

LECTURE 12

LOAD MODELS

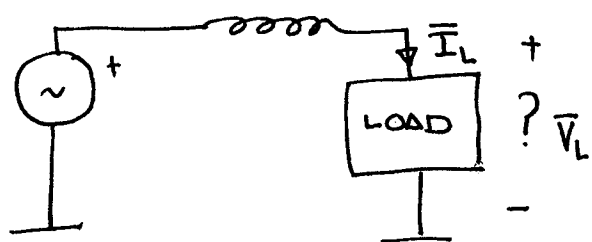
- GENERATOR MODELS

- THE POWER FLOW PROBLEM

- ADMITTANCE MATRIX

LOAD MODELS

Remember our simplest power system



How does the load look like?
• In other words, what is the appropriate model for a load in a power system?

We have three choices

- Constant impedance load: $\text{LOAD} \equiv \bar{Z}_L = R + jX : \bar{V}_L = \bar{Z}_L \bar{I}_L$
- Constant current load: $\bar{I}_L$
- Constant power load: $\bar{V}_L \cdot \bar{I}_L^* = P_L + jQ_L$

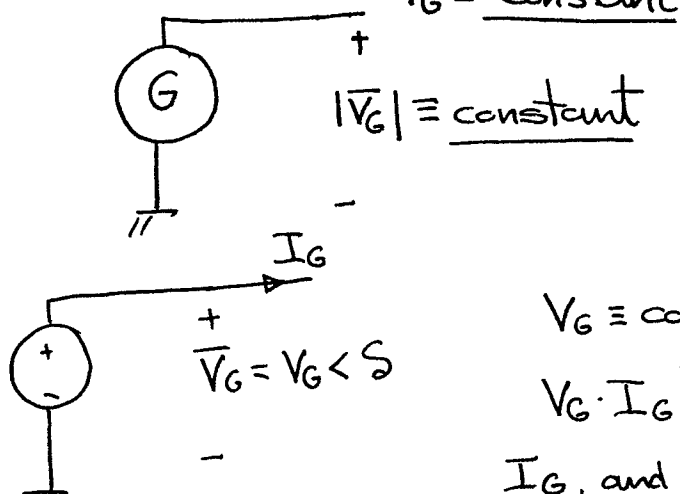
- In a real power system, loads (aggregate) are a mix, or behave, as a mix of the three above models. These are so-called ZIP models, the mix between Z, I, and P parts is not straightforward to come up with.
- The most difficult one to deal with in terms of analysis is the constant power model load.

- Constant power load models make the power flow problem formulation (which we will study next) extremely non-linear.
- Another reason why we make use of constant power load models is that forecasting the power consumption on a given node of the network is ~~that~~ in fact fairly easy from historical data and weather models.

GENERATOR MODELS

- We mainly deal with two generator models in system-level studies. You learn more about generator models (electromechanical models) in the machines course (ECE 431). In fact, you have learnt quite a bit on ECE 430 already.
- We will mainly use two models:

- A steady-state model that assumes the generator is a constant power source operating at a fixed voltage (in magnitude)

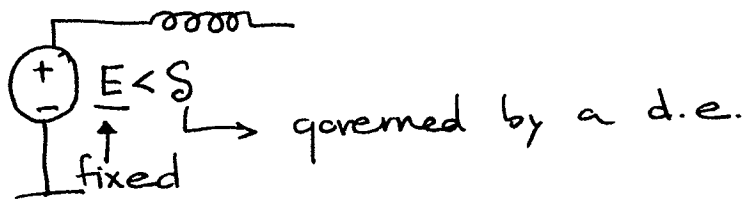


$$V_G \equiv \text{constant}$$

$$V_G \cdot I_G \cdot \cos \phi = P_G \equiv \text{constant}$$

I_G , and ϕ can vary.

