

LECTURE 14

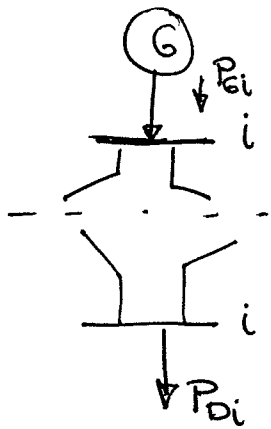
- POWER FLOW FORMULATION
- GAUSS METHOD

POWER FLOW EQUATIONS

$$\begin{cases} P_i = \sum_K V_i \cdot V_K \cdot [G_{iK} \cos(\theta_i - \theta_K) + B_{iK} \sin(\theta_i - \theta_K)] & \forall i=1,2,\dots,n \\ Q_i = \sum_K V_i \cdot V_K [G_{iK} \sin(\theta_i - \theta_K) - B_{iK} \cos(\theta_i - \theta_K)] & \forall i=1,2,\dots,n \end{cases}$$

- As we discuss, we have $2n$ equations, but $4n$ variables:
 $V_i, \theta_i, P_i, Q_i \quad \forall i=1,2,\dots,n$
- We need to fix $2n$ variables and solve for the other $2n$.
 - We are given P_i and Q_i on load buses,
 - We are given V_i and P_i on generator buses.
 - We solve for V_i, θ_i on load buses, and for Q_i, θ_i on generator buses.
- There is a problem with the above set-up. We cannot arbitrarily choose all the P_i 's on generator buses.
 - In a lossless system: $\sum_i P_{Gi} = \sum_i P_{Di}$
 - In a system with losses, $\sum_i P_{Gi} = \sum_i P_{Di} + \sum_K R_K \cdot |I_K|^2$
 - I_K depends on the flow through the transmission lines, which is a result of the power flow problem.

- Instead of fixing the power on all generator buses, we fix (or we are given) the power on all generator buses but one, which we call the "slack" generator.
- This "slack" generator is then modeled as a voltage source: we fix V_i and θ_i (usually we make it zero)
- In summary, we will have three different types of buses:
 - A voltage source (slack bus), where we know V_i and θ_i
 - $P, |V|$ sources, which are all other generator buses.
 - P, Q sources. All other load buses.



In the formulation of the power flow equations, we deal with power injections.

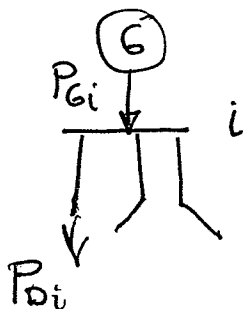
- Thus for generator buses $P_i = P_{Gi}$

- For load buses: $P_i = -P_{Di}$

- What happens if there is a bus with both generation and load?

It is still a $P, |V|$ bus:

$$P_i \triangleq P_{Gi} - P_{Di}$$



How do we solve the power flow equations?

- No analytical solution
- We need to solve them using numerical methods
 - Gauss
 - Newton-Raphson

SOLUTION OF THE THE POWER FLOW PROBLEM BY GAUSS

CASE I:

Let's assume that we only have one generator and the rest buses are loads. For convenience, we assume that the generator is in bus 1

Given: $V_1, \theta_1, \bar{S}_2, \bar{S}_3, \dots, \bar{S}_n$

find: $\bar{S}_1, V_2, \theta_2, \dots, V_n, \theta_n$

In complex form:

$$\bar{S}_1 = \bar{V}_1 \cdot \sum_{k=1}^n \bar{Y}_{1k}^* \cdot \bar{V}_k^*$$

$$\bar{S}_i = \bar{V}_i \cdot \sum_{k=1}^n \bar{Y}_{ik}^* \cdot \bar{V}_k^* \quad \forall i=2, \dots, n$$

