

LECTURE 17

FAST DECOUPLED POWER FLOW DC POWER FLOW EXAMPLES

FAST DECOUPLED POWER FLOW

Remember: that in the decoupled power flow, we assumed: $r \ll X$, which resulted in

$G_{ij} \approx 0$
and we also assumed $\theta_i \approx \theta_j \quad \forall i, j$, which resulted in $\cos(\theta_i - \theta_j) \approx 1$

$$\begin{array}{l|l} \frac{\partial P_i}{\partial \theta_j} = -V_i \cdot V_j \cdot \cancel{B_{ij}} \cos(\theta_i - \theta_j) & \frac{\partial P_i}{\partial \theta_i} = -Q_i - B_{ii} V_i^2 \\ \frac{\partial P_i}{\partial V_j} \approx 0 & \frac{\partial Q_i}{\partial \theta_i} \approx \cancel{0} \\ \frac{\partial Q_i}{\partial \theta_j} \approx 0 & \frac{\partial P_i}{\partial V_i} \approx 0 \\ \frac{\partial Q_i}{\partial V_j} = -V_i \cdot B_{ij} & \frac{\partial Q_i}{\partial V_i} = \frac{Q_i}{V_i} - B_{ii} V_i \end{array} \quad \forall i \neq j$$

In the fast decoupled power flow, we additionally assume that $V_i \approx V_j = V \quad \forall i, j$ (flat profile).

$$\begin{array}{l} \frac{\partial P_i}{\partial \theta_i} = -V_i V_j B_{ij} \\ \frac{\partial P_i}{\partial V_j} = 0 \\ \frac{\partial Q_i}{\partial \theta_j} = 0 \end{array} \quad \left\{ \begin{array}{l} \frac{\partial Q_i}{\partial V_j} = -V_i B_{ij} \\ \frac{\partial P_i}{\partial \theta_i} = -B_{ii} V_i^2 \\ \frac{\partial Q_i}{\partial V_i} = -V_i \cdot B_{ii} \end{array} \right. \quad \left\{ \begin{array}{l} \frac{\partial P_i}{\partial V_i} = 0 \\ \frac{\partial Q_i}{\partial \theta_i} = 0 \\ V_i = V_k \downarrow \\ Q_i = - \sum_k V_i V_k B_{ik} = \\ = -V_i V_k \cdot \underbrace{\sum_k B_{ik}}_0 \end{array} \right.$$

We are left with: the following:

$$\begin{bmatrix} \theta^{y+1} \\ V^{y+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \theta^y \\ V^y \end{bmatrix}}_{x^y} - J^{-1}(x^y) \cdot \begin{bmatrix} \Delta P(x^y) \\ \Delta Q(x^y) \end{bmatrix}$$

$$\text{where } J(x^y) = \begin{bmatrix} J_{11} & 0 \\ 0 & J_{22} \end{bmatrix} \quad \text{with } J_{11} = -[V] \cdot B \cdot [V] \\ J_{22} = -[V] \cdot B$$

or equivalently: $\Delta \theta^y = \theta^{y+1} - \theta^y$; $\Delta V^y = V^{y+1} - V^y$

$$+J(x^y) \begin{bmatrix} \Delta \theta^y \\ \Delta V^y \end{bmatrix} = \begin{bmatrix} \Delta P(x^y) \\ \Delta Q(x^y) \end{bmatrix}$$

$$- [V] \cdot B \cdot [V] \cdot \Delta \theta^y = \Delta P(x^y)$$

$$- [V] \cdot B \cdot \Delta V^y = \Delta Q(x^y)$$

non-zero

$[V]$ is a diagonal matrix with all its V entries equal,

One final approximation, which is consistent with the flat profile approximation is to make the second $[V]$ in $[V]B[V]$ be the identity, i.e:

$$[V] \cdot B \cdot [V] \approx [V] \cdot B, \text{ from where it follows that}$$

$$\Delta \theta^y = -B^{-1} \cdot \frac{\Delta P(x^y)}{V^y} \xrightarrow{\text{remember } V_1^y = V_2^y = \dots V_n^y = V^y}$$

$$\Delta V^y = -B^{-1} \cdot \frac{\Delta Q(x^y)}{V^y}$$

DC POWER FLOW

- Assumptions

- $r \ll x \rightarrow G_{ij} \approx 0$
- $\theta_i \sim \theta_j \rightarrow \cos(\theta_i - \theta_j) \approx 1$
- $V_1 = V_2 = \dots = V_n = 1$

