

LECTURE 13

- ADMITTANCE MATRIX REVIEW
- THE POWER FLOW PROBLEM FORMULATION

ADMITTANCE MATRIX REVIEW

By applying KCL to all the nodes in an electrical network, we can obtain a relation between the currents injected (and withdrawn) in the network and the node voltage. In matrix form this relation is given by

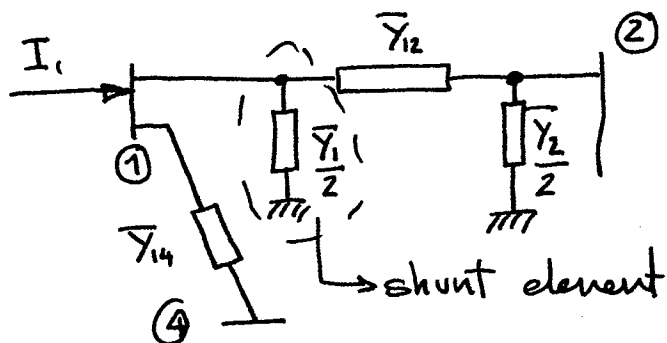
$$\mathbf{I} = \mathbf{\bar{Y}} \cdot \mathbf{V}, \text{ where } \mathbf{\bar{Y}} \text{ is the network admittance matrix.}$$

- The entries of $\mathbf{\bar{Y}}$ can be easily obtained

$\bar{Y}[i,j] \equiv$ negative of the admittance of the transmission line between nodes i & j

$\bar{Y}[i,i] \equiv$ sum of the admittances of elements connected to node i :

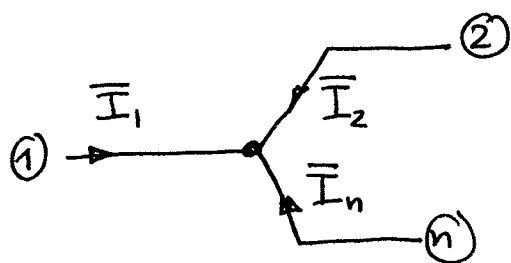
- transmission line series admittance
- transmission line shunt admittance.



$$\bar{I}_1 = \left(\frac{\bar{Y}_1}{2} + \bar{Y}_{12} + \bar{Y}_{14} \right) \bar{V}_1 - \bar{Y}_{12} \bar{V}_2 - \bar{Y}_{14} \bar{V}_4$$

THE POWER FLOW PROBLEM FORMULATION

- As previously discussed, when we are analyzing a power system, we usually have an estimate of the power (active & reactive) being consumed at a given load bus rather than the current
- Since we are assuming the system is in steady state, the way the controls work in the generators fix the voltage magnitude and the power being injected (according to a reference value fixed beforehand).
- Instead of formulating a relation between currents injected (or withdrawn) and the voltages, the natural thing to do is to formulate a relation between the power injected (or withdrawn) and the voltages.
- In Lecture 1, we talked about conservation of power:



$$\text{KCL: } \bar{I}_1 + \bar{I}_2 + \dots + \bar{I}_n = 0$$

$$\bar{V}_1 = \bar{V}_2 = \dots = \bar{V}_n \equiv \bar{V}$$

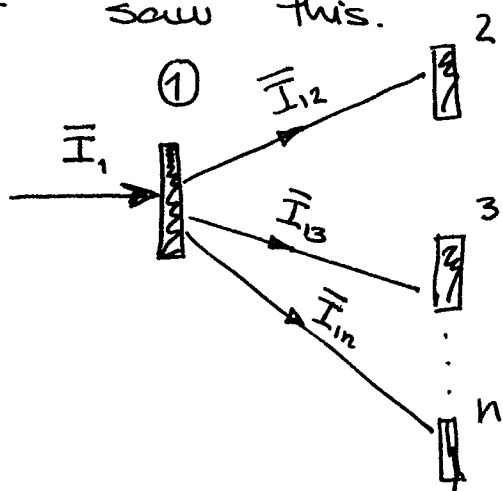
$$\bar{S}_1 + \bar{S}_2 + \dots + \bar{S}_n = \bar{V}_1 \cdot \bar{I}_1^* + \bar{V}_2 \bar{I}_2^* \dots$$

$$+ \bar{V}_n \bar{I}_n^* = \bar{V} \cdot (\bar{I}_1^* + \bar{I}_2^* + \dots + \bar{I}_n^*) = 0$$

↑

This is the most fundamental idea when formulating the power flow problem.

- With the above idea in mind, we formulate the power flow equations.
- We know that any generator or load bus, the current being injected (or withdrawn) must be equal to the current that flows into the network through the transmission lines connected to the bus.
- The same is true for the power! we just saw this.



~~$\bar{I}_1 = \bar{Y} \cdot \bar{V}$~~

Remember, in general:

$$\bar{I} = \bar{Y} \cdot \bar{V}$$

$$\bar{I}_1 = \bar{Y}_{11} \cdot \bar{V}_1 + \bar{Y}_{12} \cdot \bar{V}_2 + \dots + \bar{Y}_{1n} \bar{V}_n$$

These are the entries of matrix $\bar{Y}$

$$\bar{S}_1 = \bar{V}_1 \cdot \bar{I}_1^*$$

$$\bar{S}_1 = \bar{V}_1 \cdot (\bar{Y}_{11} \cdot \bar{V}_1 + \bar{Y}_{12} \bar{V}_2 + \dots + \bar{Y}_{1n} \bar{V}_n)^*$$

