

LECTURE 9

TRANSFORMERS

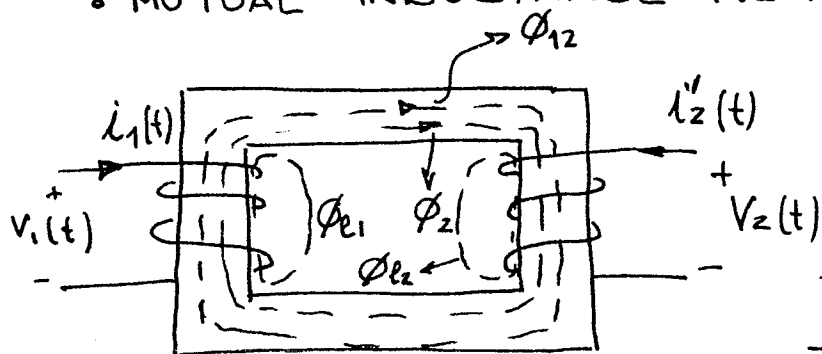
- THE NEED FOR TRANSFORMERS
- IDEAL TRANSFORMER MODEL
- REAL TRANSFORMER MODEL
- THREE - PHASE TRANSFORMERS

THE NEED FOR TRANSFORMERS

- The generation of power is usually done at voltages ranging from 11 to 13 kV (this is the norm for central power stations)
- Efficient transmission over long distances requires much higher voltages, typically 138, 230, 345, 765 kV
- Distribution of power to our neighbors and industrial plants takes place at much lower voltage: 2400-4600V
- In our houses, we have usually 240/120 V
- We need a way to step voltages up and down → The device that permits stepping ^{AC} voltages up and down is the transformer (we can step DC voltages up & down with power electronics, but this is not a subject of this course).

IDEAL TRANSFORMER MODEL (430 REVIEW)

MUTUAL INDUCTANCE REVIEW



$$\lambda_1 = N_1 \cdot \Phi_1 = N_1 (\Phi_{12} + \Phi_{21} + \Phi_{e1})$$

$$= N_1 (\Phi_{11} + \Phi_{12}) =$$

$$\boxed{\lambda_1 = L_1 i_1 + M_{12} i_2'}$$

Similarly for the second winding

$$\boxed{\lambda_2 = L_2 i_2 + M_{21} i_1'}$$

$$v_1 = \frac{d\lambda_1}{dt} \rightarrow v_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} \rightarrow v_2 = L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt}$$

Assumptions for an ideal transformer:

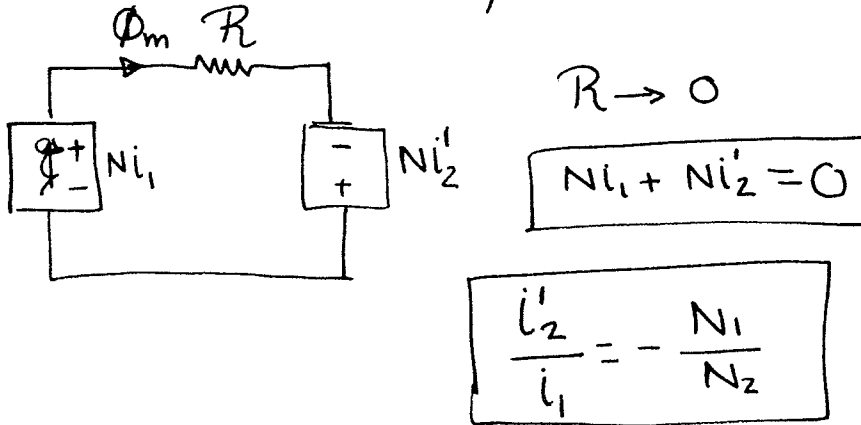
- No losses
- No leakage fluxes
- Magnetic core has infinite permeability

→ No leakage fluxes: $\Phi_{e1} = \Phi_{e2} = 0 \Rightarrow$

$$\Rightarrow \Phi_1 = \Phi_2 \Rightarrow \frac{\lambda_1}{N_1} = \frac{\lambda_2}{N_2}$$

$$\left. \begin{aligned} v_1 &= N_1 \cdot \frac{d\Phi_1}{dt} \\ v_2 &= N_2 \cdot \frac{d\Phi_2}{dt} \end{aligned} \right\} \rightarrow \boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

