

## LECTURE 15

### NEWTON-RAPSHON ALGORITHM

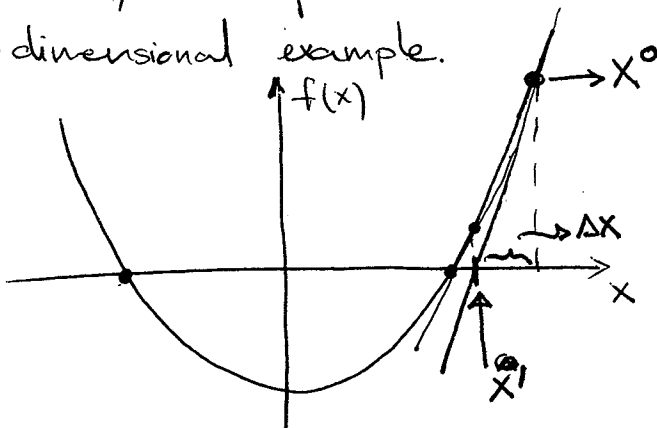
- CASE I
- CASE II

### NEWTON-RAPSHON

Remember, we are trying to solve a set of non-linear equations:

$$P_i = \sum_k V_i V_k (G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k))$$
$$Q_i = \sum_k V_i V_k (G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k))$$

- We will study now another algorithm that iteratively solves the equations above.
- This algorithm is based on the Taylor series expansion of a non-linear function.
- Let's try to gather the basic concept from a one-dimensional example.



Let's try to find the roots of  $f(x)$ :

$$f(x) = 0$$

We start with some value of  $x$ :  $x^0$ , and unless we are very lucky:  $f(x^0) \neq 0$

The Taylor series expansion of  $f(x^0)$ , allows us to write:

$$f(x^0 + \Delta x) \approx f(x^0) + \left. \frac{df}{dx} \right|_{x^0} \Delta x \leftarrow \text{This is true for small } \Delta x$$

- In practice, what we are saying is that we can approximate the nonlinear function by a straight line (in the neighborhood of  $x^0$ ).
- We could actually compute the approximation given by the straight line and obtain  $\Delta x$

$$f(x^0) + \left. \frac{df}{dx} \right|_{x^0} \Delta x = 0 \rightarrow \Delta x = -f(x^0) \left( \left. \frac{df}{dx} \right|_{x^0} \right)^{-1}$$

$$\text{but } \Delta x = x^1 - x^0 \rightarrow x^1 = x^0 - f(x^0) \left( \left. \frac{df}{dx} \right|_{x^0} \right)^{-1}$$

We can use now  $x^1$  instead of  $x^0$  and do the same:

$$f(x^1) \neq 0, \text{ but } f(x^1 + \Delta x) \approx f(x^1) + \left. \frac{df}{dx} \right|_{x^1} \underbrace{(x^2 - x^1)}_{\Delta x}$$

$$\rightarrow x^2 = x^1 - \cancel{f(x^1)} \left( \left. \frac{df}{dx} \right|_{x^1} \right)^{-1} f(x^1)$$

The iteration scheme is clear:

$$x^{y+1} = x^y - \cancel{f(x^y)} \left( \left. \frac{df}{dx} \right|_{x^y} \right)^{-1} f(x^y)$$

$x^0$  the initial guess.

In a multi-dimensional case :

$$f_1(x_1, x_2, \dots, x_n) = 0$$

$$f_2(x_1, x_2, \dots, x_n) = 0$$

$$\vdots$$

$$f_n(x_1, x_2, \dots, x_n) = 0$$

$$x \triangleq [x_1, x_2, \dots, x_n]^T$$

$$\Delta x \triangleq [\Delta x_1, \Delta x_2, \dots, \Delta x_n]^T$$

$$x^0 \triangleq [x_1^0, x_2^0, \dots, x_n^0]^T$$

Now the Taylor series expansion becomes :

$$f_1(x_1^0, x_2^0, \dots, x_n^0) :$$

$$\bullet f_1(x_1^0 + \Delta x_1, x_2^0 + \Delta x_2, \dots, x_n^0 + \Delta x_n) =$$

$$= f_1(x_1^0, x_2^0, \dots, x_n^0) + \left. \frac{\partial f_1}{\partial x_1} \right|_{x^0} \Delta x_1 + \left. \frac{\partial f_1}{\partial x_2} \right|_{x^0} \Delta x_2 + \dots + \left. \frac{\partial f_1}{\partial x_n} \right|_{x^0} \Delta x_n$$

$$= f_1(x^0) + \left[ \left. \frac{\partial f_1}{\partial x_1} \right|_{x^0} \quad \left. \frac{\partial f_1}{\partial x_2} \right|_{x^0} \quad \dots \quad \left. \frac{\partial f_1}{\partial x_n} \right|_{x^0} \right] \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

