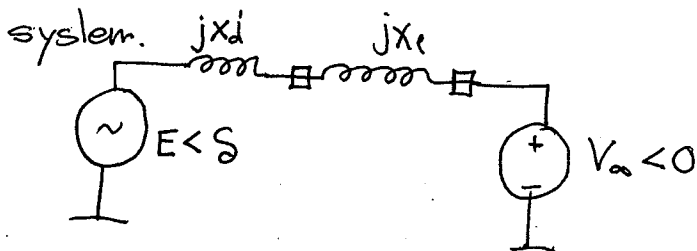


LECTURE 23

TRANSIENT STABILITY

- Small-signal stability was concerned with small deviations from the eq. point. $\Rightarrow$ we saw that linearization techniques are appropriate
- Transient stability is concerned with events that cause large excursions from the equilibrium point. Linearization techniques are not appropriate to study these phenomena.
- We will introduce all important notions in transient stability of power system with the single machine infinite bus system.



We assume an event causing the transmission line to open occurs at $t=0$. We will also assume that the system is in steady-state prior to the fault occurrence. We will neglect the friction coefficient.

$$M \cdot \frac{d^2 \delta}{dt^2} + D \frac{d\delta}{dt} = P_m - \frac{E \cdot V_\infty}{X_d' + X_l} \sin \delta$$

In steady-state $P_m - \frac{E \cdot V_\infty}{X_d' + X_l} \sin \delta = 0 \rightarrow \delta_0 = \arcsin \left(\frac{P_m}{\frac{E \cdot V_\infty}{X_d' + X_l}} \right)$

STAGE 1 $t < 0$

$$\delta = \delta_0 ; \dot{\delta} = 0 \quad P_G(\delta_0) = \frac{E \cdot V_\infty}{X_d' + X_l} \sin \delta_0 = P_m$$

STAGE 2

The transmission line opens and the system dynamics will evolve according to

$$M \cdot \frac{d^2 \delta}{dt^2} = P_M$$

$$\delta(0) = \delta_0$$

$$\dot{\delta}(0) = 0$$

The solution can be found by integration:

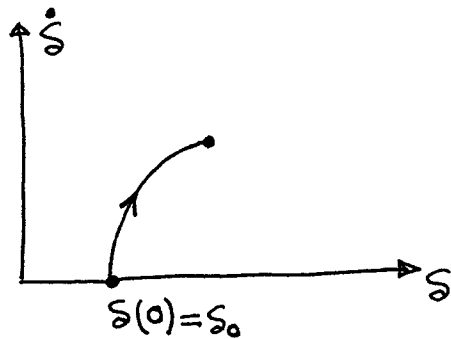
$$\dot{\delta}(t) = \frac{P_M}{M} t + \dot{\delta}(0) = \frac{P_M}{M} t \quad [\text{rad/s}]$$

$$\delta(t) = \frac{P_M}{2M} t^2 + \delta_0 \quad [\text{rad}]$$

The evolution of δ and $\dot{\delta}$ as a function of time is depicted in the phase-plane

$$t = \frac{M}{P_M} \dot{\delta}(t) \rightarrow \delta(t) - \delta_0 = \frac{P_M}{2M} \cdot \frac{M^2}{P_M^2} \dot{\delta}^2(t)$$

$$\boxed{\delta(t) - \delta_0 = \frac{M}{2P_M} \dot{\delta}^2(t)} \quad \rightarrow \text{This is a parabola}$$