→ Infinite permeability

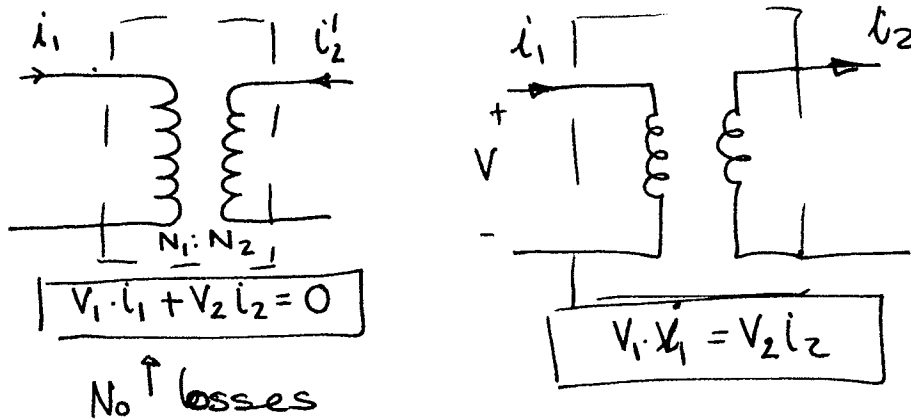


The two fundamental relations between electric variables in an ideal transformer are:

$\frac{V_2}{V_1} = \frac{N_2}{N_1}$

$\frac{i_2'}{i_1} = -\frac{N_1}{N_2}$

ELECTRIC CIRCUIT REPRESENTATION OF AN IDEAL TRANSFORMER



Power is conserved

We can redefine i_2' to be : $i_2 = -i_2'$
and then :

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{1}{a} \quad \frac{i_2}{i_1} = \frac{N_1}{N_2} = a$$

TURN RATIO : $a = \frac{N_1}{N_2}$

REAL TRANSFORMER MODEL

We relax two of the previous assumptions and let:

i) Losses

ii) Leakage flux

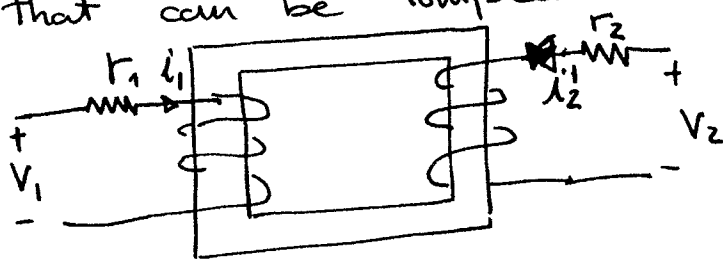
iii) Finite permeability

• With leakage we have:

$$\lambda_1 = \underbrace{N_1 \cdot \phi_{l_1}}_{\lambda_{l_1}} + \underbrace{N_1 \cdot (\phi_{12} + \phi_{21})}_{N_1 \cdot \phi_m \equiv \text{mutual flux}} = \lambda_{l_1} + N_1 \cdot \phi_m$$

$$\lambda_2 = \underbrace{N_2 \cdot \phi_{l_2}}_{\lambda_{l_2}} + N_2 \cdot (\phi_{12} + \phi_{21}) = \lambda_{l_2} + N_2 \cdot \phi_m$$

• We also assume that the wires have resistance and that can be lumped into a single element



$$V_1 = r_1 \cdot i_1 + \frac{d\lambda_1}{dt} = r_1 \cdot i_1 + \frac{d\lambda_{l_1}}{dt} + N_1 \cdot \frac{d\phi_m}{dt}$$

$$V_2 = r_2 i_2' + \frac{d\lambda_2}{dt} = r_2 i_2' + \frac{d\lambda_{l_2}}{dt} + N_2 \cdot \frac{d\phi_m}{dt}$$

