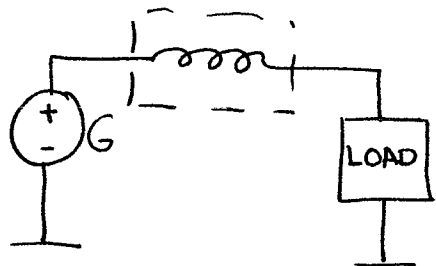


LECTURE 5

TRANSMISSION LINE MODELING

- Remember the simplest power system we talked about at the beginning of the course



- The next few lectures will be about how we build models of these elements, the transmission lines that link generator stations and loads.

The goals will be:

- 1 - Develop single models for transmission lines
- 2 - Gain an intuitive feel for how the geometry of the transmission line affects the model parameters.

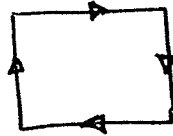
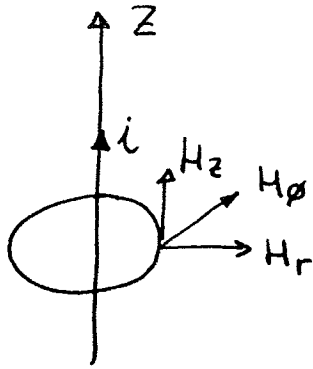
But before... we need to review a little bit about Maxwell's equations

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J}_f \cdot \vec{n} da + \frac{d}{dt} \int_S \epsilon \vec{E} \cdot \vec{n} da \quad (\text{Ampere's law})$$

$$\oint_C \vec{E} \cdot d\vec{l} = - \int_S \frac{d\vec{B}}{dt} \cdot \vec{n} da \quad (\text{Faraday's law})$$

- H : magnetic field intensity (amp-turns/meter)
- $d\vec{l}$: vector differential path length (meters)
- $\oint_C$: line integral about closed path C ($d\vec{l}$ is tangent to the path)

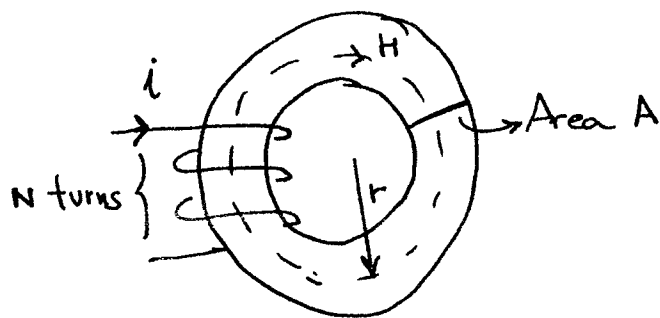
MAGNETIC FIELD FROM A SINGLE WIRE



H_z and H_r are zero by symmetry

$$H_\phi \cdot 2\pi \cdot r = i \rightarrow H_\phi = \frac{i}{2\pi r}$$

MAGNETIC FIELD INSIDE A MAGNETIC CORE



We assume H is uniform through the whole core

$$H \cdot 2\pi r = Ni$$

$$H = \frac{Ni}{2\pi r}$$

- $\mathcal{F} \triangleq Ni \rightarrow$ This is called the magneto-motive force (MMF)

MAGNETIC FLUX DENSITY (B)

$B \triangleq \mu H$ where μ is called the permeability

$$\mu = \mu_0 \cdot \mu_r$$

μ_0 : permeability of freespace : $4\pi \cdot 10^{-7} \text{ H/m}$

μ_r : relative permeability: 1 for air and 1000 for magnetic materials

units of B

Tesla [T] or Gauss [G]: $1\text{T} = 10^4 \text{ G}$

MAGNETIC FLUX DENSITY (ϕ)

Total flux passing through a surface A :

$$\phi = \int_A \vec{B} d\vec{a} \quad d\vec{a} : \text{vector with direction normal to the surface.}$$

If B is uniform and $\perp$ to A , then:

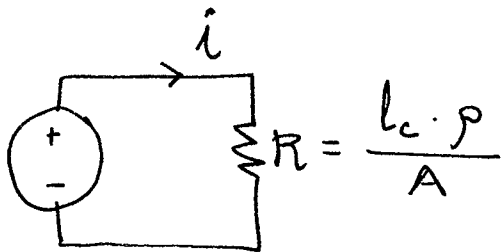
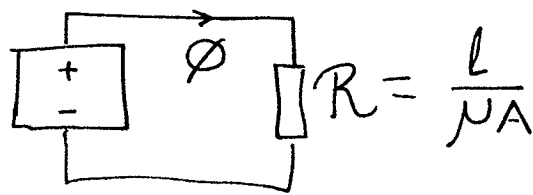
$$\boxed{\phi = BA}$$

RELUCTANCE ($\mathcal{R}$)

In the coil example, multiply by A on both sides and μ :

$$A \cdot \mu H = \mu \cdot A \cdot \frac{Ni}{2\pi R} = \frac{Ni}{\frac{l}{\mu A}}$$

$$\phi = \frac{Ni}{\mathcal{R}}$$



There is an analogy between electric circuits and magnetic circuits:

$$\begin{aligned} V &\longrightarrow Ni \\ R &\longrightarrow \mathcal{R} \\ i &\longrightarrow \phi \end{aligned}$$