

LECTURE 8

So far we
have assumed

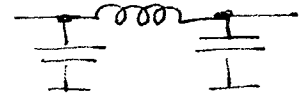
TRANSPPOSITION

REVIEW OF ELECTRIC FIELDS

- GAUSS' LAW
- VOLTAGE DIFFERENCE
- LINE CAPACITANCE

—○○○—

but indeed, the
model looks more like



→ REVIEW OF ELECTRIC FIELD CONCEPT

Similar to Ampere's law, for magnetic fields, we have Gauss' law for electric fields that allow us to compute electric fields created by charges:

$$\oint_S \vec{D} \cdot d\vec{a} = q_e$$

$\vec{D}$: Electric flux density

$d\vec{a}$: differential area

q_e : total charge in coulombs enclosed by the closed surface S

$\vec{D}$ is related to the electric field as follows

$$\vec{D} = \epsilon \vec{E}$$

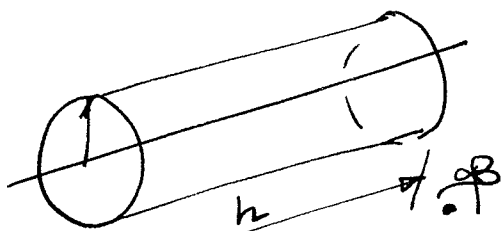
where ϵ is the permittivity of the medium in farads/m

- $\vec{E}$ [V/m]

- $\vec{D}$ [Coulombs/m²]

SINGLE WIRE

Let's compute the electric field produced by a wire of ∞ length.



q : charge density

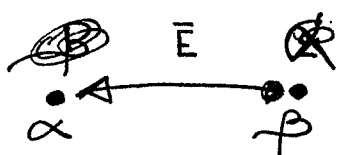
E has only radial component:

$$E_r \epsilon \cdot 2\pi R h = q_e$$

where $q_e = q \cdot h$

$$E_r = \frac{q}{2\pi \epsilon R}$$

VOLTAGE DIFFERENCE



The - comes about the following

① has more potential than ②

$$V_{\beta\alpha} = - \int_{\alpha}^{\beta} \vec{E} \cdot d\vec{l}$$

$$V_{\beta\alpha} = V_{\beta} - V_{\alpha}$$

if α is far away from the source

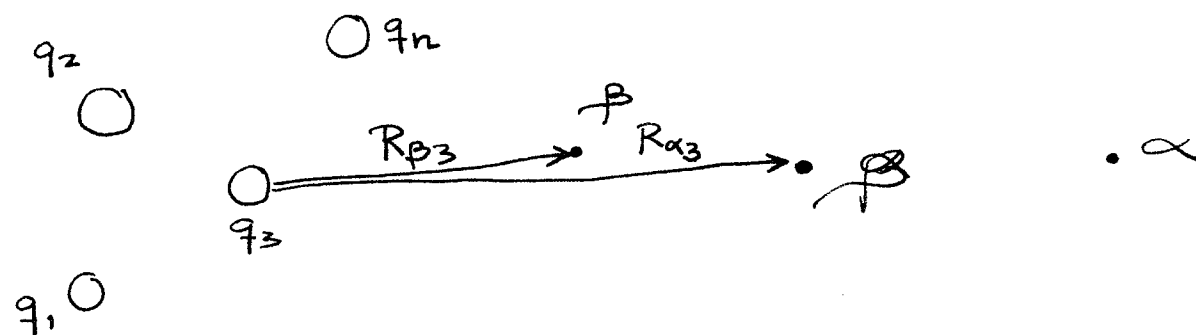
$V_{\beta} \gg V_{\alpha} \rightarrow V_{\beta\alpha} \gg 0$, but $\int_{\alpha}^{\beta} \vec{E} \cdot d\vec{l} \gg 0 \rightarrow$
~~we need the (-)~~ in front to make $V_{\beta\alpha} > 0$

In the wire example the voltage difference between two points located at R_{α} and R_{β} is

$$V_{\beta\alpha} = - \int_{R_{\alpha}}^{R_{\beta}} \frac{q}{2\pi \epsilon r} dr = \frac{q}{2\pi \epsilon} \ln \frac{R_{\alpha}}{R_{\beta}}$$

Remark. If we want to compute the voltage at which the potential is, we need to choose a reference. If α is the reference point and we locate it far away then $V_{p\alpha} \rightarrow 0$, which

MULTI-WIRE CASE



We solve the problem by superposition:

$$V_{p\alpha} = \frac{1}{2\pi\epsilon} \sum_{i=1}^n q_i \cdot \ln \frac{R_{\alpha i}}{R_{p i}} \quad \text{where } q_i \text{ is the current density on conductor } i$$

Let's assume $\sum q_i = 0$ and rewrite the expression as

$$V_{p\alpha} = \frac{1}{2\pi\epsilon} \sum_{i=1}^n q_i \ln \frac{1}{R_{p i}} + \frac{1}{2\pi\epsilon} \sum_{i=1}^n q_i \ln R_{\alpha i}$$

We can subtract the $\sum_{i=1}^n q_i R_{\alpha 1}$ (which is zero anyway)

$$V_{p\alpha} = \frac{1}{2\pi\epsilon} \sum_{i=1}^n q_i \ln \frac{1}{R_{p i}} + \frac{1}{2\pi\epsilon} \sum_{i=1}^n q_i \ln \frac{R_{\alpha i}}{R_{\alpha 1}}$$

$$\text{As } \alpha \rightarrow \infty \quad \frac{R_{\alpha i}}{R_{\alpha 1}} \rightarrow 1 \Rightarrow \ln \frac{R_{\alpha i}}{R_{\alpha 1}} \rightarrow 0 \rightarrow$$

$$\boxed{V_{p\alpha} = \frac{1}{2\pi\epsilon} \sum_{i=1}^n q_i \ln \frac{1}{R_{p i}}} \rightarrow \text{finite!}$$