- If we know, $\bar{V}_1, \bar{V}_2, \dots, \bar{V}_n$, then we can explicitly solve for $\bar{S}_1$.
- Since we already know $\bar{V}_1$, the problem is to find $\bar{V}_2, \dots, \bar{V}_n$, which can be found from equations 2, ..., n (remember that we know $\bar{S}_2, \dots, \bar{S}_n$)
- The heart of the problem is to find the solution of the implicit equations

$$\bar{S}_i = \bar{V}_i \cdot \sum_{k=1}^n \bar{Y}_{ik}^* \bar{V}_k^* \quad \forall i=2, \dots, n.$$

- We are going to rewrite these equations in a suitable form so we can iterate:

$$\bar{S}_i^* = \bar{V}_i^* \sum_{k=1}^n \bar{Y}_{ik} \bar{V}_k$$

We divide by $\bar{V}_i^*$ and separate the i,i term:

$$\frac{\bar{S}_i^*}{\bar{V}_i^*} = \bar{Y}_{ii} \cdot \bar{V}_i + \sum_{k \neq i} \bar{Y}_{ik} \cdot \bar{V}_k \rightarrow$$

$$\rightarrow \bar{V}_i = \frac{1}{\bar{Y}_{ii}} \cdot \left[\frac{\bar{S}_i^*}{\bar{V}_i^*} - \sum_{k \neq i} \bar{Y}_{ik} \cdot \bar{V}_k \right]$$

We get $n-1$ implicit nonlinear algebraic equations in the unknown complex $\bar{V}_i$:

$$\bar{V}_2 = h_2(\bar{V}_2, \bar{V}_3, \dots, \bar{V}_n)$$

$$\bar{V}_3 = h_3(\bar{V}_2, \bar{V}_3, \dots, \bar{V}_n)$$

$\vdots$

$$\bar{V}_n = h_n(\bar{V}_2, \bar{V}_3, \dots, \bar{V}_n)$$

Define $\bar{X} = [\bar{V}_1 \ \bar{V}_2 \ \dots \ \bar{V}_n]$

- In vector notation:

$$\bar{X} = h(\bar{X})$$

- We can solve this equation through an iterative scheme.

Let ν index the iteration steps:

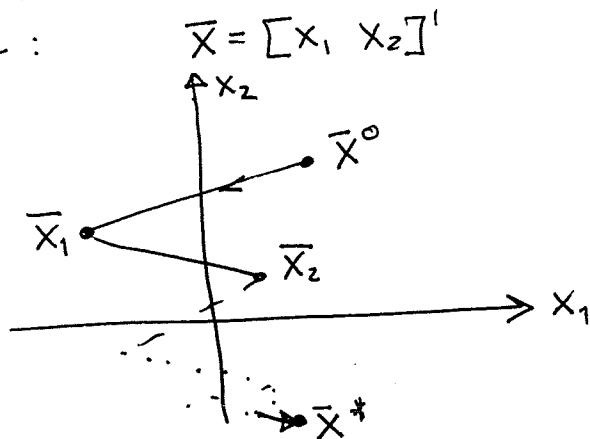
$$\bar{X}^{\nu+1} = h(\bar{X}^{\nu}) \quad \nu = 0, 1, 2, \dots$$

- We pick an initial guess for $\bar{X}$: $\bar{X}^0$,
and we generate a sequence $\bar{X}^0, \bar{X}^1, \dots, \bar{X}^n$

- If the sequence convergence $\lim_{\nu \rightarrow \infty} \bar{X}^{\nu} = \bar{X}^*$ then

$$\boxed{\bar{X}^* = h(\bar{X}^*)}$$

The procedure can be visualized in a two-dimensional space:



$\bar{X}^*$ is called a fixed point of $h(\cdot)$

In practice, we stop the iteration when $\|\bar{X}^{y+1} - \bar{X}^y\| < \epsilon$

The 2-norm for example

$$\|\bar{X}^{y+1} - \bar{X}^y\|_\infty = \max_i \{ |x_i^{y+1} - x_i^y| \}$$

$$\|\bar{X}^{y+1} - \bar{X}^y\|_2 = \sqrt{(x_1^{y+1} - x_1^y)^2 + (x_2^{y+1} - x_2^y)^2 + \dots + (x_n^{y+1} - x_n^y)^2}$$

