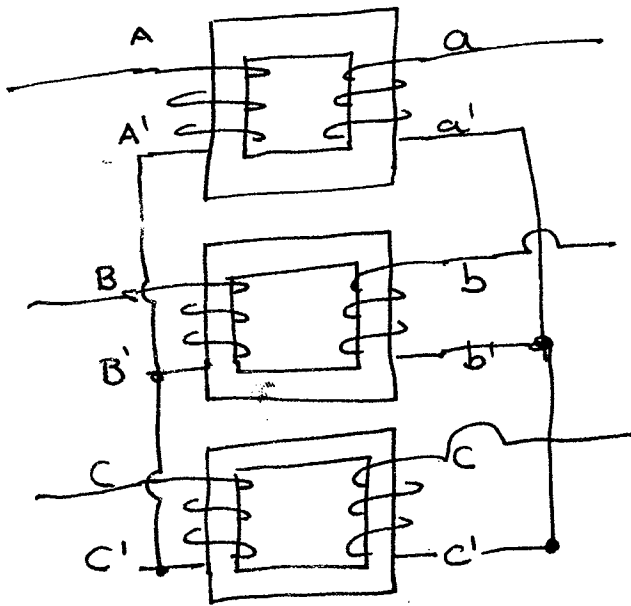


LECTURE 11

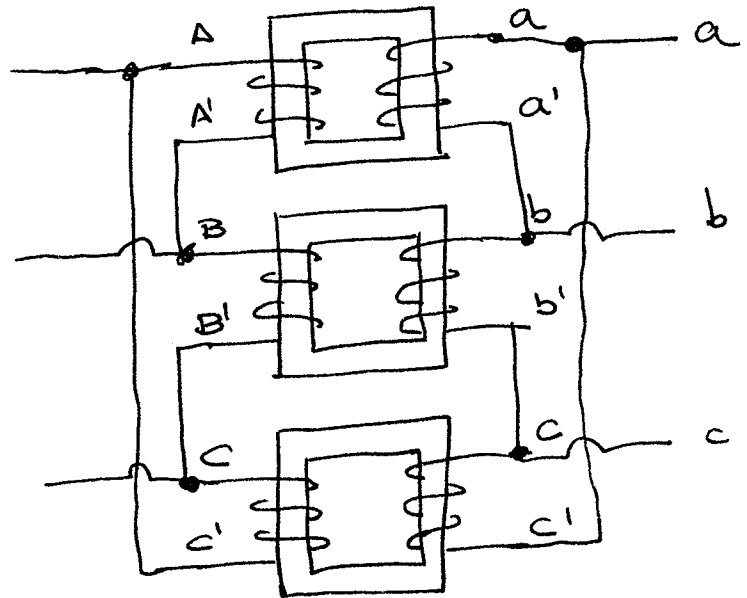
- THREE PHASE TRANSFORMERS
- PER-UNIT NORMALIZATION
 - SINGLE PHASE

THREE PHASE TRANSFORMERS

We can construct a 3ϕ transformer by interconnecting three single-phase transformers

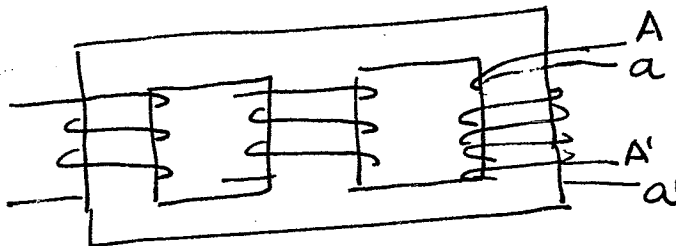


Y-Y CONNECTION



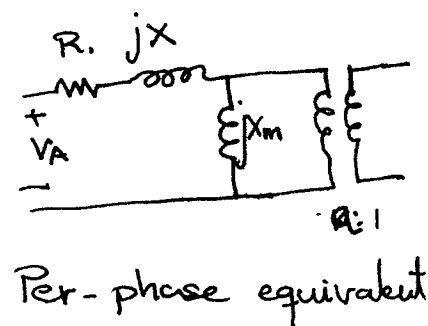
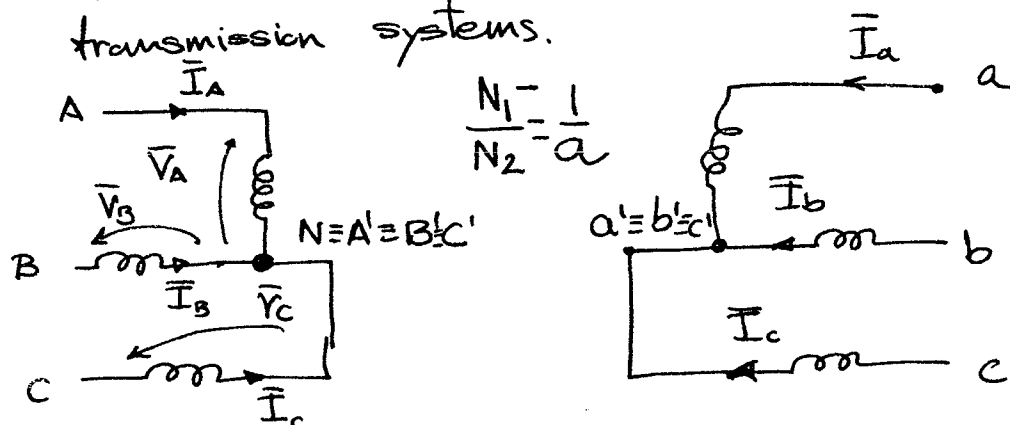
Δ - Δ CONNECTION

