

LECTURE 6

FARADAY'S LAW

INDUCTANCE OF A SINGLE WIRE

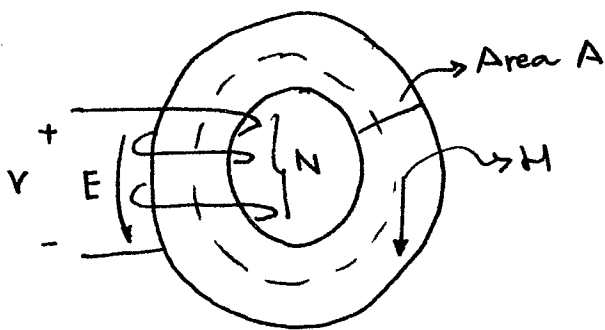
TWO-CONDUCTOR LINE INDUCTANCE

MANY-CONDUCTOR LINE INDUCTANCE

FARADAY'S LAW

$$\oint_C \vec{E} d\vec{l} = - \frac{d}{dt} \int_S \vec{B} d\vec{a}$$

Michael Faraday (1791-1867)
• No formal training



$$V + E = 0$$

$$-V = E = - \frac{d}{dt} \int_S \vec{B} d\vec{a}$$

$$V = \frac{d}{dt} \int_S \vec{B} d\vec{a} = \frac{d}{dt} \underbrace{BA \cdot N}_{\lambda}$$

$$\lambda \triangleq N \cdot \phi \rightarrow \text{flux linkages} \Rightarrow V = \frac{d\lambda}{dt}$$

- The flux linkages tells us how flux is linking an N turn coil.
- If we use Ampere's law from the previous example: $\lambda = N\phi = \frac{N^2 i}{\frac{l\mu}{A}} \rightarrow V = \frac{d}{dt} \left(\frac{N^2 i}{\frac{l\mu}{A}} \right) = \underbrace{\left(\frac{N^2}{\frac{l\mu}{A}} \right)}_L \frac{di}{dt}$

$$L \triangleq \frac{N^2}{\frac{l\mu}{A}} \rightarrow \text{inductance.}$$

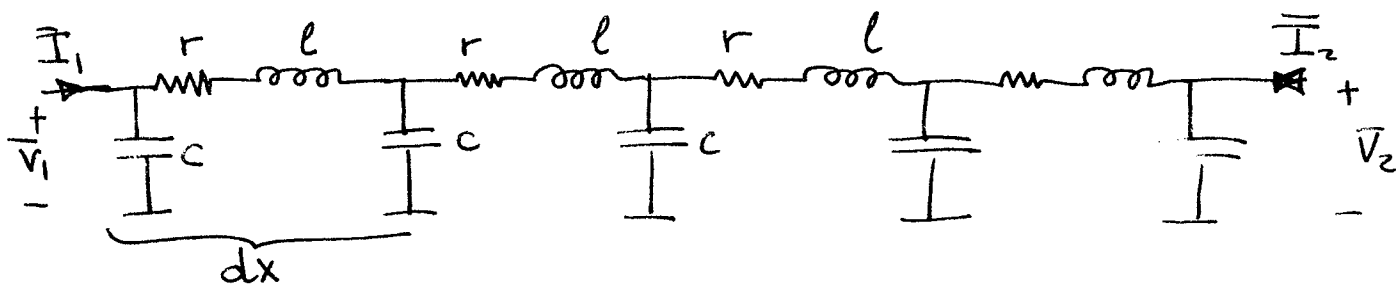
Now, taking into account that $\lambda = N\phi$, we can rewrite it as:

$$\lambda = \frac{N^2}{\frac{l\mu}{A}} i \rightarrow \boxed{\lambda = L \cdot i}$$

The significance of the inductance is the following:
In a linear circuit, the inductance relates the current and the flux linkage. It has units of Henry's: $L [H]$

