

LECTURE 19

ECONOMIC DISPATCH

- UNCONSTRAINED SOLUTION
- λ ITERATION PROCEDURE

ECONOMIC DISPATCH REVIEW

- The goal is to determine the generation dispatch that minimizes operating costs subject to some constraints:

$$\min C_T = \sum_{i=1}^m C_i(P_{Gi}) \quad \text{where } C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2$$

subject to:

- i) $P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad \forall i=1,2,\dots,m$ ~~Not~~ (Generator constraints)
- ii) $S_j = V_i \cdot \left(\sum_K Y_{ik} V_k \right)^* \quad \forall i=1,2,\dots,n$ (Network eqns)
- iii) $|P_{ij}| \leq P_{ij}^{\max}$ for all lines (flow constraints)
- iv) $V_i^{\min} \leq V_i \leq V_i^{\max}$ (Voltage constraints)
- ... (more)

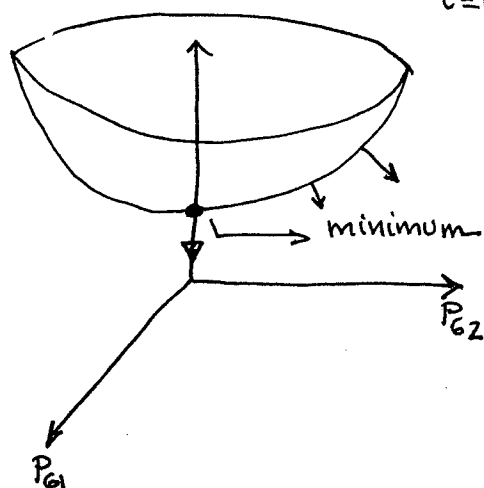
CLASSICAL ECONOMIC DISPATCH

We neglect i), iii), iv) and assume no losses. The problem simplifies to:

$$\min C_T = \sum_{i=1}^m C_i(P_{Gi}) = \sum_{i=1}^m (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2)$$
$$\text{subject to } \sum_{i=1}^m P_{Gi} = P_D = \sum_{i=m+1}^n P_{Di}$$

- This is a constrained minimization problem.
- We can reformulate the problem so as to get rid of the equality constraint.
- Let's see first what the solution would look like if it weren't for the constraints.
- Let's assume $m=2$

$$\min C_T = \sum_{i=1}^2 (\alpha_i + \beta_i P_{Gi} + \delta_i \cdot P_{Gi}^2) \quad \alpha_i, \beta_i, \delta_i > 0$$



minimum corresponds to $\nabla C_T = 0$

gradient describes the vector normal to C_T

$$\begin{bmatrix} \frac{\partial C_T}{\partial P_{G1}} & \frac{\partial C_T}{\partial P_{G2}} \end{bmatrix} = 0$$

$$\beta_i + 2\delta_i \cdot P_{Gi} = 0 \rightarrow$$

$$\rightarrow P_{Gi} = -\frac{\beta_i}{2\delta_i}$$

in our setup this solution does not make physical sense!

In general in the unconstrained case:

$$\boxed{\frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} = 0} \quad \forall i=1, 2, \dots, m$$

$\frac{\partial C_i(P_{Gi})}{\partial P_{Gi}}$ is the so-called "incremental cost" for generator i . It represents the increase in cost per increase in MWh of produced electricity.

Let's put the constraint back:

$$\min C_T \triangleq \sum_{i=1}^m C_i(P_{Gi})$$

$$\text{subject to } P_{G1} + P_{G2} + \dots + P_{Gm} = P_D$$

The key idea is to turn the above constrained minimization problem into an unconstrained minimization problem.

This can be done by introducing a so-called Lagrange multiplier λ and reformulate the problem as follows:

$$\min_{P_{G1}, P_{G2}, \dots, P_{Gm}, \lambda} C_T + \lambda (P_D - \sum_{i=1}^m P_{Gi})$$

$$\min_{P_{G1}, P_{G2}, \dots, P_{Gm}, \lambda} \sum_{i=1}^m (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \lambda (P_D - \sum_{i=1}^m P_{Gi})$$

$\uparrow$
 λ becomes a decision variable.