- Depending on the application, we choose the corresponding connection
- It is very common (specially for small and medium size) transformers to build a single core



Y-Y TRANSFORMER

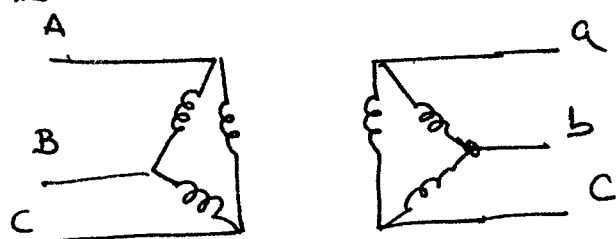
- Turn ratio of individual transformers is K
- No phase-shifting introduces and possibility of grounding both sides of the transformer
- Y-Y connections are very common in transmission systems.



$$\frac{\bar{V}_A}{\bar{V}_a} = \frac{\bar{V}_B}{\bar{V}_b} = \frac{\bar{V}_C}{\bar{V}_c} = a$$

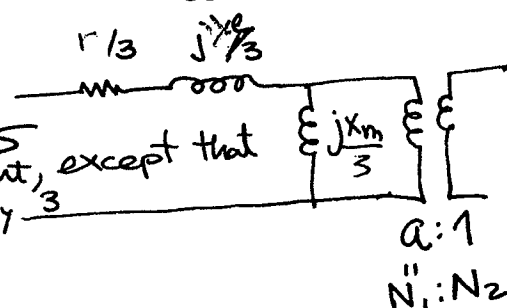
$$\frac{\bar{I}_A}{\bar{I}_a} = \frac{\bar{I}_B}{\bar{I}_b} = \frac{\bar{I}_C}{\bar{I}_c} = \frac{1}{a}$$

Δ-Δ TRANSFORMER

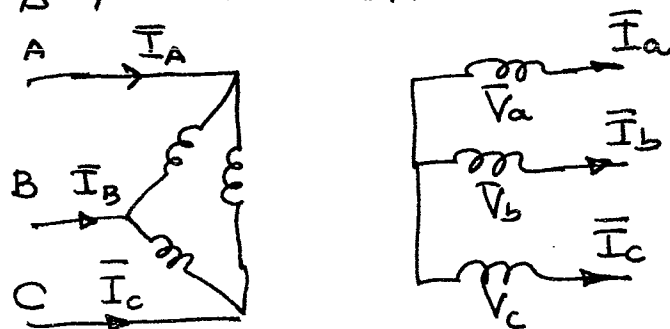


$$\frac{\bar{V}_{AB}}{\bar{V}_{ab}} = K \quad \frac{\bar{V}_{BC}}{\bar{V}_{bc}} = K \quad \frac{\bar{V}_{CA}}{\bar{V}_{ca}} = K$$

• Similar to the Y-Y per-phase equivalent, except that impedances are divided by 3



Δ-Y CONNECTION



$$\frac{\bar{V}_{AB}}{\bar{V}_a} = \frac{\bar{V}_{BC}}{\bar{V}_b} = \frac{\bar{V}_{CA}}{\bar{V}_c} = K$$

$$\bar{V}_{ab} = \sqrt{3} \bar{V}_a \angle -30^\circ$$

$$\boxed{\bar{V}_{ab} = \sqrt{3} \frac{\bar{V}_{AB}}{a} \angle -30^\circ} \quad \boxed{\bar{V}_a = \sqrt{3} \frac{\bar{V}_A}{a} \angle -30^\circ}$$