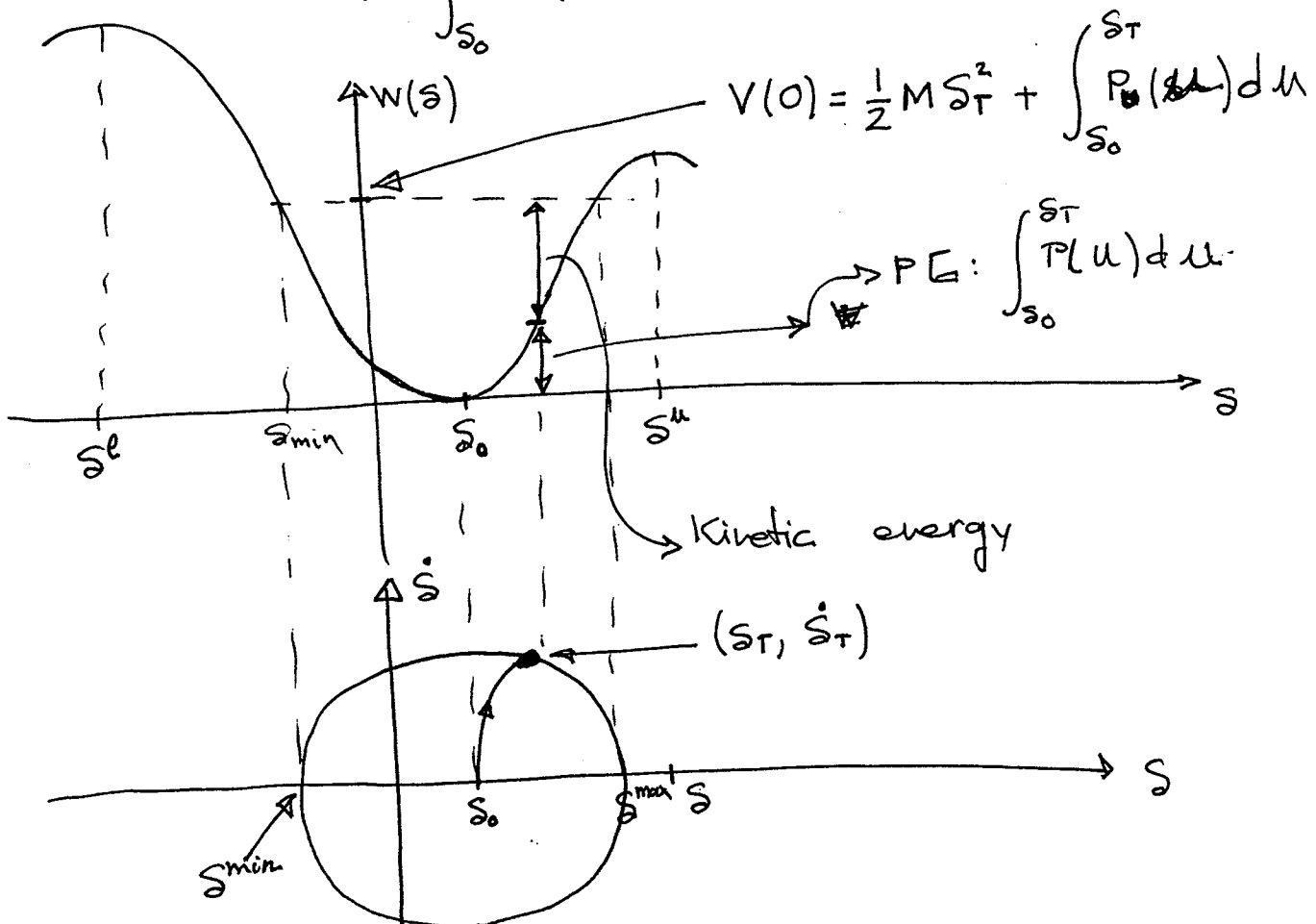
- Before anything happens (STAGE 1) $V=0$ ($\dot{S}=0$ and $\int_{S_0}^{S_0} P(u) du = 0$)

- In stage 2, the system acquires both Kinetic and potential energy and at the beginning of STAGE 3 the system has some initial energy.