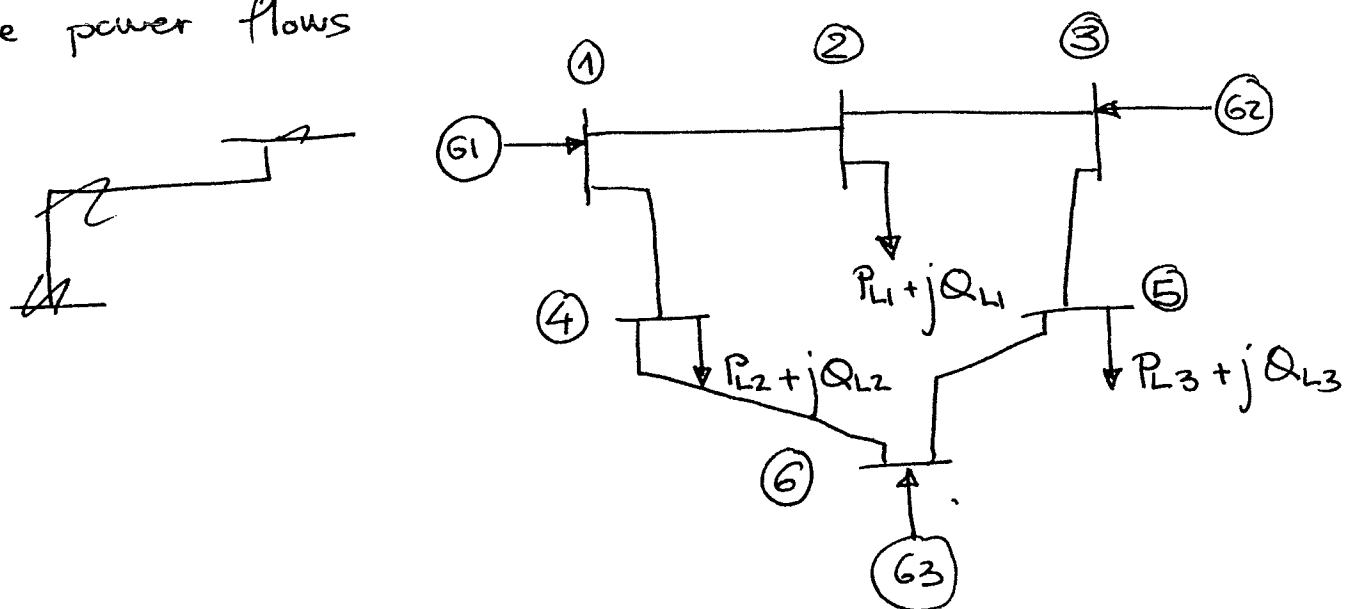
- A second model we are going to use is a transient model, that treats the generator as a constant voltage source behind a reactance, where the phase angle of the source is governed by a differential equation describing the machine (generator) rotational motion.



THE POWER FLOW PROBLEM

- This is a fundamental problem in power systems.
- In this analysis, the transmission system is modeled by a set of buses (or nodes) interconnected by transmission lines.
- Generators and loads, connected to various nodes of the system, inject and remove power from the transmission system.
- The analysis is appropriate for solving for the steady-state powers and voltages of the transmission system.
- The calculation is analogous to the familiar problem of solving for the steady-state voltages and currents in a circuit.

