

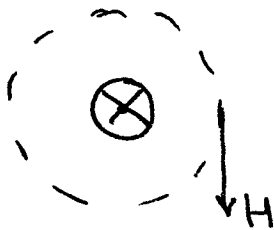
LECTURE 7

- REVIEW
- MANY-CONDUCTOR LINE INDUCTANCE
- CONDUCTOR BUNDLING
- TRANSPOSITION

REVIEW

In previous lecture, we computed the per-unit length inductance of a transmission line with only two wires

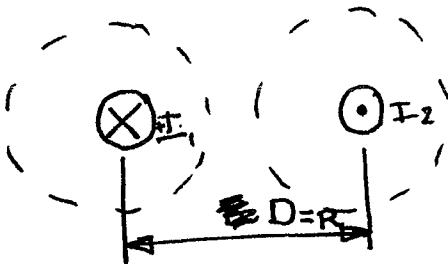
- We started with the single wire



i) We saw that

$$L \triangleq \frac{\mu}{2\pi} \ln \frac{R_p}{r'}, \text{ where } r' = r e^{-\mu r/4}$$

- remember that r is the wire radius, the correction factor $e^{-\mu r/4}$ takes into account the magnetic field inside the wire
- since $R_p \rightarrow \infty \Rightarrow L$ is not well-defined



ii) We saw that

$$L \triangleq \frac{\mu}{2\pi} \ln \frac{D}{r'}, \text{ now } L \text{ is well defined as } R \text{ is finite.}$$

Remember that to obtain $\lambda_1 = \lambda_{11} + \lambda_{12}$
 λ_{11} is flux linked by conductor one due to its own current
 λ_{12} is flux linked by conductor one due to current two

$$\lambda_{11} = \frac{\mu}{2\pi} \ln \frac{R_p}{r'} I_1 + \frac{\mu}{2\pi} \ln \frac{R_p - R}{R} I_2$$

$$\text{but } I_1 = -I_2 = I$$

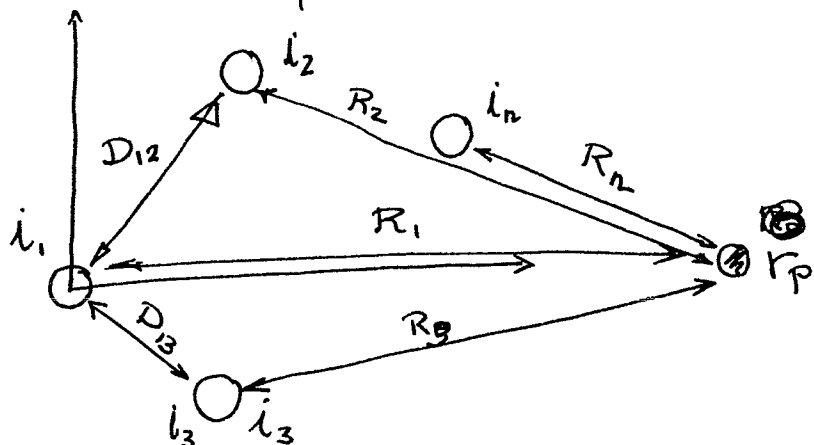
$$\lambda_1 = \frac{\mu}{2\pi} \ln \frac{D}{r'} I \rightarrow L = \frac{\mu}{2\pi} \ln \frac{D}{r'}$$

$$\lambda_1 = \left(\frac{\mu}{2\pi} \ln \frac{D}{r'} + \frac{\mu}{2\pi} \ln \frac{R_p}{R_p - R} \right) I$$

$$\text{as } R_p \rightarrow \infty$$

MANY-CONDUCTOR LINE INDUCTANCE

We will use the same trick as before, assuming the return paths are located @ ∞ and apply superposition