Now /we are in a position to define the potential at any point in the space (except inside the wire).

$$V_P = \frac{1}{2\pi\epsilon} \sum_{i=1}^n q_i \ln \frac{1}{R_{pi}}$$

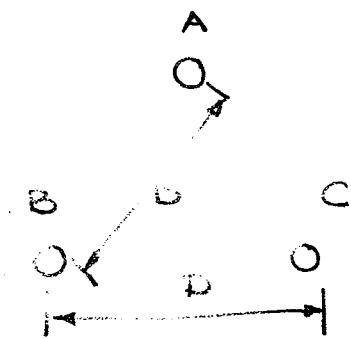
The main application of this expression is to relate transmission-line voltages and charges.

For example, in the previous case:

$$V_1 = \frac{1}{2\pi\epsilon} \cdot \left(q_1 \cdot \ln \frac{1}{R_{11}} + q_2 \cdot \ln \frac{1}{R_{12}} + \dots + q_n \ln \frac{1}{R_{1n}} \right)$$

where R_{11} is the radius of conductor 1

EXAMPLE



r : radius of conductor

$$V_a = \frac{1}{2\pi\epsilon} \left(q_a \ln \frac{1}{r} + q_b \ln \frac{1}{D} + q_c \ln \frac{1}{D} \right)$$

as before $q_b + q_c = -q_a$

$$V_a = \frac{q_a}{2\pi\epsilon} \ln \frac{D}{r}$$

LINE CAPACITANCE

- For a single line relates the charge of the line and the line potential.

$$q_i = C_i V_i$$

- In the case of multiple lines, ~~the~~ the charge of the line is related to all the potentials of the lines present.

$$\begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} C_{11} & \dots & C_{1n} \\ \vdots & & \vdots \\ C_{n1} & & C_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_n \end{bmatrix}$$

- In this course, we do not consider mutual capacitance. As we have seen in the symmetric case, they can be eliminated.

- In general they can be eliminated with in uniformly transposed lines.

In the previous cases

$$\textcircled{g} \quad V_a = \frac{q_a}{2\pi\epsilon} \ln \frac{D}{r} \rightarrow$$

$$\rightarrow \boxed{q_a = \frac{2\pi\epsilon}{\ln D/r} V_a} \rightarrow$$

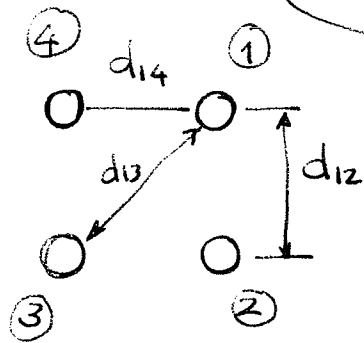
$$\rightarrow \boxed{C = \frac{2\pi\epsilon}{\ln D/r}}$$

BUNDLED CAPACITOR

As in the case of bundled capacitors, we use the original capacitance equation and just substitute by the ~~equivalent~~ radius

$$R_B^c = (r d_{12} \dots d_{1b})^{1/b} \quad b \geq 2$$

Geometric mean

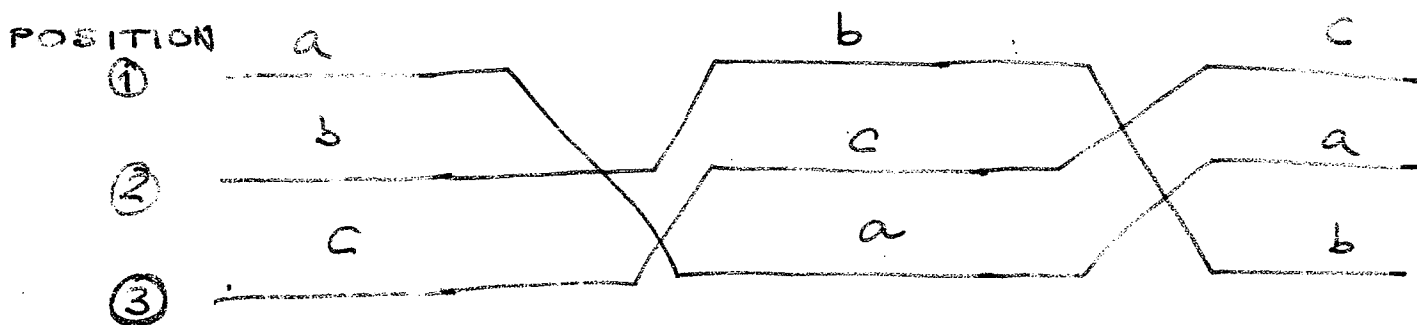


$\rightarrow r$ not $r'!!$

In the case of uniformly transposed lines

$$Z = \frac{2\pi E}{\ln D_m / R_B^c}, \text{ where}$$

$$D_m = [d_{12} d_{13} d_{23}]^{1/3}$$



RESISTANCE

- Line resistance per unit length is given by

$$R = \frac{\rho}{A} \quad \text{where } \rho \text{ is the resistivity.}$$

- Because AC current tends to flow towards the surface of a conductor, the resistance of a 60Hz line is slightly higher than at DC (~~Q~~ Skin effect)
- Resistivity (and therefore ρ) increases as Temperature increases
- Actual resistance, inductance and capacitance needs to be determined from tables because conductors are actually stranded.