- Since the system is conservative (no friction), the energy should be conserved during the system evolution on STAGE 3

• The key factor determining system behavior is the potential energy curve.

$$W(S) = \int_{S_0}^S P(u) du = -P_m (S - S_0) - \frac{E \cdot V_{\infty}}{X_d + X_l} [\cos S - \cos S_0]$$



STAGE 3

The transmission line is connected at $t=T$, and the system will evolve according to a new differential eqn.:

$$M \frac{d^2 S}{dt^2} = P_M - \frac{E \cdot V_{\infty}}{X_d' + X_l} \sin S$$

The initial conditions are the final conditions of STAGE 2:

$$\left\{ \begin{aligned} S(0) &= S(T) = \frac{P_M}{2M} T^2 + S_0 \\ \dot{S}(0) &= \frac{P_M}{M} T \end{aligned} \right.$$

$$\left\{ \begin{aligned} \dot{S}(0) &= \frac{P_M}{M} T \end{aligned} \right. \rightarrow P(S)$$

$$\boxed{\begin{aligned} M \frac{d^2 S}{dt^2} + \frac{E \cdot V_{\infty}}{X_d' + X_l} \sin S - P_M &= 0 \\ S(0) &= \frac{P_M}{2M} T^2 + S_0 \\ \dot{S}(0) &= \frac{P_M}{M} T \end{aligned}}$$

$$M \frac{d^2 S}{dt^2} = \underbrace{P_M - \frac{E \cdot V_{\infty}}{X_d' + X_l} \sin S}_{\text{This is the force}}$$

We are now interested in the properties of

$$\left\{ \begin{aligned} M \frac{d^2 S}{dt^2} + P(S) &= 0 \\ S(0) &= \frac{P_M}{2M} T^2 + S_0 \\ \dot{S}(0) &= \frac{P_M}{M} T \end{aligned} \right.$$

$\rightarrow$ This is an isolated system and we can use energy techniques to understand the system evolution.

- The initial energy of the system can be calculated since we know $S(0)$ and $\dot{S}(0)$

- In general, we can calculate the energy as the sum of the kinetic and potential energy

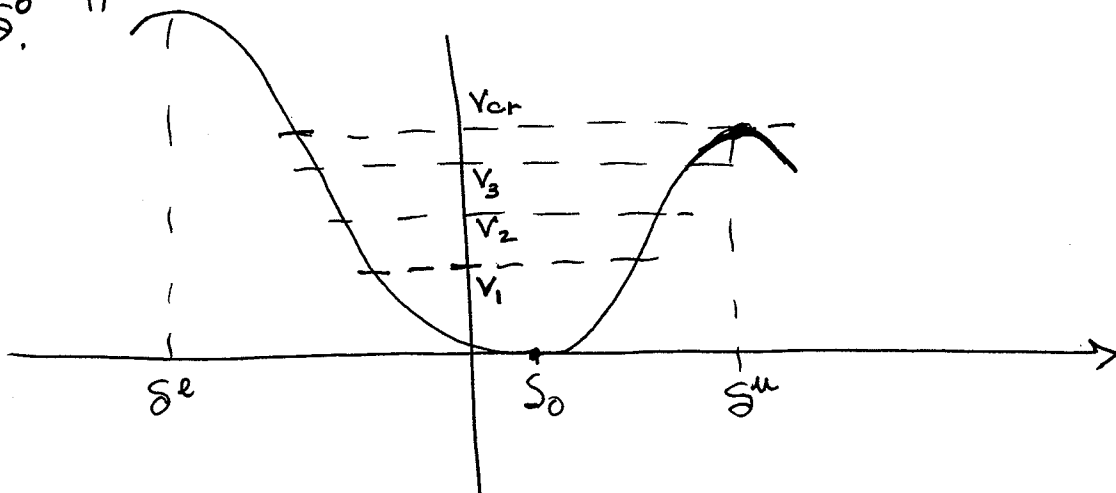
$$V(S) = \underbrace{\frac{1}{2} M \dot{S}^2}_{\text{Kinetic energy}} + \underbrace{\int_{S_0}^S P(u) du}_{\text{Potential energy}}$$

- Let's assume that we don't neglect the friction (i.e., $D \neq 0$)
- It is no longer true that $V(S(t)) = V(S_1) \forall t > 0$.
- What will happen is that the energy will dissipate as time evolves. Let's check this out by looking at the sign of $\dot{V}(S(t))$

$$\begin{aligned}\dot{V}(S(t)) &= \frac{\partial V}{\partial \dot{S}} \cdot \frac{d\dot{S}}{dt} + \frac{\partial V}{\partial S} \cdot \frac{dS}{dt} = M \cdot \dot{S} \cdot \ddot{S} + P(S) \cdot \dot{S} \\ &= \underbrace{[M \cdot \ddot{S} + P(S)]}_{-D \cdot \dot{S}} \dot{S} = -D \cdot \dot{S}^2 \leq 0\end{aligned}$$

EQUAL AREA CRITERIA

When we studied the evolution of the system by looking at the potential energy barrier, we assume that the ~~total~~ energy at the beginning of STAGE 3 was smaller than the top of the hills, in fact if the energy at the beginning of the period is larger than the maximum of the potential energy barrier, the system will never return to S^0 .