INDUCTANCE OF A SINGLE WIRE

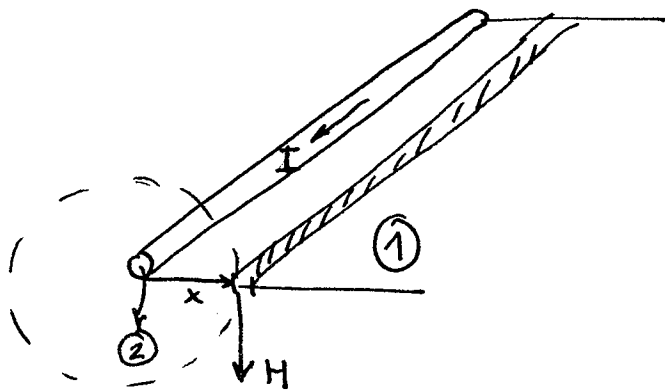
To develop models of transmission lines, we need to calculate the per-unit length inductance and capacitance.



We need to first find out the values of r , l , and c

- We are going to use Ampere's law to first obtain l
- We will talk later about r and c .
- We are going to start with a single infinitely long wire.
- We are going to compute the flux linkages and relate them to the current through the wire.
- Then assuming linearity, we will obtain l

- We will assume that the current density through the wire is uniform and the wire has a radius r
- To compute the flux linkages, we first need to compute them inside the wire and then outside the wire.



i) Flux linkage through a surface outside the wire, infinitely long and of width R

$$H_x \cdot 2\pi \cdot x = I \rightarrow H_x = \frac{I}{2\pi x}$$

$$\Phi_1 = \int_r^R \vec{B}_1 \cdot d\vec{a} = \int_r^R \frac{\mu_0 I}{2\pi x} dx =$$

$$= l \cdot \int_r^R \frac{\mu_0 I}{2\pi x} dx = \frac{\mu_0 I}{2\pi} l \cdot \ln \frac{R}{r}$$

Since we only have one turn, we get:

$$\lambda_1 = \Phi_1 = \frac{\mu_0 l I}{2\pi} \ln \frac{R}{r}$$

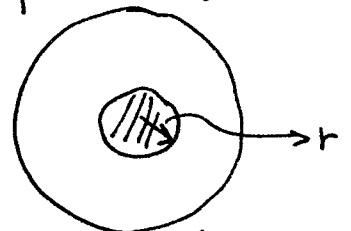
Since $l \rightarrow \infty$, $\lambda \rightarrow \infty$, let's define the per unit flux linkage $\lambda_1/l = \frac{\mu_0 I}{2\pi} \ln \frac{R}{r}$

ii) Flux linkage inside the wire.

$$\Phi_2 = \int_0^r \vec{B}_2 \cdot d\vec{a}$$

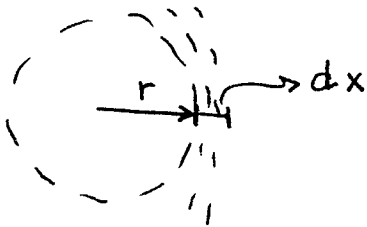
$$B_2 = \mu \cdot \frac{I_2}{2\pi x} = \frac{\mu \cdot I \cdot x^2/r^2}{2\pi x}$$

$$B_2 = \mu \cdot \frac{x}{2\pi r^2} I$$



$$\lambda_2 = \int_0^r d\lambda = \int_0^r \underbrace{\frac{\mu \cdot x \ell}{2\pi r^2}}_{B_2} \cdot \underbrace{\left(\frac{x^2}{r^2} \right)}_{\substack{\text{"Number of turns"} \\ \downarrow}} dx = \frac{\mu I}{2\pi r^4} \int_0^r x^3 dx$$

$$\lambda_2 = \frac{\mu I}{2\pi r^4} \cdot \frac{r^4}{4} = \frac{\mu I}{8\pi} \ell \rightarrow \lambda_2 / \ell = \frac{\mu I}{8\pi}$$