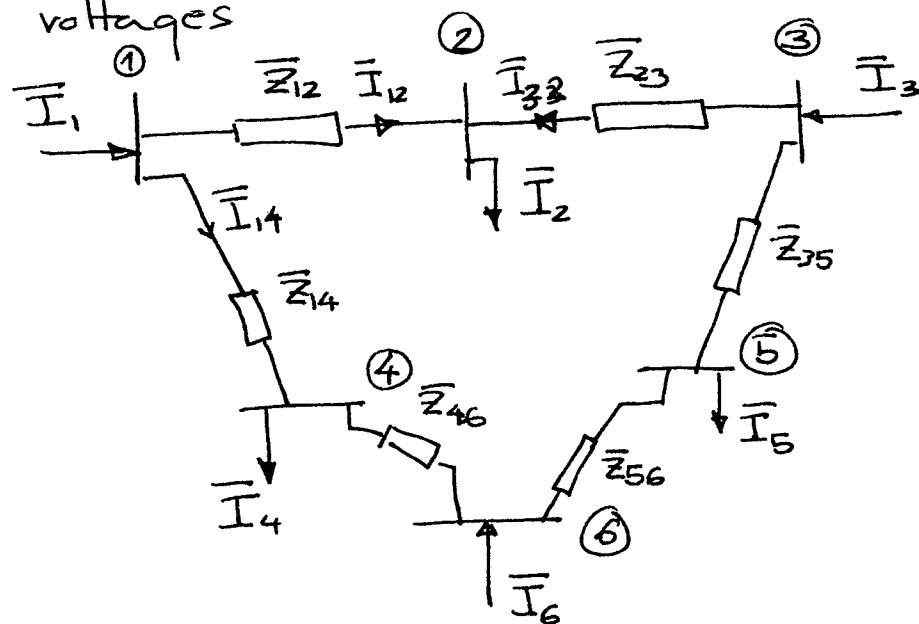
In a power flow problem, we are usually given load consumptions and generations, and we want to solve for the voltages on the network and the power flows



- Let's take a step back and solve a more fundamental problem. Instead of ~~the~~ treating injections and withdraws of power, let's talk about current injections and withdraws.
- We are going to do this, to develop a fundamental ingredient of the power flow problem formulation, the ADMITTANCE MATRIX

BUS ADMITTANCE MATRIX

→ It gives the relationships between current injections (and withdrawals) and the bus voltages



It is obtained by applying KCL at each bus and then relate the current injections to bus voltages and ~~impedances~~ admittances

Current injected at node 1

$$\bar{I}_1 = \bar{I}_{12} + \bar{I}_{14}$$

$$\bar{I}_1 = \frac{\bar{V}_1 - \bar{V}_2}{\bar{Z}_{12}} + \frac{\bar{V}_1 - \bar{V}_4}{\bar{Z}_{14}} \rightarrow \bar{I}_1 = \left(\frac{1}{\bar{Z}_{12}} + \frac{1}{\bar{Z}_{14}} \right) \bar{V}_1 - \frac{1}{\bar{Z}_{12}} \bar{V}_2 - \frac{1}{\bar{Z}_{14}} \bar{V}_4$$

$$\bar{I}_2 = \bar{I}_{12} + \bar{I}_{32} = \frac{\bar{V}_1 - \bar{V}_2}{\bar{Z}_{12}} + \frac{\bar{V}_3 - \bar{V}_2}{\bar{Z}_{23}} = \frac{\bar{V}_1}{\bar{Z}_{12}} - \bar{V}_2 \left(\frac{1}{\bar{Z}_{12}} + \frac{1}{\bar{Z}_{23}} \right) + \frac{1}{\bar{Z}_{23}} \bar{V}_3$$

Let's use the opposite convention for withdrawn currents:

$$-\bar{I}_2 = -\frac{\bar{V}_1}{\bar{Z}_{12}} + \bar{V}_2 \left(\frac{1}{\bar{Z}_{12}} + \frac{1}{\bar{Z}_{23}} \right) - \frac{1}{\bar{Z}_{23}} \bar{V}_3$$

$\uparrow \qquad \qquad \uparrow \qquad \uparrow \qquad \uparrow$

These are all admittances.