- return path.
- $r_i' \equiv$ equivalent radius of conductor i
 - $R_i \equiv$ distance of conductor i to its return path
 - $D_{ij} \equiv$ distance between conductors i and j

$$\lambda_1 = \lambda_{11} + \lambda_{12} + \dots + \lambda_{1n}$$

$$\lambda_{11} = \frac{\mu_0}{2\pi} \ln \frac{R_1}{r_1'} i_1; \quad \lambda_{12} = \frac{\mu_0}{2\pi} \ln \frac{R_2}{D_{12}} i_2; \dots, \quad \lambda_{1n} = \frac{\mu_0}{2\pi} \ln \frac{R_n}{D_{1n}} i_n$$

$$\lambda_1 = \frac{\mu_0}{2\pi} \left[i_1 \ln \frac{R_1}{r_1'} + i_2 \ln \frac{R_2}{D_{12}} + \dots + i_n \ln \frac{R_n}{D_{1n}} \right] =$$

$$= \frac{\mu_0}{2\pi} \left[i_1 \ln \frac{R_1}{r_1'} + \ln \frac{1}{D_{12}} i_2 + \dots + i_n \ln \frac{1}{D_{1n}} \right] + \underbrace{\frac{\mu_0}{2\pi} \left[\ln R_2 i_2 + \dots + \ln R_n i_n \right]}_{\approx -i_1 R_1}$$

$$\lambda_1 = \frac{\mu_0}{2\pi} \left[i_1 \ln \frac{R_1}{r_1'} + \sum_{j=2}^n i_j \ln \frac{1}{D_{1j}} \right] + \frac{\mu_0}{2\pi} \sum_{j=2}^n i_j \ln R_j$$

$$\sum_{j=2}^n i_j = -i_1 \quad \text{as} \quad R_1 \rightarrow \infty \quad \boxed{R_1 \equiv R_2 \equiv \dots \equiv R_j = R_1}$$

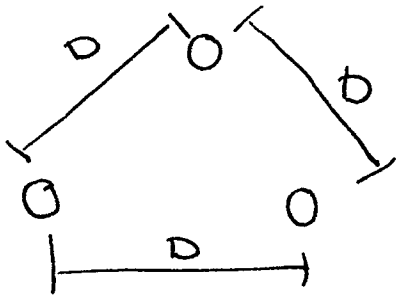
$$\lambda_1 = \frac{\mu_0}{2\pi} \left[i_1 \ln \frac{R_1}{r_1'} + \sum_{j=2}^n i_j \ln \frac{1}{D_{1j}} \right] - \frac{\mu_0}{2\pi} i_1 R_1$$

$$\lambda_1 = \frac{\mu_0}{2\pi} \left[i_1 \ln \frac{1}{r_1'} + i_2 \ln \frac{1}{D_{12}} + \dots + i_n \ln \frac{1}{D_{1n}} \right]$$

$$\boxed{\lambda_1 = L_{11} i_1 + L_{12} i_2 + \dots + L_{1n} i_n} \rightarrow \text{There is coupling, mutual inductances, no good!}$$

REMEMBER THE ASSUMPTION WE MADE WHEN WE TALKED ABOUT PER-PHASE EQUIVALENTS, NO COUPLING!

EXAMPLE



$$\lambda_a = i_a L_{aa} + i_b L_{ab} + i_c L_{ac}$$

$$= \frac{\mu_0}{2\pi} \left[i_a \ln \frac{1}{r'} + i_b \ln \frac{1}{D} + i_c \ln \frac{1}{D} \right]$$

$$\text{but } i_a = -i_b - i_c$$

$$\lambda_a = \frac{\mu_0}{2\pi} i_a \left[\ln \frac{1}{r'} - \ln \frac{1}{D} \right] i_a$$

$$\lambda_a = \frac{\mu_0}{2\pi} \cdot \ln \frac{D}{r'} \cdot i_a \rightarrow$$

$$\rightarrow L_a \triangleq \frac{\mu_0}{2\pi} \cdot \ln \frac{D}{r'}$$

Even if there was coupling between the phases, in a symmetric configuration, the magnetic coupling can be modeled using self-inductances: GOOD!