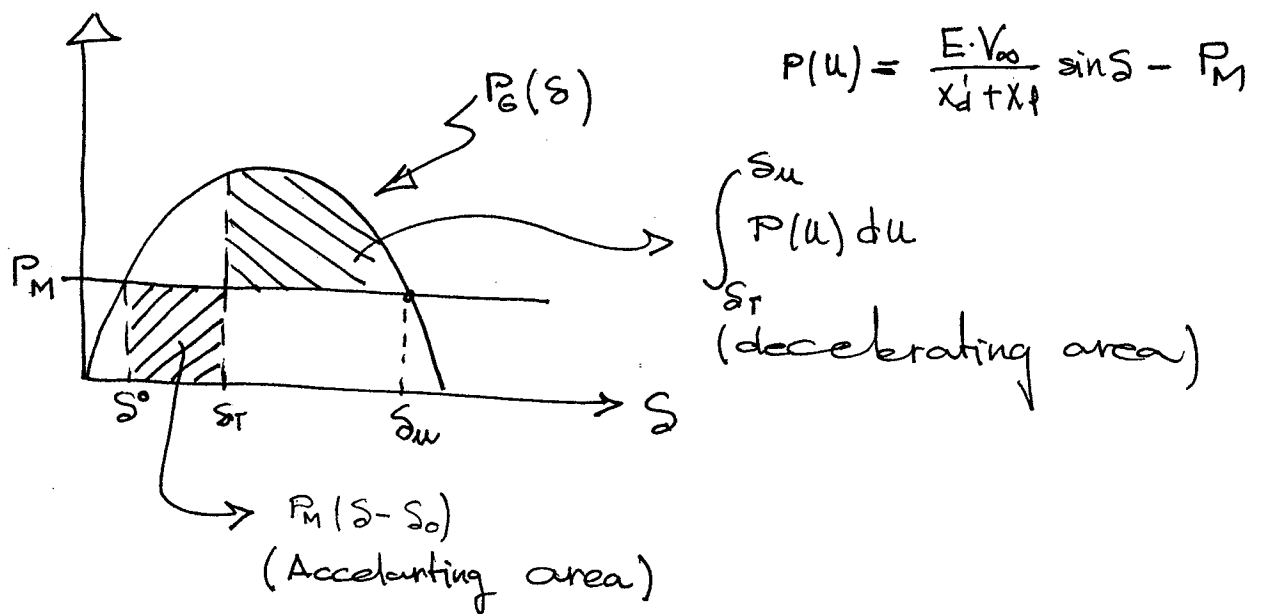
if $V > V_{cr} \rightarrow$ the system will not remain in sync.

$$\frac{1}{2} M \dot{S}_T^2 + \int_{S_0}^{S_T} P(u) du < \int_{S_0}^{S_u} P(u) du \rightarrow$$

$$\rightarrow \boxed{\frac{1}{2} M \dot{S}_T^2 < \int_{S_T}^{S_u} P(u) du} \rightarrow \text{The system remains stable.}$$

We also saw that at the end of STAGE 2

$$S - S^0 = \frac{M}{2P_M} \dot{S}^2 \rightarrow \frac{1}{2} M \dot{S}^2 = (S - S_0) P_M$$



CRITICAL RECLOSURE TIME

There is a time T_{cr} when

$$\boxed{\frac{1}{2} M \dot{S}_T^2 = \int_{S_T}^{S_u} P(u) du}$$

if $T < T_{cr} \rightarrow$ system remains stable

if $T > T_{cr} \rightarrow$ system will not be unstable.