Total flux linkage per unit of length is:

$$\lambda = \lambda_1 + \lambda_2$$

$$\lambda = \frac{\mu_0 I}{2\pi} \ln \frac{R}{r} + \frac{\mu I}{8\pi} = \frac{\mu_0}{2\pi} \left(\ln \frac{R}{r} + \frac{\mu_r}{4} \right) I =$$

$$= \frac{\mu_0}{2\pi} \left(\ln R - \ln r - \ln e^{-\mu_r/4} \right) = \frac{\mu_0}{2\pi} \ln \frac{R}{r'} I$$

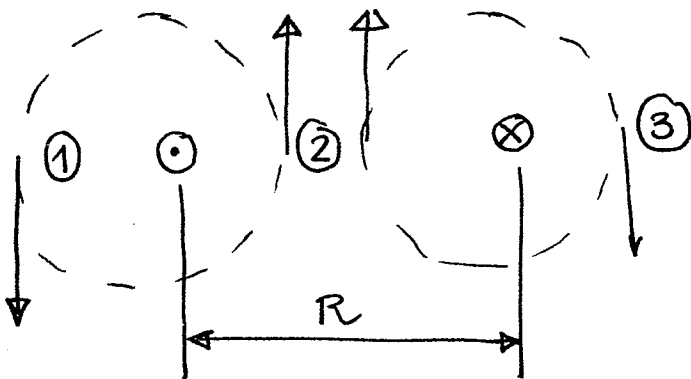
$$\text{where } r' = e^{-\mu_r/4}$$

$$L \triangleq \frac{\mu_0}{2\pi} \cdot \ln \frac{R}{r'}$$

$\rightarrow L$ defined this way is infinite.

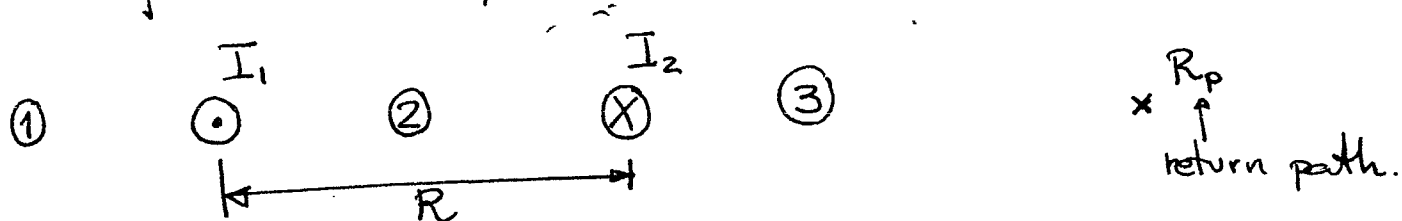
$\rightarrow$ we assume the return path was at ∞ .

TWO-CONDUCTOR LINE INDUCTANCE



To calculate the inductance of each wire, we will do as before, except that now we will have some field cancellation

Let's think about two wires with the return path at infinite, then the flux linked by the first one is the sum of the linkages created by itself $\oplus$ the linkages created by the second wire



$$\lambda_1 = \lambda_{11} + \lambda_{12}$$

$$\lambda_{11} = \frac{\mu_0}{2\pi} \ln \frac{R_p}{r'} I_1$$

$$\lambda_{12} = \frac{\mu_0}{2\pi} \ln \frac{R_p - R}{R} I_2$$

only at a distance greater than R , otherwise it does not link the wire.

but $I_1 = -I_2 = I$

$$\lambda_1 = \left(\frac{\mu_0}{2\pi} \ln \frac{R_p}{r'} - \frac{\mu_0}{2\pi} \ln \frac{R_p - R}{R} \right) I$$

$$\lambda_1 = \frac{\mu_0}{2\pi} \ln \cdot \frac{R}{r'} \cdot \frac{R_p}{R_p - R} I$$

$$\lim_{R_p \rightarrow \infty} \lambda_1 = \frac{\mu_0}{2\pi} \ln \frac{R}{r'} \cdot \frac{R_p}{R_p - R} \Rightarrow$$

$$\Rightarrow \boxed{L_1 = \frac{\mu_0}{2\pi} \ln \frac{R}{r'}} [H/m] \quad \rightarrow \text{now, it is finite.}$$