

LECTURE 20

ECONOMIC DISPATCH

- CONSTRAINTS ON GENERATORS
- NUMERICAL EXAMPLES
- INCLUSION OF LOSSES

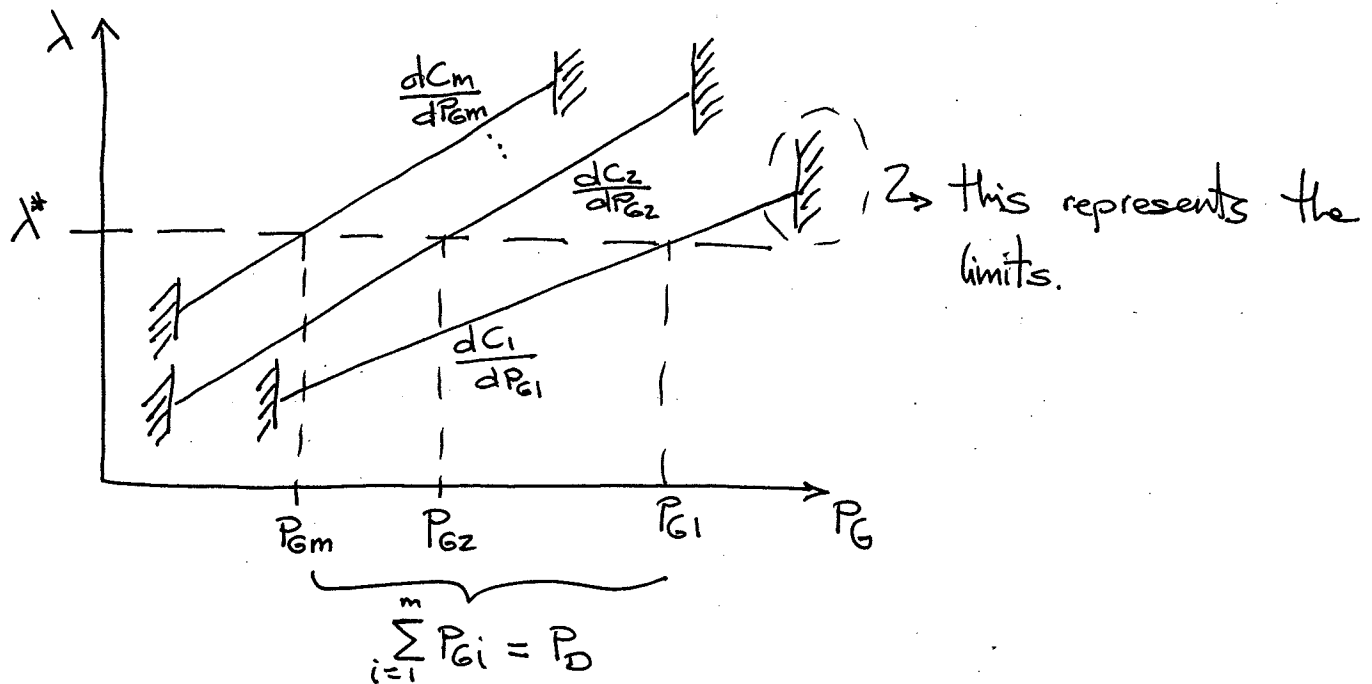
ECONOMIC DISPATCH PROBLEM WITH GENERATION CONSTRAINTS

$$\min C_T = \sum_{i=1}^m C_i(P_{Gi}) = \sum_{i=1}^m (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) \quad (1)$$

$$\text{subject to } \sum_{i=1}^m P_{Gi} = P_D \quad (2)$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad \forall i=1, 2, \dots, m \quad (3)$$

It is obvious that if the solution to the unconstrained case discussed in Lecture 19 satisfies (2) and (3), then it must also satisfy (1).



Now if P_D starts increasing slowly, so will λ and for large enough increase, we will hit the upper limit of generator ①. At this point we can not squeeze more power out of G1.

We fix its power at $P_{Gi}^{\max}$ and continue the procedure with the other generators, if we hit other limits, we repeat the procedure.

• In the lambda iteration procedure, we include this as follows

• Initial guess λ_H^0 and λ_L^0 . We repeat for each generator

• if $P_{Gi}(\lambda_L^0) < P_{Gi}^{\min}$ then $P_{Gi}(\lambda_L^0) = P_{Gi}^{\min}$

• if $P_{Gi}(\lambda_H^0) > P_{Gi}^{\max}$ then $P_{Gi}(\lambda_H^0) = P_{Gi}^{\max}$

while $|\lambda_H^v - \lambda_L^v| > \epsilon$

$$\lambda_M^v = \frac{\lambda_H^v + \lambda_L^v}{2}$$

• if $P_{Gi}(\lambda_M^v) < P_{Gi}^{\min}$ then $P_{Gi}(\lambda_M^v) = P_{Gi}^{\min}$

