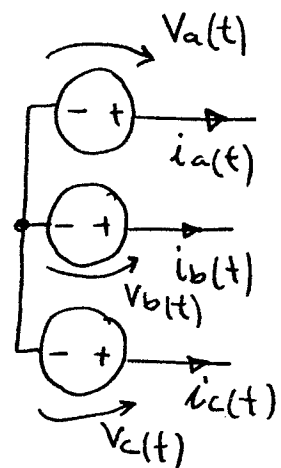


LECTURE 4

- REVIEW: COMPLEX POWER IN 3 ϕ SYSTEMS
- Y- Δ CONNECTION
- Δ -Y TRANSFORMATION
- PER-PHASE EQUIVALENTS
- EXAMPLES

COMPLEX POWER (AND INSTANTANEOUS POWER)



$$P(t) = V_a(t) i_a(t) + V_b(t) i_b(t) + V_c(t) i_c(t)$$

$$V_a(t) = \frac{V\sqrt{2}}{\sqrt{2}} \cos \omega t \quad i_a(t) = \frac{I\sqrt{2}}{\sqrt{2}} \cos(\omega t - \theta)$$

$$V_b(t) = \frac{V\sqrt{2}}{\sqrt{2}} \cos(\omega t - 120^\circ) \quad i_b(t) = \frac{I\sqrt{2}}{\sqrt{2}} \cos(\omega t - 120^\circ - \theta)$$

$$V_c(t) = \frac{V\sqrt{2}}{\sqrt{2}} \cos(\omega t + 120^\circ) \quad i_c(t) = \frac{I\sqrt{2}}{\sqrt{2}} \cos(\omega t + 120^\circ - \theta)$$

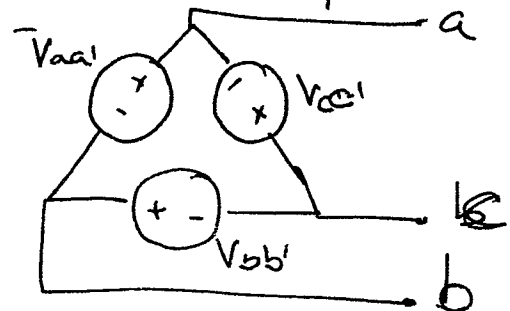
$$P(t) = 3VI \cos \theta \quad \leftarrow \text{CONSTANT!}$$

$$\bar{S}_{3\phi} = \bar{V}_a \bar{I}_a^* + \bar{V}_b \bar{I}_b^* + \bar{V}_c \bar{I}_c^* = 3 \cdot V \cdot I \cdot \cos \theta + j 3VI \sin \theta$$

But $I = I_L = I_\phi$ (in this case) and $V_L = \sqrt{3} V_\phi = \sqrt{3} V$

$$\bar{S}_{3\phi} = \sqrt{3} V_L \cdot I_L \cos \theta + j \sqrt{3} V_L \cdot I_L \sin \theta \quad \leftarrow \text{angle between } V_\phi \text{ and } I_\phi$$

→ This expression is true for any connection



$$\bar{S}_{3\phi} = 3 \cdot V_\phi \cdot \frac{I_L}{\sqrt{3}} \cos \theta + j 3 V_\phi I_\phi \sin \theta$$

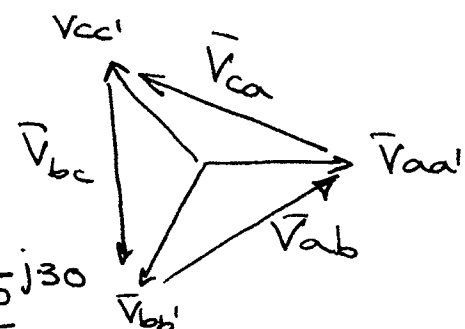
$\uparrow$
 V_L $\frac{I_L}{\sqrt{3}}$ $\uparrow$
 angle between V_ϕ and I_ϕ

$$\bar{S} = \sqrt{3} \bar{V}_L \bar{I}_L^* \leftarrow \text{wrong}$$

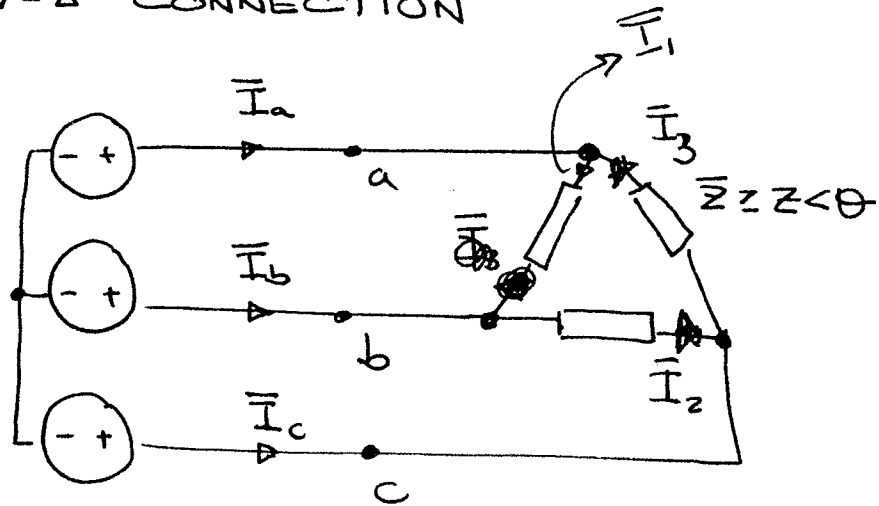
$$\bar{S} = 3 \cdot \bar{V}_\phi \cdot \bar{I}_\phi^*$$