CONDUCTOR BUNDLING

In reality, transmission lines have more than one conductor per phase:

- Increases the capacity of the line
- Typical values are two conductors for 345KV lines, three for 500KV and four for 765KV

$$\begin{matrix} 1 & 2 \\ 0 & 0 \\ 3 & 4 \\ 0 & 0 \end{matrix} \left\{ A \right.$$

We will use the same notation as before

r_i' $\equiv$ equivalent radius of conductor i

D_{ij} $\equiv$ distances between conductors i and j

As before:

$$\lambda_1 = \sum_{i=1}^{12} \lambda_{1i}$$

$$\begin{matrix} 9 & 10 \\ 0 & 0 \\ 11 & 12 \\ 0 & 0 \end{matrix} \left\{ C \right.$$

$$\begin{matrix} 5 & 6 \\ 0 & 0 \\ 7 & 8 \\ 0 & 0 \end{matrix} \left\{ B \right.$$

Let's build on the previous result for n conductors:

$$\lambda_1 = \frac{N_0}{2\pi} \left[i_1 \cdot \ln \frac{1}{r_1'} + \sum_{j=2}^{12} i_j \cdot \ln \frac{1}{D_{ij}} \right]$$

A few observations: ~~the~~

- The current i_1 is as assumed to be $1/4$ of the current flowing through phase A.
- Similar for the other wires

$$\lambda_1 = \frac{N_0}{2\pi} \left[\frac{i_a}{4} \ln \left(\frac{1}{r_1'} \right) + \sum_{j=2}^4 i_j \ln \frac{1}{D_{ij}} \right] +$$

$$+ \sum_{j=5}^8 i_j \ln \frac{1}{D_{ij}} + \sum_{j=9}^{12} i_j \ln \frac{1}{D_{ij}}$$

$$\lambda_1 = \frac{N_0}{2\pi} \left[\frac{i_a}{4} \left[\ln \frac{1}{r_1'} + \ln \frac{1}{D_{12}} + \ln \frac{1}{D_{13}} + \ln \frac{1}{D_{14}} \right] \right.$$

$$+ \frac{N_0}{2\pi} \left[\frac{i_b}{4} \left[\ln \frac{1}{D_{15}} + \ln \frac{1}{D_{16}} + \ln \frac{1}{D_{17}} + \ln \frac{1}{D_{18}} \right] \right.$$

$$+ \frac{N_0}{2\pi} \left[\frac{i_c}{4} \left[\ln \frac{1}{D_{19}} + \ln \frac{1}{D_{110}} + \ln \frac{1}{D_{111}} + \ln \frac{1}{D_{112}} \right] \right]$$

Let's introduce two definitions that will simplify the notation.

i) Geometric mean radius (GMR) of the bundle (R_b)

$$R_b = \sqrt[4]{r' d_{12} d_{13} d_{14}} \quad (\text{for this example})$$

$$R_b = (r' d_{12} d_{13} d_{14} \dots d_{1b})^{1/b} \text{ for a conductor with } b \text{ wires}$$

ii) Geometric mean distance (GMD) of conductor 1 to phase b & c:

Geometric M.
between phase:

$$D_{1b} = \sqrt[4]{d_{15} d_{16} d_{17} d_{18}} \simeq D_{2b} \simeq D_{3b} \simeq D_{4b} \equiv D_{ab}$$

$$D_{1c} = \sqrt[4]{d_{19} d_{110} d_{111} d_{12}} \simeq D_{2c} \simeq D_{3c} \simeq D_{4c} \equiv D_{ac}$$