We can write all the relations for each node in matrix form

$$\begin{bmatrix} \bar{I}_1 \\ -\bar{I}_2 \\ \bar{I}_3 \\ -\bar{I}_4 \\ -\bar{I}_5 \\ \bar{I}_6 \end{bmatrix} = \begin{bmatrix} \bar{Y}_{12} + \bar{Y}_{14} & -\bar{Y}_{12} & 0 & -\bar{Y}_{14} & 0 & 0 \\ -\bar{Y}_{12} & \bar{Y}_{12} + \bar{Y}_{23} & -\bar{Y}_{23} & 0 & 0 & 0 \\ 0 & -\bar{Y}_{23} & \bar{Y}_{23} + \bar{Y}_{35} & 0 & -\bar{Y}_{35} & 0 \\ -\bar{Y}_{14} & 0 & 0 & \bar{Y}_{14} + \bar{Y}_{46} & 0 & -\bar{Y}_{46} \\ 0 & 0 & -\bar{Y}_{35} & 0 & \bar{Y}_{35} + \bar{Y}_{56} & -\bar{Y}_{56} \\ 0 & 0 & 0 & -\bar{Y}_{46} & -\bar{Y}_{56} & \bar{Y}_{46} + \bar{Y}_{56} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \\ \bar{V}_5 \\ \bar{V}_6 \end{bmatrix}$$

$\underbrace{\hspace{15em}}_{\text{Admittance matrix} \equiv \bar{Y}}$

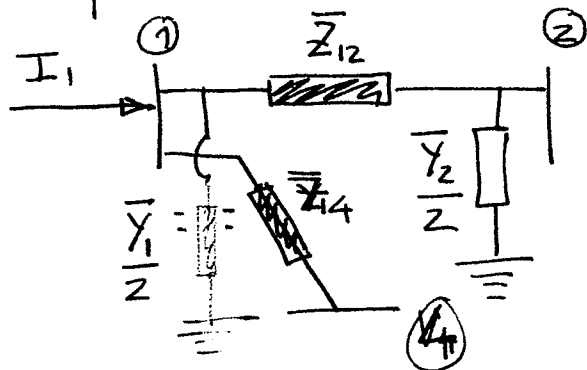
In short-form: $\bar{I} = \bar{Y} \cdot \bar{V}$

The entries of $\bar{Y}$ are easily obtained

$\bar{Y}[i, j] \equiv$ the negative of the admittance of the transmission line between nodes i and j

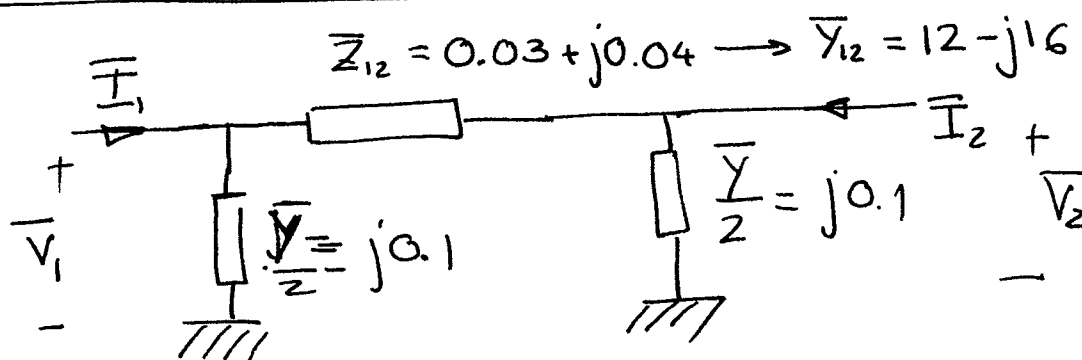
$\bar{Y}[i, i] \equiv$ the sum of admittances of ~~the~~ the transmission lines connected to node i .

- What happens when we also have shunt capacitances, we just add them on the diagonal entries of the admittance matrix.



$$\bar{I}_1 = \left(\frac{\bar{Y}_1}{2} + \bar{Y}_{12} + \bar{Y}_{14} \right) \bar{V}_1 + \bar{Y}_{12} \bar{V}_2 + \bar{Y}_{14} \bar{V}_4$$

TWO BUS EXAMPLE:



$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} 12 - j16 + j0.1 & -12 + j16 \\ -12 + j16 & 12 - j16 + 0.1 \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

If the currents are known, then we could just invert $\bar{Y}$ to get the voltages

$$\boxed{\bar{V} = \bar{Y}^{-1} \cdot \bar{I}}$$