 0 - \left(\frac{0.1}{\omega_{syn}}\right) \left(\frac{d\delta_t}{dt}\right)$$

$$(c) \delta_{t+\Delta t} = \delta_t + \frac{d\delta_t}{dt} \Delta t = \delta_t + \Delta t (\omega_t - \omega_{syn})$$

$$\omega_{t+\Delta t} = \omega_t + \frac{d\omega_t}{dt} \Delta t = \omega_t + \Delta t \left[\frac{4\omega_{syn} - 0.1 \frac{d\delta_t}{dt}}{2H\omega_{pu}} \right] = \omega_t + \Delta t \left[\frac{4\omega_{syn} - 0.1 (\omega_t - \omega_{syn})}{2H\omega_{pu}} \right]$$

$$\Delta t = \frac{1}{60} = 0.01667 \text{ s}$$

$$\omega_0 = \omega_{syn} = 2\pi 60 = 377$$

$$\delta_0 = 18.2629^\circ = 0.3187 \text{ rad from (a).}$$

$$\delta_{0.01667} = 0.3187 + (377 - 377)(0.01667) = 0.3187$$

$$\omega_{0.01667} = 377 + (0.01667) \left[\frac{4 \times 377 - 0.1(0)}{2(20)(1)} \right] = 377.628$$

$$\delta_{0.0333} = 0.3187 + (0.01667)(377.628 - 377) = 0.329 \text{ rad} = 18.86^\circ$$

$$\omega_{0.0333} = 377.628 + (0.01667) \left[\frac{4 \times 377 - 0.1(0.628)}{2(20)(1)} \right] = 378.26 \text{ rad/s}$$

11.20 The critical clearing time for a solid fault midway between buses 1 and 3 is about 0.18 seconds. Below is a plot of the generator rotor angle with the fault cleared at 0.18 seconds.