Now if $D_{ab} = D_{bc} = D_{ac} = D$ and $i_a = -i_b - i_c$

We get:

$$\lambda_1 = \frac{N_0}{2\pi} \left[i_a \ln \frac{1}{R_b} - i_a \ln \frac{1}{D} \right] = \frac{N_0}{2\pi} \ln \frac{D}{R_b} i_a$$

$$\text{but } i_a = 4 i_1 \rightarrow$$

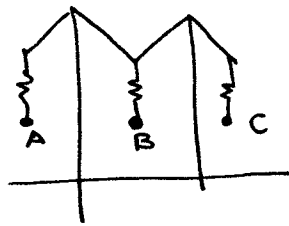
$$\lambda_1 = \frac{N_0}{2\pi} 4 \cdot \ln \frac{D}{R_b} i_1 \rightarrow \boxed{\ell_1 \triangleq \frac{N_0}{2\pi} 4 \ln \frac{D}{R_b}}$$

But remember that phase a are 4 wires in parallel: $\ell_1 \equiv \ell_2 = \ell_3 = \ell_4$

$$\Rightarrow \boxed{\ell_A = \frac{N_0}{2\pi} \ln \frac{D}{R_b}} \quad \rightarrow \text{The formula is consistent with previous expressions}$$

TRANSPOSITION

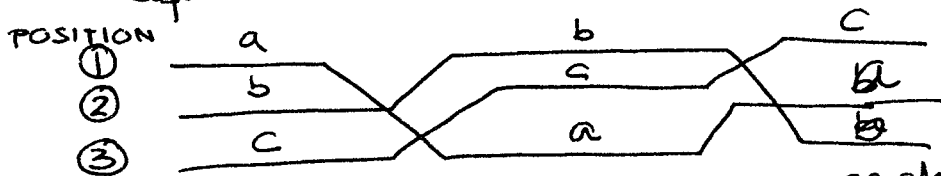
- What happens if the tower configuration is not symmetric? (GMD)



$$D_{ab} \neq D_{bc} \neq D_{ca}$$

→ Unless we do something, we will get unbalanced phases.

- To keep the system balanced over the length of a transmission line, the conductors are rotated so each phase occupies each position of the tower for an equal distance. This is known as TRANSPOSITION



Flux linkages (assume that ~~the line~~ each phase physically occupies position 1, 2, and 3 the same length:

$$\lambda_a = \frac{1}{3} \frac{N_0}{2\pi} \left[l_a \ln \frac{1}{r_1} + l_b \ln \frac{1}{D_{12}} + l_c \ln \frac{1}{D_{13}} \right] \text{ "a is in position 1"}$$

$$+ \frac{1}{3} \frac{N_0}{2\pi} \left[l_a \ln \frac{1}{r_1} + l_b \ln \frac{1}{D_{13}} + l_c \ln \frac{1}{D_{23}} \right] \text{ "a is in position 2"}$$

$$+ \frac{1}{3} \frac{N_0}{2\pi} \left[l_a \ln \frac{1}{r_1} + l_b \ln \frac{1}{D_{23}} + l_c \ln \frac{1}{D_{12}} \right] \text{ "a is in position 3"}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$3l_a \ln \frac{1}{r_1} \quad l_b \ln \frac{1}{D_{12} D_{13} D_{23}} \quad l_c \ln \frac{1}{D_{13} D_{12} D_{23}}$$

$$\lambda_a = \frac{N_0}{2\pi} \left[l_a \ln \frac{1}{r_1} + l_b \ln \frac{1}{\sqrt{D_{12} D_{13} D_{23}}} + l_c \ln \frac{1}{\sqrt{D_{12} D_{13} D_{23}}} \right]$$

$$D_m \triangleq \sqrt{D_{12} D_{13} D_{23}} \quad \rightarrow \text{Geometric mean distance between physical positions}$$

$$L_a \triangleq \frac{N_0}{2\pi} \ln \frac{D_m}{r_1}$$

(Assuming $l_b + l_c = l_a$)