We can do something similar for the others, and we get :

$$f(x^0 + \Delta x) \simeq f(x^0) + \underbrace{\begin{bmatrix} \left. \frac{\partial f_1}{\partial x_1} \right|_{x^0} & \left. \frac{\partial f_1}{\partial x_2} \right|_{x^0} & \dots & \left. \frac{\partial f_1}{\partial x_n} \right|_{x^0} \\ \left. \frac{\partial f_2}{\partial x_1} \right|_{x^0} & \left. \frac{\partial f_2}{\partial x_2} \right|_{x^0} & \dots & \left. \frac{\partial f_2}{\partial x_n} \right|_{x^0} \\ \vdots & \vdots & \ddots & \vdots \\ \left. \frac{\partial f_n}{\partial x_1} \right|_{x^0} & \left. \frac{\partial f_n}{\partial x_2} \right|_{x^0} & \dots & \left. \frac{\partial f_n}{\partial x_n} \right|_{x^0} \end{bmatrix}}_{J(x^0)} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = 0$$

The iteration scheme becomes  $J(x^0) \leftarrow$  Jacobian matrix

$$x^{y+1} = x^y - J^{-1}(x^y) \cdot f(x^y)$$

# APPLICATION OF NEWTON-RAPHSON TO SOLVE THE POWER FLOW PROBLEM

Remember:

$$P_i = \sum_k V_i \cdot V_k [G_{ik} \cdot \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_k V_i \cdot V_k [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)]$$

In general  $P_i = P_{Gi} - P_{Di}$  and  $Q_i = Q_{Gi} - Q_{Di}$ ,  
we can rewrite the equations as:

$$\sum_k V_i V_k [G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)] - P_{Gi} + P_{Di} = 0$$

$$\sum_k V_i V_k [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)] - Q_{Gi} + Q_{Di} = 0$$

## CASE I

Given,  $V_1, \theta_1, \bar{S}_2, \bar{S}_3, \dots, \bar{S}_n$

find,  $\bar{S}_1, V_2, \theta_2, \dots, V_n, \theta_n$ .

As before, if we focus on the nodes  $2, \dots, n$   
we can solve the problem.

Define  $X = [\theta_2, \dots, \theta_n, V_2, \dots, V_n]$ ,

the problem is to find a solution to

$$P_2(X) - P_{G2} + P_{D2} = 0, \text{ where } P_2(X) = \sum_k V_2 V_k \cdot (G_{ik} \dots$$

$$P_3(X) - P_{G3} + P_{D3} = 0$$

$\vdots$

$$P_n(X) - P_{Gn} + P_{Dn} = 0$$

$$Q_2(X) - Q_{G2} + Q_{D2} = 0$$

$\vdots$

$$Q_n(X) - Q_{Gn} + Q_{Dn} = 0$$

$$\left\{ \begin{array}{l} P_2(X) - P_{G2} + P_{D2} \\ P_3(X) - P_{G3} + P_{D3} \\ \vdots \\ P_n(X) - P_{Gn} + P_{Dn} \\ Q_2(X) - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n(X) - Q_{Gn} + Q_{Dn} \end{array} \right\} = 0$$

$P(X)$

$$X^{y+1} = X^y - J^{-1}(X^y) \cdot f(X^y)$$

$$J(X^y) = \begin{bmatrix} \frac{\partial P_2}{\partial x_2} \big|_{X^y} & \frac{\partial P_2}{\partial x_3} \big|_{X^y} & \dots & \frac{\partial P_2}{\partial x_n} \big|_{X^y} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial x_2} \big|_{X^y} & \frac{\partial P_n}{\partial x_3} \big|_{X^y} & \dots & \frac{\partial P_n}{\partial x_n} \big|_{X^y} \\ \frac{\partial Q_2}{\partial x_2} \big|_{X^y} & \frac{\partial Q_2}{\partial x_3} \big|_{X^y} & \dots & \frac{\partial Q_2}{\partial x_n} \big|_{X^y} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial x_2} \big|_{X^y} & \frac{\partial Q_n}{\partial x_3} \big|_{X^y} & \dots & \frac{\partial Q_n}{\partial x_n} \big|_{X^y} \end{bmatrix}$$

## CASE II

- What happens when we have more than one generator?
- The voltage magnitude in PV buses is fixed, so we don't include it in the formulation of  $x$ , i.e.,  $x$  only contains the voltage magnitudes that are unknown and all the voltage angles
- We don't include the reactive power equation in the formulation.

→ In other words:  $n$  buses,  $m$  generators (PV),  $n-m+1$  loads  
 For PQ buses: ( $m$  in total)

$$\left. \begin{aligned} P_i(x) - P_{Gi} + P_{Di} &= 0 \\ Q_i(x) - Q_{Gi} + Q_{Di} &= 0 \end{aligned} \right\} \rightarrow \begin{matrix} \text{2 equations} \\ 2(n-m+1) \end{matrix}$$

For PV buses

$$P_i(x) - P_{Gi} + P_{Di} = 0 \rightarrow n-1 \text{ equations (we don't include the slack bus)}$$

$x \rightarrow$  contains  $\left\{ \begin{array}{l} \underline{V_i, \theta_i} \text{ for the load buses} \\ 2(n-m+1) \text{ unknowns} \end{array} \right.$

$\left\{ \begin{array}{l} \underline{\theta_i} \text{ for the generator buses (except the slack bus)} \\ n-1 \text{ unknowns.} \end{array} \right.$