• if $P_{Gi}(\lambda_M^v) > P_{Gi}^{\max}$ then $P_{Gi}(\lambda_M^v) = P_{Gi}^{\max}$

• if $\sum_{i=1}^n P_{Gi}(\lambda_M^v - P_D) < 0$ then $\begin{cases} \lambda_L^{v+1} = \lambda_M^v \\ \lambda_H^{v+1} = \lambda_H^v \end{cases}$

else $\begin{cases} \lambda_L^{v+1} = \lambda_L^v \\ \lambda_H^{v+1} = \lambda_M^v \end{cases}$

NUMERICAL EXAMPLES

i) Two-generator economic dispatch with no constraints

$$\min_{P_{G1}, P_{G2}} (100 + 20P_{G1} + 0.01P_{G1}^2) + (400 + 15P_{G2} + 0.03P_{G2}^2)$$

subject to $P_{G1} + P_{G2} = 500 \text{ MW}$

We can solve the problem using the Lagrange multipliers method. Thus the above optimization is equivalent to

$$\min_{P_{G1}, P_{G2}, \lambda} \underbrace{(100 + 20P_{G1} + 0.01P_{G1}^2) + (400 + 15P_{G2} + 0.03P_{G2}^2) + \lambda(500 - P_{G1} - P_{G2})}_{\hat{C}_T(P_{G1}, P_{G2}, \lambda)}$$

we know the solution is :

$$\frac{\partial \hat{C}_T}{\partial P_{G1}} = 0 \rightarrow 20 + 0.02P_{G1} - \lambda = 0$$

$$\frac{\partial \hat{C}_T}{\partial P_{G2}} = 0 \rightarrow 15 + 0.06P_{G2} - \lambda = 0$$

$$\frac{\partial \hat{C}_T}{\partial P_{G3}} = 0 \quad 500 - P_{G1} - P_{G2} = 0$$

$$\begin{bmatrix} 0.02 & 0 & -1 \\ 0 & 0.06 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ 500 \end{bmatrix} \rightarrow \begin{bmatrix} P_{G1}^* \\ P_{G2}^* \\ \lambda \end{bmatrix} = \begin{bmatrix} 312.5 \text{ MW} \\ 187.5 \text{ MW} \\ 26.2 \text{ \$/MWh} \end{bmatrix}$$

$$\bullet \frac{\partial \hat{C}_T}{\partial P_{G1}} = 0 \Leftrightarrow \frac{\partial C_1}{\partial P_{G1}} = \lambda \quad (IC_1(P_{G1})) \Rightarrow 20 + 0.02P_{G1} = \lambda$$

$$\bullet \frac{\partial \hat{C}_T}{\partial P_{G2}} = 0 \Leftrightarrow \frac{\partial C_2}{\partial P_{G2}} = \lambda \quad (IC_2(P_{G2})) \Rightarrow$$

ii) Three-generator example using bmbada iteration.

$$C_1 = 400 + 15P_{G1} + 0.01P_{G1}^2$$

$$C_2 = 300 + 20P_{G2} + 0.005P_{G2}^2$$

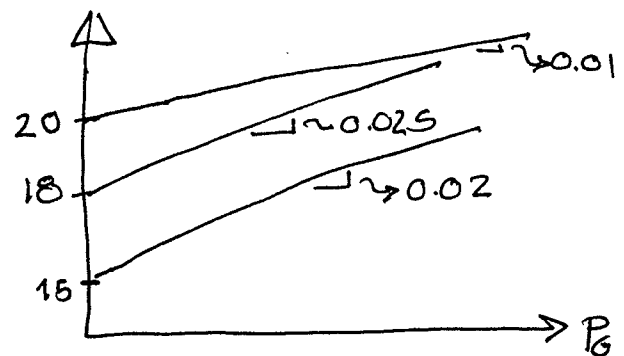
$$C_3 = 200 + 18P_{G3} + 0.0125P_{G3}^2$$

$$\text{with } P_{G1} + P_{G2} + P_{G3} = 1000 \text{ MW}$$

$$IC_1 = \frac{\partial C_1}{\partial P_{G1}} = 15 + 0.02P_{G1}$$

$$IC_2 = \frac{\partial C_2}{\partial P_{G2}} = 20 + 0.01P_{G2}$$

$$IC_3 = \frac{\partial C_3}{\partial P_{G3}} = 18 + 0.025P_{G3}$$



$$P_{G1}(\lambda) = \frac{\lambda - 15}{0.02} ; \quad P_{G2}(\lambda) = \frac{\lambda - 20}{0.01} ; \quad P_{G3}(\lambda) = \frac{\lambda - 18}{0.025}$$