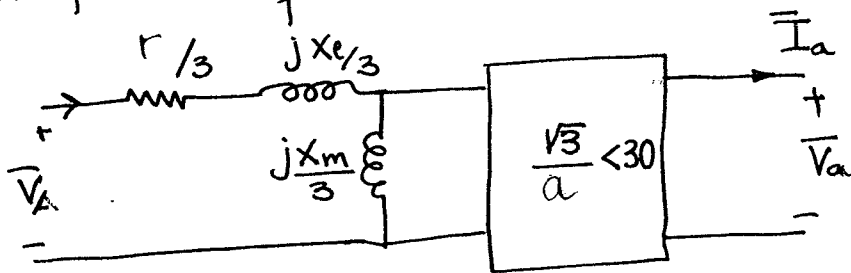
For current:

$$\frac{\bar{I}_{AB}}{\bar{I}_{ab}} = \frac{1}{K} \rightarrow \bar{I}_A = \sqrt{3} \bar{I}_{AB} \angle -30^\circ \rightarrow$$

$$\rightarrow \boxed{\bar{I}_a = K \cdot \frac{1}{\sqrt{3}} \bar{I}_A \angle 30^\circ}$$

- Δ -Y connection introduces a 30° phase shift

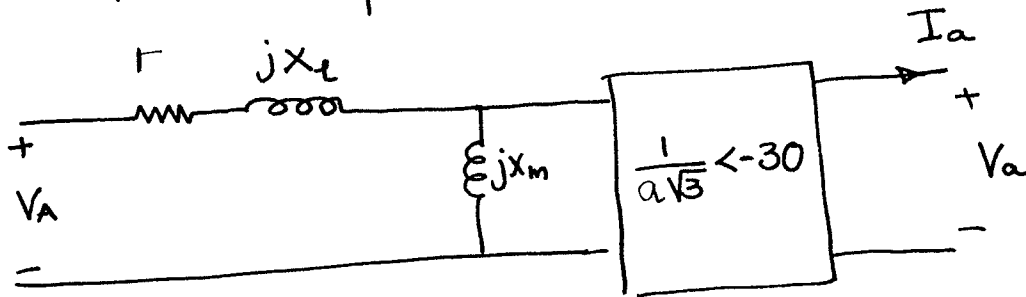
Per-phase equivalent circuit:



- Common in transmission/distribution step-down since there is a neutral on the low voltage side
- Even if $a=1$, there is a $\sqrt{3}$ step-up ratio.

Y- Δ CONNECTION

A similar analysis to the Δ -Y connection. The per-phase equivalent circuit:



- As the Δ -Y connection, it introduces a 30° phase shift.
- It is used for stepping down at the end of the transmission line when there is no need for grounding on the low voltage side.
- Even if $a=1$, it introduces an additional $\sqrt{3}$ step-down ratio.

PER-UNIT NORMALIZATION

- Power system quantities such as voltage, current, power, and impedance are often expressed in per-unit or % of specified base values. ¶
- One advantage of the per-unit system is that by properly specifying base quantities, the transformer equivalent circuit can be simplified.
- The ideal transformer windings on the equivalent circuit can be eliminated, such that voltages, currents and external impedances and admittances expressed in per-unit do not change when they are referred from one side of a transformer to the other

$$\text{quantity in per-unit} = \frac{\text{actual quantity}}{\text{base value quantity}}$$

- A vital point is that the bases must be picked to satisfy the same kind of relationships to the actual variables

$$\bar{V} = \bar{Z} \cdot \bar{I} \quad (\text{actual quantities})$$

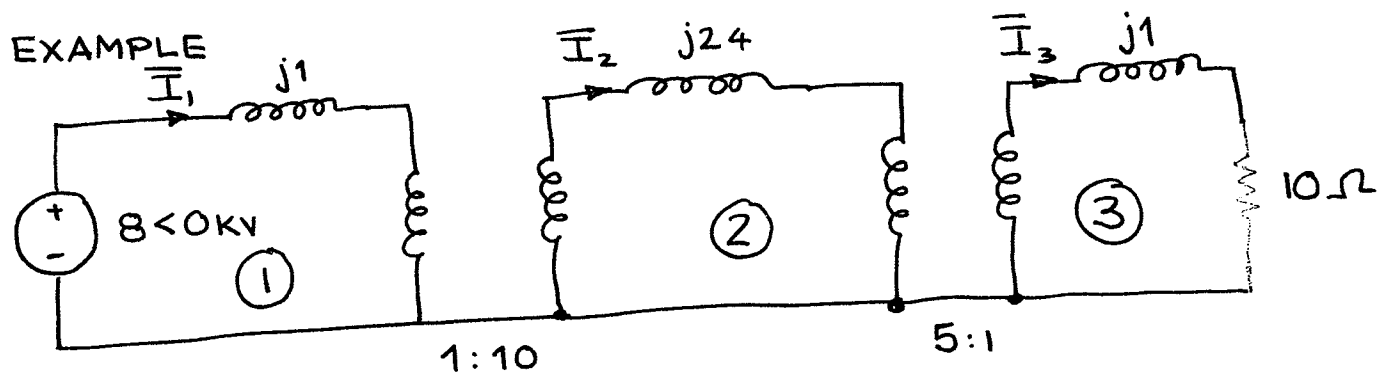
$$\bar{V}_B = \bar{Z}_B \cdot \bar{I}_B \quad (\text{base value quantities})$$