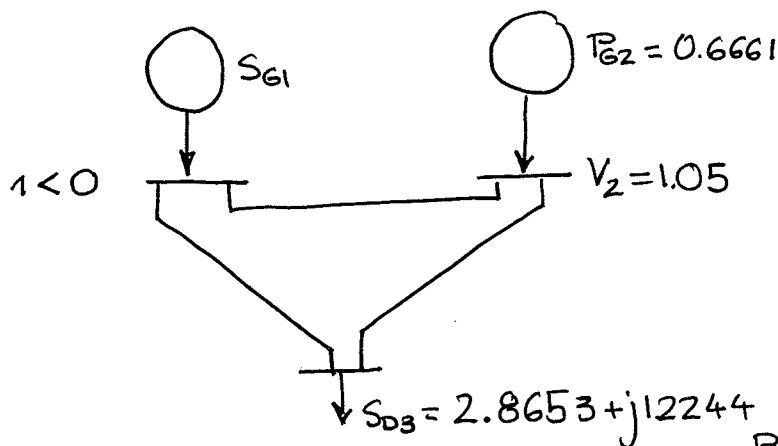
We don't need to iterate:

$$\theta = -B^{-1} \cdot P$$

→ Power injections or withdrawals

EXAMPLE (SEE NEXT PAGE)

EXAMPLE



$$Z_L = j0.1$$

$$Y_c = j0.01$$

$$\bar{Y} = \begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix} \begin{matrix} P \\ Q \end{matrix} \left\{ \begin{array}{l} V_2 \cdot V_1 \cdot B_{21} \cdot \sin(\theta_2 - \theta_1) + V_2 \cdot V_3 \cdot B_{23} \sin(\theta_2 - \theta_3) \\ = P_{G2} \\ -V_3 \cdot V_1 \cdot B_{31} \cos(\theta_3 - \theta_1) - V_3 \cdot V_1 \cdot B_{32} \cos(\theta_3 - \theta_2) \\ -V_3^2 \cdot B_{23} = -Q_{D3} \end{array} \right.$$

$$x^{y+1} = x^y - \bar{J}^{-1}(x^y) \cdot f(x)$$

$$x = [\theta_2 \ \theta_3 \ V_3]^T$$

$$\left\{ \begin{array}{l} V_3 \cdot V_1 \cdot B_{31} \sin(\theta_3 - \theta_1) + V_3 \cdot V_2 \cdot B_{32} \sin(\theta_3 - \theta_2) \\ = -P_{D3} \end{array} \right.$$

Bus ②

$$10.5 \cdot \sin \theta_2 + 10.5 \cdot V_3 \sin(\theta_2 - \theta_3) = 0.6661$$

Bus ③

$$10 \cdot V_3 \sin \theta_3 + 10.5 \cdot V_3 \cdot \sin(\theta_3 - \theta_2) = -\cancel{1.2244} - 2.8653$$

$$-10 \cdot V_3 \cdot \cos \theta_3 - 10.5 V_3 \cos(\theta_3 - \theta_2) - 19.98 V_3^2 = -1.2244$$

We can rewrite the above equations as:

$$\underbrace{10.5 \cdot \sin \theta_2 + 10.5 V_3 \cdot \sin(\theta_3 - \theta_2)}_{P_2(x)} - 0.6661 = 0$$

$$\underbrace{10 \cdot V_3 \sin \theta_3 + 10.5 V_3 \sin(\theta_3 - \theta_2)}_{P_3(x)} + 2.8653 = 0$$

$$\underbrace{-10 V_3 \cos \theta_3 - 10.5 V_3 \cos(\theta_3 - \theta_2)}_{Q_3(x)} - 19.98 V_3^2 + 1.2244 = 0$$

→ This is now in the form $f(x) = 0$

Jacobian elements:

$$(1) \frac{\partial P_2}{\partial \theta_2} = 10.5 \cos \theta_2 + 10.5 V_3 \cos (\theta_2 - \theta_3)$$

$$(2) \frac{\partial P_2}{\partial \theta_3} = -10.5 V_3 \cos (\theta_2 - \theta_3)$$

$$(3) \frac{\partial P_2}{\partial V_3} = 10.5 \sin (\theta_2 - \theta_3)$$

$$(4) \frac{\partial P_3}{\partial \theta_2} = -10.5 V_3 \cos (\theta_3 - \theta_2)$$

$$(5) \frac{\partial P_3}{\partial \theta_3} = 10 V_3 \cos \theta_3 + 10.5 V_3 \cos (\theta_3 - \theta_2)$$

$$(6) \frac{\partial P_3}{\partial V_3} = 10 \sin \theta_3 + 10.5 \sin (\theta_3 - \theta_2)$$

$$(7) \frac{\partial Q_3}{\partial \theta_2} = -10.5 V_3 \sin (\theta_3 - \theta_2)$$

$$(8) \frac{\partial Q_3}{\partial \theta_3} = 10 V_3 \sin \theta_3 + 10.5 V_3 \sin (\theta_3 - \theta_2)$$

$$(9) \frac{\partial Q_3}{\partial V_3} = -10 \cos \theta_3 - 10.5 \cos (\theta_3 - \theta_2) + 39.96 V_3$$