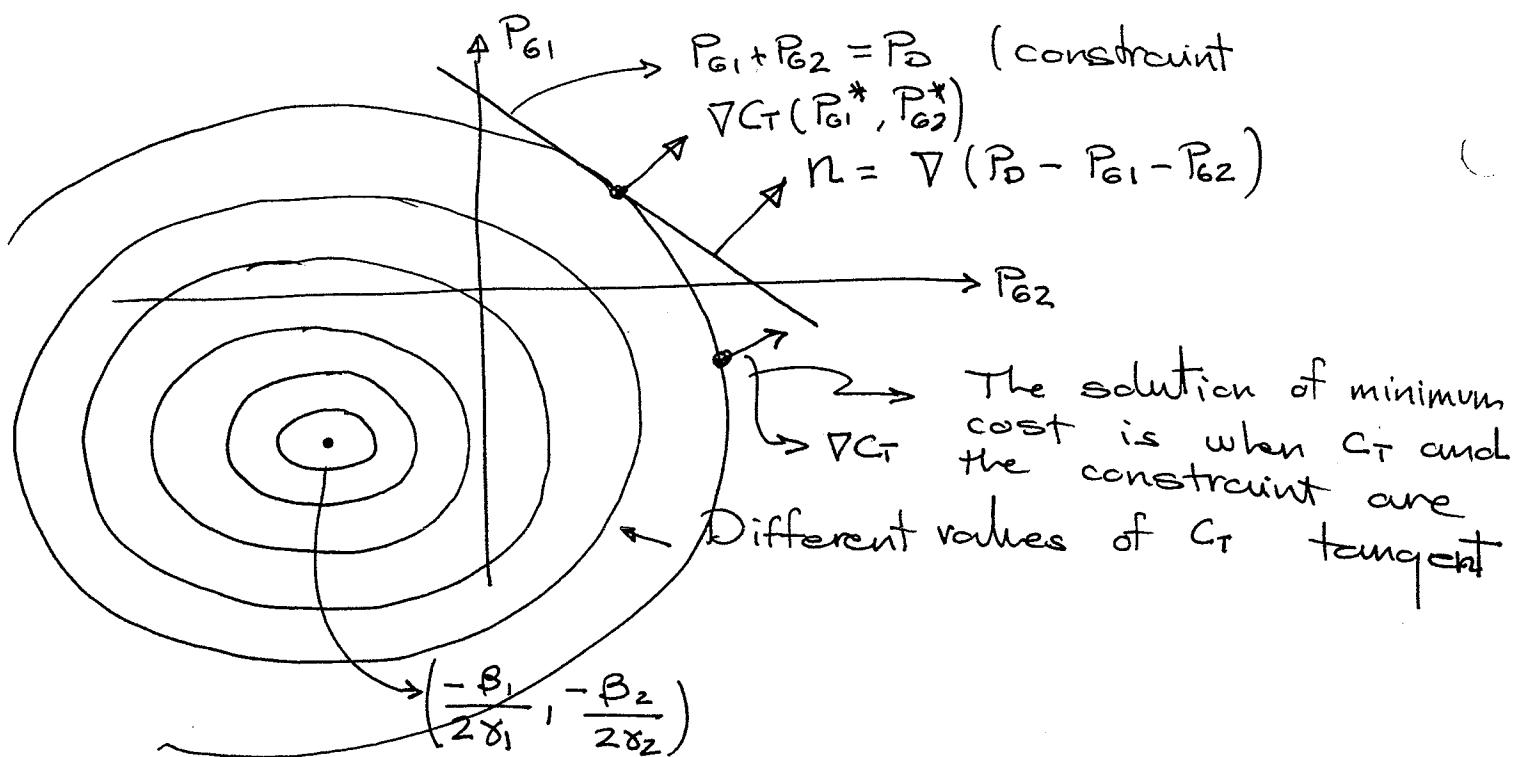
Let's try to understand why this works by looking at the geometry of the problem.

