

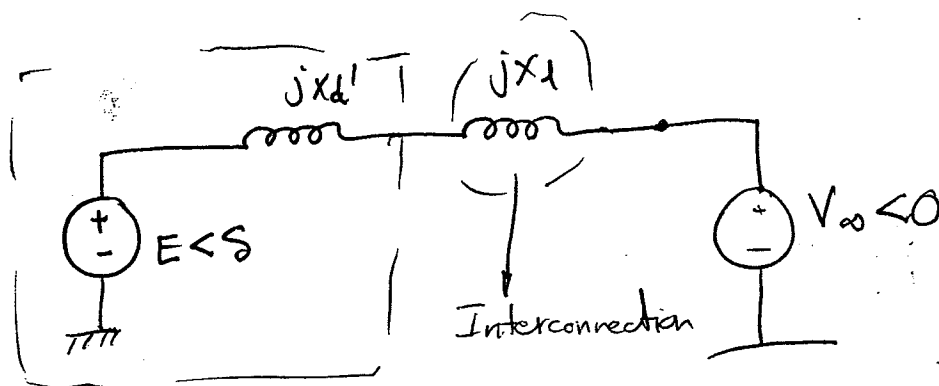
LECTURE 22

SMALL SIGNAL STABILITY TRANSIENT STABILITY

The canonical problem we are trying to solve is to determine whether or not the system remains stable after a disturbance. We have two types of disturbances:

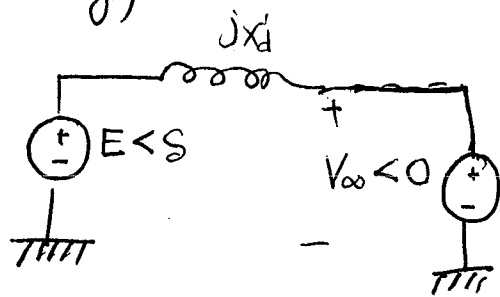
- small disturbances
- large disturbances

Although power systems are extremely large nonlinear dynamical systems, most stability issues can be appreciated with a very simplistic model: the single machine infinite bus, which assumes a synchronous machine (the stability of which we want to study) is connected to a voltage source that models the rest of the system



SINGLE-MACHINE INFINITE BUS

- In the previous lecture, we derived a machine model suited for studying dynamic phenomena in power systems - the so-called swing model (basically a nonlinear spring)



- We derive the equation that governs the evolution of S (the torque angle)

- In per-unit:

$$M \ddot{S} + D \dot{S} = P_m - \frac{E \cdot V_{\infty}}{X_d'} \sin S$$

- We can rewrite this equations in state-space (430 material)

$$\dot{S} = \omega_m - \omega_s$$

$$\dot{\omega}_m = \omega_m$$

$$\frac{dS}{dt} = \omega_m - \omega_s$$

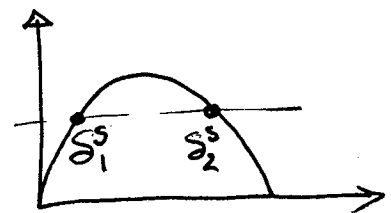
$$\frac{d\omega_m}{dt} = -\frac{D}{M} (\omega_m - \omega_s) - \frac{E \cdot V_{\infty}}{M X_d'} \sin S + \frac{1}{M} P_m$$

which is of the form $\dot{x} = f(x)$, where $x = [S, \omega_m]'$

SMALL-SIGNAL STABILITY

- We can think of small disturbances as for example, small variations in V_{∞} or P_m (the mechanical power slightly fluctuates or the voltage at the infinite bus).
- In this case, we can study the stability of the linearized system around the equilibrium point.
- Eq. point $\dot{S} = \dot{W}_m = 0 \rightarrow W_m - W_s = 0$

$$\delta^s = \arcsin \left(\frac{P_m}{\frac{E \cdot V_{\infty}}{X_d + X_L}} \right)$$



Linearization

Remember from 430: $\dot{x} = f(x) \rightarrow$

$$\rightarrow x \approx x^s + \Delta x \rightarrow \Delta \dot{x} = f'(x^s) + \left. \frac{df(x)}{dx} \right|_{x_s} \Delta x$$

If we have inputs, we have an additional term for the input: $\dot{x} = f(x, u) \leftrightarrow f(x_s, u_n) = 0$

$$\Delta \dot{x} = \left. \frac{\partial f(x, u)}{\partial x} \right|_{x_s, u_n} \Delta x + \left. \frac{\partial f(x, u)}{\partial u} \right|_{x_s, u_n} \Delta u$$

Let's study the case when the system is in equilibrium for some value V_{∞}^s and P_m^s and let's perturb it slightly. The linearized model is:

$$S = S_s + \Delta S \longrightarrow W = W_s + \Delta W$$

Let's denote $W_r = W_m - W_s$

$$\frac{dS}{dt} = W_r$$

$$\frac{dW_r}{dt} = -\frac{DW_r}{M} - \frac{E \cdot V_{\infty}}{M(X_d' + X_l)} \sin S + \frac{1}{M} P_m$$

$$\frac{d}{dt} \begin{bmatrix} \Delta S \\ \Delta W_r \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{E V_{\infty} \cos S^s}{M(X_d' + X_l)} & -\frac{D}{M} \end{bmatrix}}_A \begin{bmatrix} \Delta S \\ \Delta W_r \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -\frac{E \cdot V_{\infty}}{M(X_d' + X_l)} \sin S^s \end{bmatrix}}_B \Delta V_{\infty}$$

Eigenvalues of A

$$\lambda^2 + \frac{D}{M} \lambda + \frac{E \cdot V_{\infty} \cdot \cos S^s}{M(X_m' + X_l)} = 0$$

$$\lambda = \frac{-\frac{D}{M} \pm \sqrt{\left(\frac{D}{M}\right)^2 - 4 \cdot \frac{E \cdot V_{\infty} \cos S^s}{M(X_m' + X_l)}}}{2}$$

if $\cos S^s > 0$ both $\lambda < 0 \rightarrow$ the system is small-signal stable.

if $\cos S^s < 0$ one eigenvalue is $> 0 \rightarrow$ system is unstable