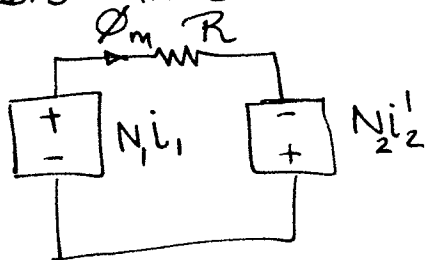
• Let's assume linearity of the magnetic core:

$$\lambda_{l_1} = L_{l_1} \cdot i_1$$

$$\lambda_{l_2} = L_{l_2} i_2$$

$$\boxed{\begin{aligned} V_1 &= r_1 i_1 + L_{l_1} \frac{di_1}{dt} + N_1 \frac{d\phi_m}{dt} \\ V_2 &= r_2 i_2' + L_{l_2} \frac{di_2'}{dt} + N_2 \frac{d\phi_m}{dt} \end{aligned}}$$

- Let's introduce finite permeability, then



$$N_1 i_1 + N_2 i_2' = R \phi_m$$

- Let us consider the situation when $i_2' = 0$, which results from the transformer secondary open-circuit

Then : $i_1 = \frac{R}{N_1} \phi_m$

- In other words, there is still current flowing through the primary. We called this current primary magnetization current:

$$i_m \triangleq \frac{R}{N_1} \phi_m$$

$$i_1 + \frac{N_2}{N_1} i_2' = \underbrace{\frac{R}{N_1} \phi_m}_{i_m} \Rightarrow \boxed{i_1 = i_m - \frac{N_2}{N_1} i_2'}$$

We can now rename i_2' as before to be:

$$\boxed{i_2 = i_2'}$$

And we get : $\boxed{i_1 = i_m + \frac{N_2}{N_1} i_2}$

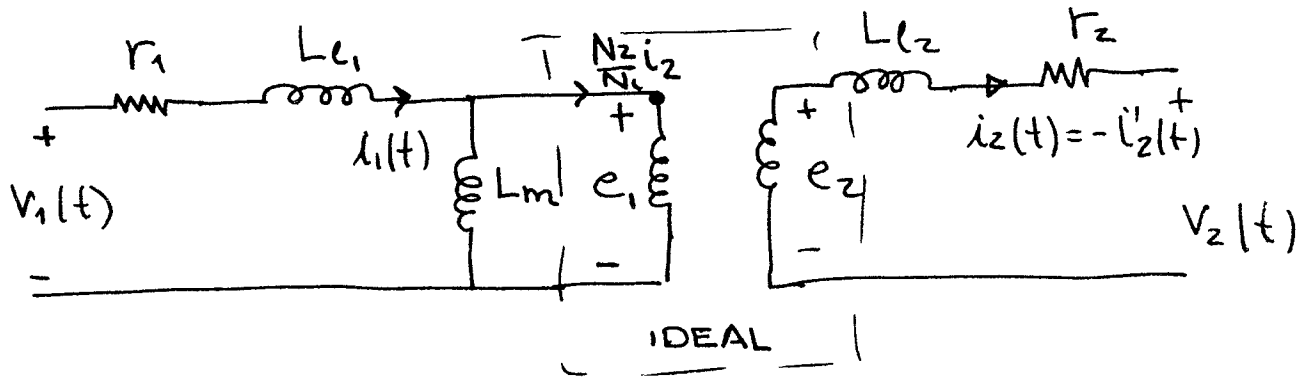
We can ~~to~~ now define two auxiliary voltage quantities:

$$e_1 \triangleq N_1 \frac{d\phi_m}{dt} = L_m \frac{di_m}{dt} \quad \cancel{e_2 \triangleq N_2 \frac{d\phi_m}{dt} = L_m \frac{di_m}{dt}}$$

