

LECTURE 16

POWER FLOW JACOBIAN STRUCTURE

DECOUPLED POWER FLOW

FAST DECOUPLED POWER FLOW

DC POWER FLOW

POWER FLOW JACOBIAN STRUCTURE

$$X^{y+1} = X^y - J^{-1}(X^y) \cdot f(X^y) \quad X = [\underbrace{\theta_2 \ \theta_3 \dots \theta_n}_{\theta'} \underbrace{V_2 \ V_3 \dots V_n}_{V'}]$$

$$f(x) = \begin{bmatrix} P_2(x) - P_2 \\ P_3(x) - P_3 \\ \vdots \\ P_n(x) - P_n \\ Q_2(x) - Q_2 \\ \vdots \\ Q_n(x) - Q_n \end{bmatrix}$$

$$P_i(x) = \sum_k V_i \cdot V_k \cdot [G_{ik} \cdot \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)]$$

$$Q_i(x) = \sum_k V_i \cdot V_k [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)]$$

$$J(x) = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \dots & \frac{\partial P_2}{\partial \theta_n} & \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} & \dots & \frac{\partial P_2}{\partial V_n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial P_n}{\partial \theta_2} & \frac{\partial P_n}{\partial \theta_3} & \dots & \frac{\partial P_n}{\partial \theta_n} & \frac{\partial P_n}{\partial V_2} & \frac{\partial P_n}{\partial V_3} & \dots & \frac{\partial P_n}{\partial V_n} \\ \frac{\partial Q_2}{\partial \theta_2} & \frac{\partial Q_2}{\partial \theta_3} & \dots & \frac{\partial Q_2}{\partial \theta_n} & \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_3} & \dots & \frac{\partial Q_2}{\partial V_n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial Q_n}{\partial \theta_2} & \frac{\partial Q_n}{\partial \theta_3} & \dots & \frac{\partial Q_n}{\partial \theta_n} & \frac{\partial Q_n}{\partial V_2} & \frac{\partial Q_n}{\partial V_3} & \dots & \frac{\partial Q_n}{\partial V_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}$$

MAKE SURE YOU HAVE THE EQUATIONS ON THE
↓ BLACKBOARD

The terms in the Jacobian are:

$$\theta_{ij} \triangleq \theta_i - \theta_j$$

$$\left\{ \begin{array}{l} \frac{\partial P_i}{\partial \theta_j} = V_i \cdot V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad \forall j \neq i \\ \frac{\partial P_i}{\partial V_j} = V_i (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ \frac{\partial Q_i}{\partial \theta_j} = -V_i \cdot V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ \frac{\partial Q_i}{\partial V_j} = V_i \cdot (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\partial P_i}{\partial \theta_i} = -Q_i - B_{ii} \cdot V_i^2 \\ \frac{\partial Q_i}{\partial \theta_i} = P_i - G_{ii} \cdot V_i^2 \\ \frac{\partial P_i}{\partial V_i} = \frac{P_i}{V_i} + G_{ii} \cdot V_i \\ \frac{\partial Q_i}{\partial V_i} = \frac{Q_i}{V_i} - B_{ii} \cdot V_i \end{array} \right.$$

DECOUPLED POWER FLOW

$$X^{y+1} = X^y - J^{-1}(X^y) \cdot f(X^y), \text{ where } X^y = [\underbrace{\theta_2, \theta_3, \dots, \theta_n}_{\theta} \underbrace{V_2, V_3, \dots, V_n}_{V}]$$

$$J(X^y) = \begin{bmatrix} \frac{\partial P}{\partial \theta} \big|_{X^y} & \frac{\partial P}{\partial V} \big|_{X^y} \\ \frac{\partial Q}{\partial \theta} \big|_{X^y} & \frac{\partial Q}{\partial V} \big|_{X^y} \end{bmatrix} \rightarrow \begin{bmatrix} \theta^{y+1} \\ V^{y+1} \end{bmatrix} = \begin{bmatrix} \theta^y \\ V^y \end{bmatrix} - \begin{bmatrix} \frac{\partial P}{\partial \theta} \\ \frac{\partial Q}{\partial \theta} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

The decoupled power flow assumes $\frac{\partial P}{\partial V} \big|_{X^y} = \frac{\partial Q}{\partial \theta} \big|_{X^y} \approx 0$

Thus:

$$\theta^{y+1} = \theta^y - \left[\frac{\partial P}{\partial \theta} \big|_{X^y} \right]^{-1} \cdot \Delta P^y \quad V^{y+1} = V^y - \left[\frac{\partial Q}{\partial V} \big|_{X^y} \right]^{-1} \Delta Q^y$$

The justification to this approximation is justified if $r \ll X \rightarrow \cancel{G_{ij} \approx 0}$ and $\theta_i - \theta_j \approx 0$

$$\frac{\partial P_i}{\partial V_j} = V_i \cdot (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \approx 0$$

$$\frac{\partial Q_i}{\partial \theta_j} = -V_i \cdot V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \approx 0$$

FAST DECOUPLED POWER FLOW

$$G_{ij} = 0$$

$$V_i = V_j \quad \forall i \neq j$$

$$\sin \theta_{ij} = 0$$

$$\cos \theta_{ij} = 1$$

$$\Delta \theta^y = B^{-1} \cdot \frac{\Delta P(x^y)}{V^y}$$

where B is the imaginary part of the admittance matrix Y

$$\Delta V^y = B^{-1} \cdot \frac{\Delta Q(x^y)}{V^y}$$

Justification:

$$\frac{\partial P_i}{\partial \theta_i} = -B_{ii} \cdot V_i^2$$

$$Q_i \approx \overset{\text{equal}}{\sum} V_i V_j \sum B_{ik} = 0$$

$$\frac{\partial P_i}{\partial \theta_j} = V_i \cdot V_j \cdot B_{ij}$$

$$\frac{\partial Q_i}{\partial V_i} = -V_i \cdot B_{ii}$$

$$\frac{\partial Q_i}{\partial V_j} = -V_i \cdot B_{ij}$$

$$B \triangleq \begin{bmatrix} B_{22} & B_{23} & \dots & B_{2n} \\ \vdots & \vdots & & \vdots \\ B_{n2} & B_{n3} & \dots & B_{nn} \end{bmatrix}$$

B is obtained from Y by stripping away the first row and columns and taking the imaginary part

$$- \begin{bmatrix} J_{11} & 0 \\ 0 & J_{22} \end{bmatrix} = \begin{bmatrix} \Delta \theta^y \\ \Delta V^y \end{bmatrix} = \begin{bmatrix} \Delta P(x^y) \\ \Delta Q(x^y) \end{bmatrix}$$

$$J_{11} = -[V] \cdot B [V] \quad J_{22} = -[V] \cdot B, \text{ where } [V] = \begin{bmatrix} V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix}$$

The flat profile assumption: $V_2 = V_3 = \dots V_n$
allows ~~for~~ rewrite:

$$- [V^y] \cdot B \Delta \theta^y = \Delta P(x^y)$$

$$- B \Delta \theta^y = [V^y]^{-1} \cdot \Delta P(x^y)$$

DC POWER FLOW

• The crudest approximation

$$V_1 = V_2 = \dots = V_n = 1 \quad r = 0$$

$$\Delta \theta^y = - B^{-1} \cdot \Delta P(x^y) \leftarrow \text{There is no iteration!}$$