$$C_T = \alpha_1 + \beta_1 P_{G1} + \gamma_1 P_{G1}^2 + \alpha_2 + \beta_2 P_{G2} + \gamma_2 P_{G2}^2$$

It is easy to show that we can write the above expression as:

$$C_T = \gamma_1 \left(P_{G1} + \frac{\beta_1}{2\gamma_1} \right)^2 + \gamma_2 \left(P_{G2} + \frac{\beta_2}{2\gamma_2} \right)^2 - \frac{\beta_1^2}{4\gamma_1} - \frac{\beta_2^2}{4\gamma_2}$$

This is the equation of an ellipse.



For the ellipse to be tangent to the constraint, it implies that the normal vector to the constraint $n = [1, 1]$ and the gradient of the ellipse must be parallel, which implies that: there exists some λ such that:

$$\nabla C_T - \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0 \quad \leftarrow \quad \nabla C_T + \lambda \cdot \nabla(P_D - P_{G1} - P_{G2}) = 0$$

$$\frac{\partial C_T}{\partial P_{G1}} - \lambda = 0$$

$$\frac{\partial C_T}{\partial P_{G2}} - \lambda = 0$$

$$\nabla(C_T + \lambda(P_D - P_{G1} - P_{G2})) = 0$$

↓

This is equivalent to

$$\min_{P_{G1}, P_{G2}, \lambda} (C_T + \lambda(P_D - P_{G1} - P_{G2})) = \infty$$

In order to solve the problem, we need to solve:

$$\left. \begin{aligned} \frac{\partial C_T}{\partial P_{G1}} - \lambda &= 0 \rightarrow \beta_1 + 2\gamma_1 P_{G1} - \lambda = 0 \\ \frac{\partial C_T}{\partial P_{G2}} - \lambda &= 0 \rightarrow \beta_2 + 2\gamma_2 P_{G2} - \lambda = 0 \\ P_{G1} + P_{G2} &= P_D \end{aligned} \right\} \rightarrow$$

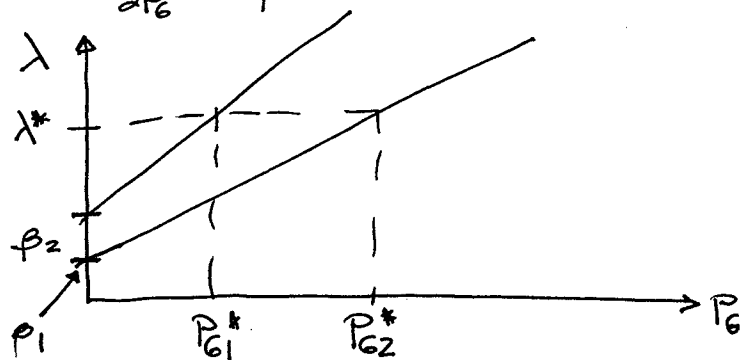
$$\rightarrow \begin{bmatrix} 2\gamma_1 & 0 & -1 \\ 0 & 2\gamma_2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -\beta_1 \\ -\beta_2 \\ P_D \end{bmatrix} \rightarrow \boxed{P_{G1}^*, P_{G2}^*, \lambda^*}$$