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}$$

We can now start the iterative process

$$x^{y+1} - x^y = -J^{-1}(x^y) f(x^y)$$

$$\underbrace{\begin{bmatrix} \theta_2^{y+1} \\ \theta_3^{y+1} \\ V_3^{y+1} \end{bmatrix} - \begin{bmatrix} \theta_2^y \\ \theta_3^y \\ V_3^y \end{bmatrix}}_{\Delta x^y} = -J^{-1}(x^y) \cdot \begin{bmatrix} P_2(x^y) - P_{D2} \\ P_3(x^y) - P_{D3} \\ Q_3(x^y) - Q_{D3} \end{bmatrix}$$

$$v = 0 :$$

We need an initial guess. An easy one is $\theta_2 = \theta_3 = 0$, $V_3 = 1$ (this is referred to as a flat start)

$$\begin{bmatrix} P_2(x^0) - P_{02} \\ P_3(x^0) + P_{03} \\ Q_3(x^0) + Q_{03} \end{bmatrix} = \begin{bmatrix} 0 & -0.661 \\ 0 & +2.8653 \\ -0.52 & +1.2244 \end{bmatrix} = \begin{bmatrix} -0.661 \\ 2.8653 \\ 0.7044 \end{bmatrix}$$

$$J^0 = \begin{bmatrix} 21 & -10.5 & 0 \\ -10.5 & 20.5 & 0 \\ 0 & 0 & 19.46 \end{bmatrix} \rightarrow (J^0)^{-1} = \begin{bmatrix} 0.0640 & 0.0328 & 0 \\ 0.0328 & 0.0656 & 0 \\ 0 & 0 & 0.0514 \end{bmatrix}$$

$$\begin{aligned} \text{Now:} \\ \begin{bmatrix} \Delta\theta_2^0 \\ \Delta\theta_3^0 \\ \Delta V_3^0 \end{bmatrix} &= -(J^0)^{-1} \begin{bmatrix} P_2(x^0) - P_{02} \\ P_3(x^0) + P_{03} \\ Q_3(x^0) + Q_{03} \end{bmatrix} = - \begin{bmatrix} 0.0640 & 0.0328 & 0 \\ 0.0328 & 0.0656 & 0 \\ 0 & 0 & 0.0514 \end{bmatrix} \begin{bmatrix} -0.661 \\ 2.8653 \\ 0.7044 \end{bmatrix} \\ &= \begin{bmatrix} -2.9393^\circ \\ -9.5111^\circ \\ -0.0362 \end{bmatrix} \rightarrow \begin{bmatrix} \theta_2^1 \\ \theta_3^1 \\ V_3^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2.9395 \\ -9.5111 \\ -0.0362 \end{bmatrix} = \begin{bmatrix} -2.9395 \\ -9.5111 \\ 0.9638 \end{bmatrix} \end{aligned}$$

$$v = 1$$

$$\begin{bmatrix} P_2(x^1) - P_{02} \\ P_3(x^1) + P_{03} \\ Q_3(x^1) + Q_{03} \end{bmatrix} = \begin{bmatrix} -0.0463 \\ 0.1145 \\ 0.2251 \end{bmatrix} \quad J^1 = \begin{bmatrix} 20.5396 & -10.0534 & 1.2017 \\ -10.0534 & 19.5589 & -2.8541 \\ 1.1582 & -2.7508 & 18.2199 \end{bmatrix}$$

$$\begin{aligned} (J^1)^{-1} &= \begin{bmatrix} 0.0651 & 0.0336 & 0.0010 \\ 0.0336 & 0.0696 & 0.0087 \\ 0.0009 & 0.0084 & 0.0561 \end{bmatrix} \rightarrow \begin{bmatrix} \Delta\theta_2^1 \\ \Delta\theta_3^1 \\ \Delta V_3^1 \end{bmatrix} \rightarrow \\ &\rightarrow \begin{bmatrix} \theta_2^2 \\ \theta_3^2 \\ V_3^2 \end{bmatrix} = \begin{bmatrix} -3.0023 \\ -9.9924 \\ 0.9502 \end{bmatrix} \end{aligned}$$

- The exact answer is $\begin{bmatrix} \theta_2 \\ \theta_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} -3 \\ -10 \\ 0.95 \end{bmatrix}$, so

in a couple of iterations we already got very close! The largest error is $\sim 0.8\%$.

- Obviously in the actual problem, we do not know the answer, so we continue iterating until

$$|P_2(x^{r+1}) - P_{02}| < \epsilon$$

$$|P_3(x^{r+1}) + P_{03}| < \epsilon$$

$$|Q_3(x^{r+1}) + Q_{03}| < \epsilon$$

Once we have a good solution (close enough), we need to compute $S_{01} = P_{01} + jQ_{01}$, and Q_{02}

$$P_{01} = V_1 \cdot V_2 \cdot B_{12} \cdot \sin(\theta_1 - \theta_2) + V_1 \cdot V_3 \cdot B_{13} \sin(\theta_1 - \theta_3)$$

$$= 10.5 \cdot \sin 3.0023 + 9.502 \cdot \sin 9.9924 = 2.1987$$

$$Q_{01} = 0.1365$$

$$Q_{02} = -1.6395$$

same way