A VARIATION OF GAUSS: GAUSS-SEIDEL

Note that if we process the equations sequentially, after the first equation is processed, we already have the next estimate for x_2 , thus instead of using the old estimate of x_2 when processing the second equation, we can use the new one instead, this can be generalized to all subsequent equations. This is the so-called Gauss-Seidel iteration, which usually converges faster than the Gauss iteration.

$$\begin{aligned} \bar{V}_2^{y+1} &= h_2(\bar{V}_2^y, \bar{V}_3^y, \dots, \bar{V}_n^y) \rightarrow \text{use } \bar{V}_2^{y+1} \\ \bar{V}_3^{y+1} &= h_3(\bar{V}_2^{y+1}, \bar{V}_3^y, \dots, \bar{V}_n^y) \rightarrow \text{use } \bar{V}_2^{y+1} \text{ and } \bar{V}_3^{y+1} \\ &\vdots \\ \bar{V}_n^{y+1} &= h_n(\bar{V}_2^{y+1}, \bar{V}_3^{y+1}, \dots, \bar{V}_{n-1}^{y+1}, \bar{V}_n^y) \end{aligned}$$

CASE II

- What happens when we have more than one generator?
- without loss of generality, let's assume that the first m buses are generators and the rest loads (we can always reorder them):
- We know:
$$\begin{cases} V_1, \theta_1, P_2, V_2, P_3, V_3, \dots, P_m, V_m \\ P_{m+1}, Q_{m+1}, P_{m+2}, Q_{m+2}, \dots, P_n, Q_n \end{cases}$$

As Before:

$$\bar{V}_i = \frac{1}{\bar{Y}_{ii}} \left[\frac{\bar{S}_i^*}{\bar{V}_i^*} - \sum_{k \neq i} \bar{Y}_{ik} \bar{V}_k \right] \quad \forall i$$

or $\bar{S}_i = P_i + jQ_i$, thus:

$$\bar{V}_i = \frac{1}{\bar{Y}_{ii}} \left[\frac{P_i - jQ_i}{\bar{V}_i^*} - \sum_{k \neq i} \bar{Y}_{ik} \bar{V}_k \right] \quad \forall i$$

- For the load buses ($i = m+1, \dots, n$), we have the same scheme as before since P_i and Q_i are known thus:

$$\bar{V}_i^{v+1} = \frac{1}{\bar{Y}_{ii}} \left[\frac{P_i - jQ_i}{(\bar{V}_i^v)^*} - \sum_{k \neq i} \bar{Y}_{ik} \bar{V}_k^v \right]$$

Fixed

- What happens with the generator buses?
- We know P_i , but we don't know Q_i
- We also know $V_i \quad \forall i = 1, \dots, m$

→

We still don't know V_i

How do we handle Q_i ?

- We do first a side calculation based on the $\bar{V}_k^{\gamma}$ estimates:

$$Q_i^{\gamma} = \text{Im} \left\{ \bar{V}_i^{\gamma} \cdot \sum_{k \neq i} \bar{Y}_{ik}^* (\bar{V}_k^{\gamma})^* \right\}$$

• We now use the value of Q_i^{γ} to obtain $\tilde{\bar{V}}_i^{\gamma+1}$:

$$\tilde{\bar{V}}_i^{\gamma+1} = \frac{1}{\bar{Y}_{ii}} \left[\frac{P_i^{\gamma+1} - j Q_i^{\gamma+1}}{(\bar{V}_i^{\gamma})^*} - \sum_{k \neq i} \bar{Y}_{ik} \bar{V}_k^{\gamma} \right]$$

$$\tilde{\bar{V}}_i^{\gamma+1} = \tilde{V}_i^{\gamma+1} < \theta_i^{\gamma+1}$$

Since we know the value of V_i (this is a generator bus), we throw away the value of $\tilde{\bar{V}}_i^{\gamma+1}$, and just keep $\theta_i^{\gamma+1}$. The new estimate for $\bar{V}_i$ is then:

$$\boxed{\bar{V}_i^{\gamma+1} = V_i < \theta_i^{\gamma+1}}$$