We pick λ_L^0 such that $\sum_{i=1}^3 P_{Gi}(\lambda_L^0) - 1000 < 0$ and
 λ_H^0 such that $\sum_{i=1}^3 P_{Gi}(\lambda_H^0) - 1000 > 0$

For example $\lambda_L^0 = 20$ results in $\sum_{i=1}^3 P_{Gi}(\lambda_L^0) - 1000 = -670 \text{ MW}$
 $\lambda_H^0 = 30$ results in $\sum_{i=1}^3 P_{Gi}(\lambda_H^0) - 1000 = 1230 \text{ MW}$

Let's pick convergence tolerance $\epsilon = 0.05$

$$\lambda_M^0 = \frac{\lambda_H^0 + \lambda_L^0}{2} = 25$$

$$\text{since } \sum_{i=1}^3 P_{Gi}(25) - 1000 = 280 \rightarrow \lambda_H^1 = \lambda_M^0 = 25$$

$$\lambda_L^1 = \lambda_L^0 = 20$$

$$|25 - 20| > 0.05 \dots$$

$$\lambda_M^1 = \frac{\lambda_H^1 + \lambda_L^1}{2} = \frac{20 + 25}{2} = 22.5$$

$$\sum_{i=1}^3 P_{Gi}(22.5) - 1000 = -195 \rightarrow \lambda_L^2 = 22.5 \quad \lambda_M^1$$

Continue the iteration until $\lambda^* = 23.53 \text{ \$/MWh}$

$$P_{G1}(23.5) = \frac{23.53 - 15}{0.02} = 426 \text{ MW}$$

$$P_{G2}(23.5) = \dots = 353 \text{ MW}$$

$$P_{G3}(23.5) = \dots = 221 \text{ MW}$$

iii) Three-generator example with gen limits.

→ Same example as before, but now assume

$$0 \leq P_{G1} \leq 300 \text{ MW}$$

$$100 \leq P_{G2} \leq 500 \text{ MW}$$

$$200 \leq P_{G3} \leq 600 \text{ MW}$$

$$\lambda_L^0 = 20 \rightarrow \sum_{i=1}^3 P_{Gi} - 1000 = P_{G1}(20) + P_{G2}(20) + P_{G3}(20) - 1000 = 250 + 100 + 200 = -670$$

$$\lambda_H^0 = 20 \rightarrow \sum_{i=1}^3 P_{Gi} - 1000 = 300 + 500 + 400 - 1000 = 200 \text{ MW}$$

$\uparrow$ limit $\uparrow$ limit
 $\uparrow$ limit $\uparrow$ limit

The final solution is $\lambda^* = 24.43 \text{ \$/MWh}$.

The presence of limits will always cause λ to either increase or remain the same.

$$P_{G1}(24.43) = 300 \text{ MW (limit)}$$

$$P_{G2}(24.43) = 443 \text{ MW}$$

$$P_{G3}(24.43) = 257 \text{ MW}$$

INCLUSION OF TRANSMISSION LINE LOSSES

- So far, we have considered the case with no losses:

$\sum_{i=1}^m P_{Gi} = P_D$, which is reasonable if all generators are very closely located to each other.

- If the power plants are geographically dispersed, then it is necessary to include losses.

• We can write now $\sum_{i=1}^m P_{Gi} = P_D + P_L$, where P_L are the "losses" in the system.

• P_L is a function of $P_{G2}, P_{G3}, \dots, P_{Gm}$, not P_{G1} explicitly as $P_{G1} = P_D + P_L(P_{G2}, \dots, P_{Gm})$

$$\min C_T = \sum_{i=1}^m C_i(P_{Gi})$$

$$\text{subject to } \sum_{i=1}^m P_{Gi} - P_L(P_{G2}, P_{G3}, \dots, P_{Gm}) - P_D = 0$$

Which we can solve as

$$\min_{P_{G1}, P_{G2}, \dots, P_{Gm}, \lambda} \sum_{i=1}^m C_i(P_{Gi}) + \lambda \left(P_D - \sum_{i=1}^m P_{Gi} + P_L(P_{G2}, P_{G3}, \dots, P_{Gm}) \right)$$

$$\Rightarrow \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda \cdot \left(1 - \frac{\partial P_L}{\partial P_{Gi}} \right) = 0 \quad \forall i=2, 3, \dots, m$$

$$\rightarrow \frac{dC_i(P_{Gi})}{dP_{Gi}} = \lambda \cdot \left(1 - \frac{\partial P_L}{\partial P_{Gi}} \right)$$

Penalty factor: $L_i = \left(1 - \frac{\partial P_L}{\partial P_{Gi}} \right)^{-1}$ thus:

$$L_i \cdot \frac{dC_i(P_{Gi})}{dP_{Gi}} = \lambda \quad \forall i=2, \dots, m$$

$$\frac{dC_1(P_{G1})}{dP_{G1}} = \lambda \quad (L_1 = 1)$$