$$\frac{\bar{V}}{\bar{V}_B} = \frac{\bar{Z}}{\bar{Z}_B} \cdot \frac{\bar{I}}{\bar{I}_B} \longrightarrow \boxed{\bar{V}_{p.u.} = \bar{Z}_{p.u.} \cdot \bar{I}_{p.u.}}$$

~~per-phas~~
~~per-unit relation~~
relation between per-unit quantities (the same as with the actual quantities)

PER-UNIT NORMALIZATION FOR ONE-PHASE SYSTEMS

- Pick a base power for the entire system
- Pick a base voltage for each different voltage level, but keeping in mind that the voltage base on both sides of a transformer must be related by transformer turn ratio (otherwise we defeat the purpose of the per-unit normalization)
- Calculate base impedances for each voltage level as
$$Z_B = \frac{V_B^2}{S_B} \quad (\text{magnitudes, not phasors})$$
- Calculate base current for each voltage level
$$I_B = \frac{V_B}{Z_B}$$
- Convert actual values to per-unit.
- The equivalent circuit topology in p.u. is exactly the same as the actual quantities circuit topology except for the ideal transformers that disappear



- Let's pick the base power to be

$$S_B = 100 \text{ MVA}$$

$$I_{1B} = \frac{100}{8}$$

- Let's pick $V_{1B} = 8 \text{ kV}$, then

$$V_{2B} = 10 \cdot V_{1B} = 80 \text{ kV} \rightarrow V_{2B} = 80 \text{ kV} \quad I_{2B} = \frac{100}{80}$$

$$V_{3B} = \frac{1}{5} \cdot V_{2B} = 16 \text{ kV} \rightarrow V_{3B} = 16 \text{ kV} \quad I_{3B} = \frac{100}{16}$$

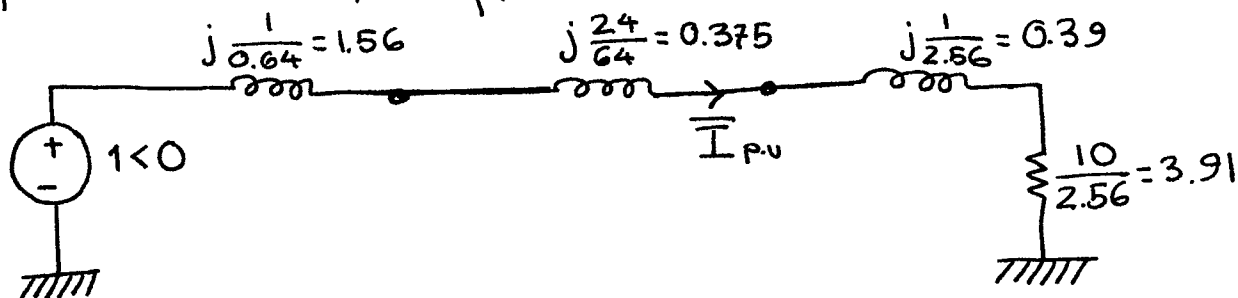
- Let's calculate the base impedances:

$$Z_{1B} = \frac{(8 \cdot 10^3)^2}{100 \cdot 10^6} = 0.64 \Omega \rightarrow Z_{1B} = 0.64 \Omega$$

$$Z_{2B} = \frac{(80 \cdot 10^3)^2}{100 \cdot 10^6} = 64 \Omega \rightarrow Z_{2B} = 64 \Omega$$

$$Z_{3B} = \frac{(16 \cdot 10^3)^2}{100 \cdot 10^6} = 2.56 \Omega \rightarrow Z_{3B} = 2.56 \Omega$$

per-unit circuit representation



$$\bar{I}_{p.u.} = \frac{1 \angle 0}{j(1.56 + 0.375 + 0.39) + 3.91} = 0.22 \angle -30.8$$

$$\bar{I}_1 = \frac{100}{8} \cdot 0.22 \angle -30.8$$

$$\bar{I}_2 = \frac{100}{80} \cdot 0.22 \angle -30.8$$

$$\bar{I}_3 = \frac{100}{16} \cdot 0.22 \angle -30.8$$