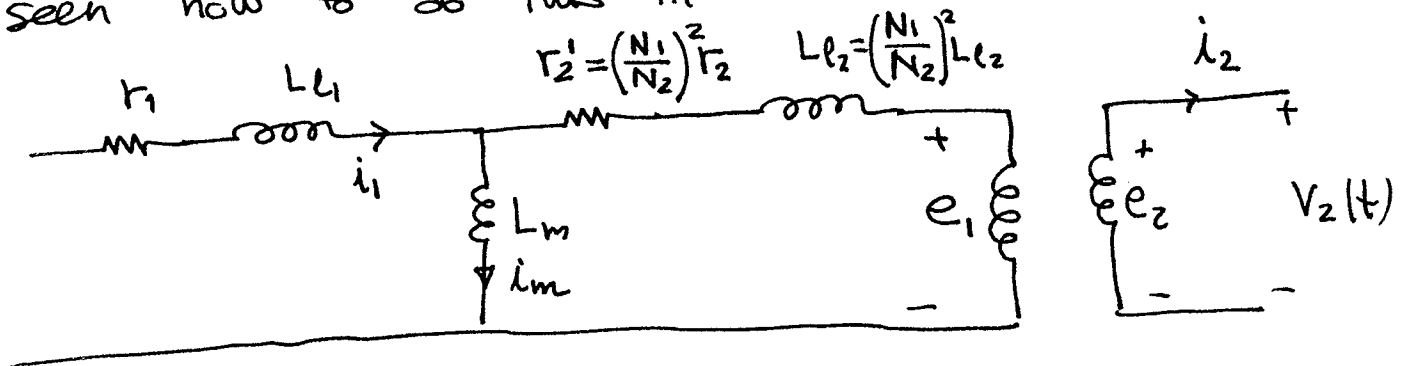
$$V_1 = r_1 i_1 + L_{l1} \frac{di_1}{dt} + L_m \frac{di_m}{dt} \quad \overbrace{e_1}^{e_1}$$

$$N_2 \frac{d\phi_m}{dt} = \frac{N_2}{N_1} \overbrace{N_1 \frac{d\phi_m}{dt}}^{e_1}$$

$$V_2 = r_2 i_2' + L_{l2} \frac{di_2'}{dt} + N_2 \frac{d\phi_m}{dt} = -r_2 i_2 - L_{l2} \frac{di_2}{dt} + \frac{N_2}{N_1} e_1$$

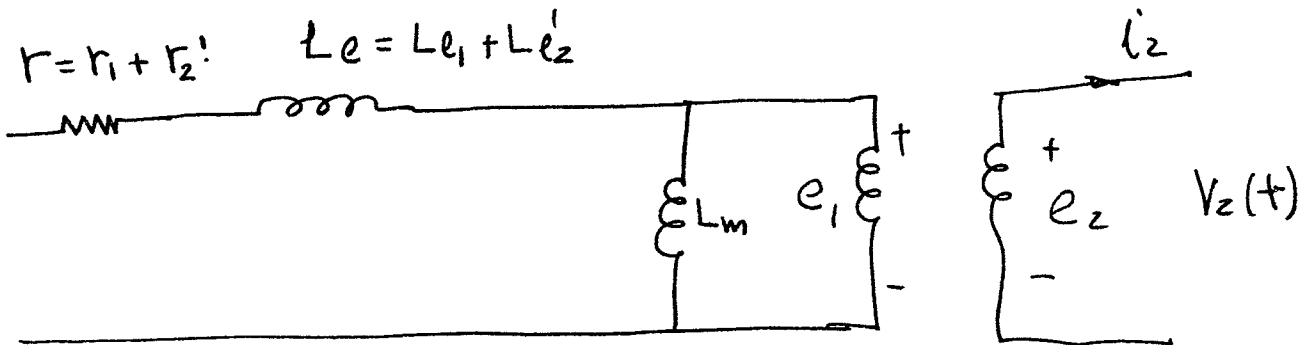


If we refer L_{l2} and r_2 to the primary (you have seen how to do this in 430)

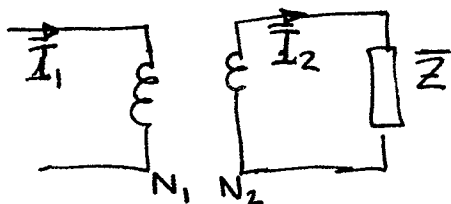


Since $L_m \uparrow$ and $r_2', L_{l2} \downarrow$ therefore

$$r = r_1 + r_2' \quad L_e = L_{l1} + L_{l2}'$$



IMPEDANCE REFERRED TO PRIMARY



$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$\frac{V_2}{I_1} = Z \rightarrow \left(\frac{N_2}{N_1} \right) \frac{V_1}{\frac{N_1}{N_2} I_1} = Z$$

$$\frac{V_1}{I_1} = \left(\frac{N_1}{N_2} \right)^2 Z = Z'$$