↑
optimal solution

GRAPHICAL INTERPRETATION

$$\lambda = \frac{\partial C_T}{\partial P_{G1}} = \beta_1 + 2\gamma_1 P_{G1} \quad P_{G1} + P_{G2} = P_D$$

$$\lambda = \frac{\partial C_T}{\partial P_{G2}} = \beta_2 + 2\gamma_2 P_{G2}$$

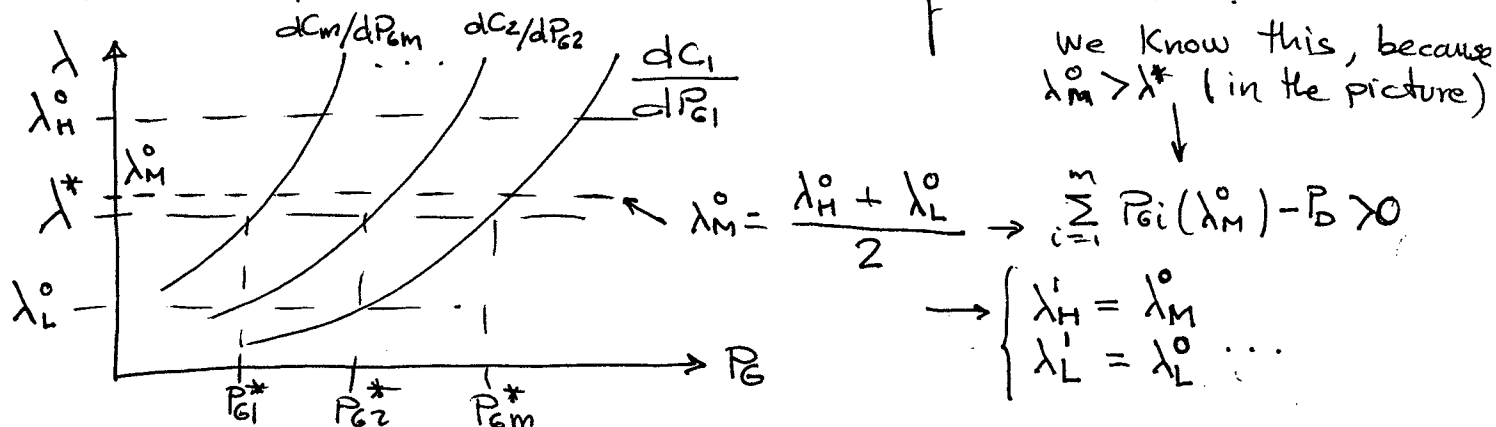


such that $P_{G1}^* + P_{G2}^* = P_D$

$$\lambda^* \text{ solves: } \begin{cases} \frac{dC_i}{dP_{Gi}} = \lambda \quad \forall i=1,2,\dots,m \\ P_D - \sum_{i=1}^m P_{Gi} = 0 \end{cases}$$

A MORE GENERAL APPROACH: LAMBDA ITERATION

- Imagine the cost functions are no longer quadratic, then the problem is not longer equivalent to solve a set of linear equations.
- We argued that in ECE 476, we would only care about quadratic ^{cost} functions, however, by introducing non-quadratic cost functions, we can introduce a more general iterative solution method that will be useful later on when introducing constraints.



λ^* is still the value that solves
$$\begin{cases} \frac{dC_i}{dP_{Gi}} = \lambda \quad \forall i=1,2,\dots,m \\ P_D - \sum_{i=1}^m P_{Gi} = 0 \end{cases}$$

- How do we find λ^* ?

- We start with two values for λ_H^0, λ_L^0 such that

$$\sum_{i=1}^m P_{Gi}(\lambda_L^0) - P_D < 0$$

$$\sum_{i=1}^m P_{Gi}(\lambda_H^0) - P_D > 0$$

while $|\lambda_H^v - \lambda_L^v| > \epsilon$
$$\lambda_M^v = \frac{\lambda_L^v + \lambda_H^v}{2}$$

if $\sum_{i=1}^m P_{Gi}(\lambda_M^v) - P_D < 0$ then
$$\begin{cases} \lambda_L^{v+1} = \lambda_M^v \\ \lambda_H^{v+1} = \lambda_H^v \end{cases}$$

else
$$\begin{cases} \lambda_L^{v+1} = \lambda_L^v \\ \lambda_H^{v+1} = \lambda_M^v \end{cases}$$