In general, for any node i in the network, if we have the admittance matrix $\bar{Y}$, we can write:

$$\boxed{\bar{S}_i = \bar{V}_i \cdot \left(\sum_{k=1}^n \bar{Y}_{ik} \cdot \bar{V}_k \right)^*}$$

$$\bar{S}_i = \bar{V}_i \cdot \left(\sum_{k=1}^n \bar{Y}_{ik} \bar{V}_k \right)^*$$

We can write this complex equation in its real and imaginary parts with some additional work.

$$\begin{aligned} \text{Define: } \bar{S}_i &= P_i + jQ_i & \bar{V}_i &= V_i e^{j\theta_i} \\ \bar{Y}_{ik} &= G_{ik} + jB_{ik} & \theta_{ik} &= \theta_i - \theta_k \end{aligned}$$

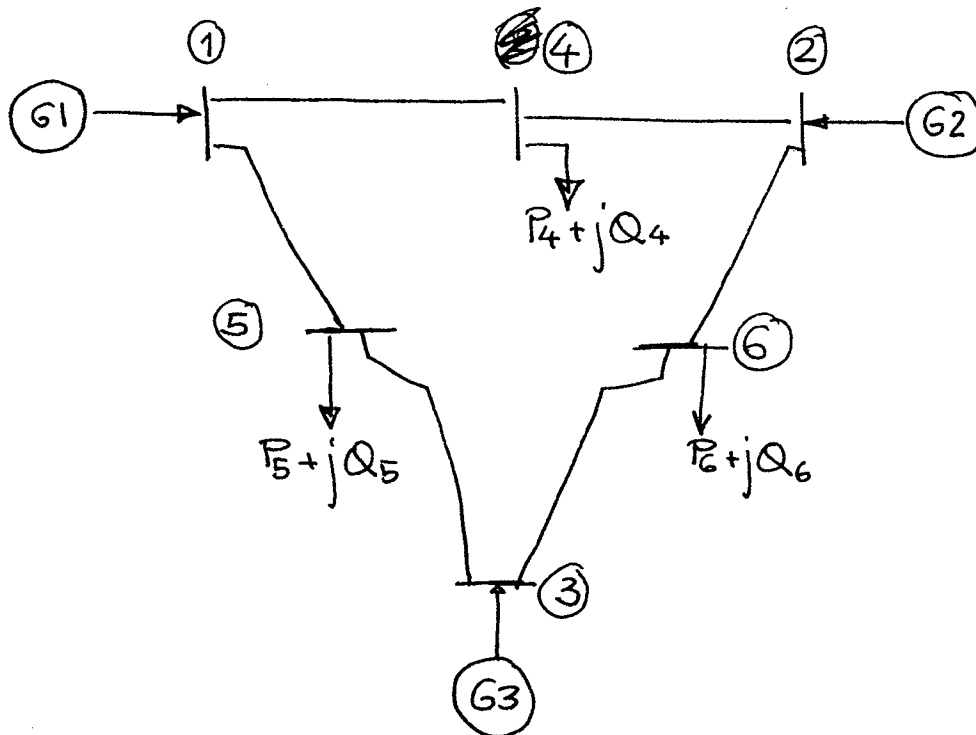
$$\begin{aligned} \bar{S}_i &= P_i + jQ_i = V_i e^{j\theta_i} \cdot \left(\sum_{k=1}^n (G_{ik} + jB_{ik}) \cdot V_k e^{j\theta_k} \right)^* = \\ &= \sum_{k=1}^n (G_{ik} - jB_{ik}) \cdot V_i \cdot V_k e^{j(\theta_i - \theta_k)} \\ &= \sum_{k=1}^n (G_{ik} - jB_{ik}) \cdot V_i \cdot V_k \cdot [\cos(\theta_i - \theta_k) + j\sin(\theta_i - \theta_k)] \\ &= \sum_{k=1}^n V_i V_k \cdot [G_{ik} \cos(\theta_{ik}) + B_{ik} \sin(\theta_{ik})] + \\ &\quad + j \sum_{k=1}^n V_i V_k \cdot [G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}] \end{aligned}$$

$$\boxed{\begin{aligned} P_i &= \sum_{k=1}^n V_i \cdot V_k \cdot [G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}] \\ Q_i &= \sum_{k=1}^n V_i \cdot V_k \cdot [G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}] \end{aligned}}$$

↑
These are the most fundamental equations in this course!!

- We obtain $2 \times n$ equations (two equations per bus)
- We have $4 \times n$ variables (4 variables per bus)
 $\boxed{P_i, Q_i, V_i, \theta_i}$

THE POWER FLOW PROBLEM



- We have 6 nodes.
 - We formulate 12 equations (two per-node)
 - We have 24 variables (4 per-node)
- We need to fix 12 variables and solve for the other 12 variables
- The variables that we fix are either Known (loads) or we have control over (voltages on generators and active power on generators)
 - - Operational considerations indicate that at a generator bus the active power P_{Gi} and the voltage magnitude might be specified.
 - In general we cannot specify all the generator P_{Gi} 's independently!

- In a lossless power system:

$$\boxed{\sum P_{Gi} = \sum P_{Di}}$$

- In a system with losses $\sum_i P_{Gi} = \sum_j P_{Dj} + \sum_k R_k I_k^2$

- If we don't know the losses beforehand, this becomes a problem.

- An easy way to solve this is to assume that one generator is a voltage source, i.e. we fix the voltage magnitude and the angle instead of the V and P . This bus is called the slack bus.

In summary, we will have three types of buses:

- A voltage source (slack bus), where we know V_1 and θ_1 (usually we pick it to be zero) ^{rather}
- P , $|V|$ sources, which are all the generator buses
- P , Q sources. All load buses.