For a γ connection $\bar{V}_\phi = \frac{\bar{V}_L}{\sqrt{3}} \cdot e^{-j30}$

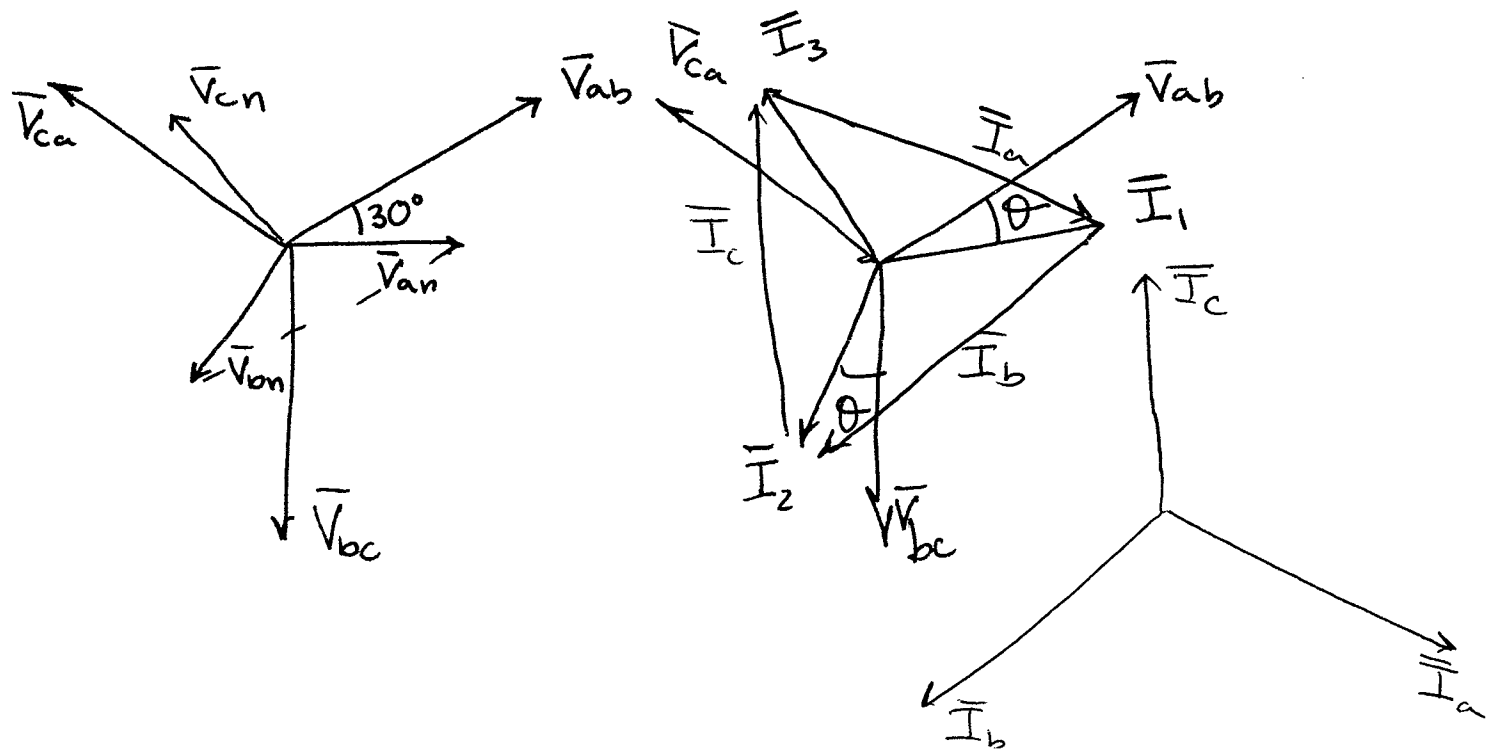
$$\bar{S} = 3 \cdot \frac{\bar{V}_L}{\sqrt{3}} \bar{I}_L^* \cdot e^{-j30} = \sqrt{3} \cdot \bar{V}_L \cdot \bar{I}_L^* \cdot e^{-j30}$$



Y- Δ CONNECTION

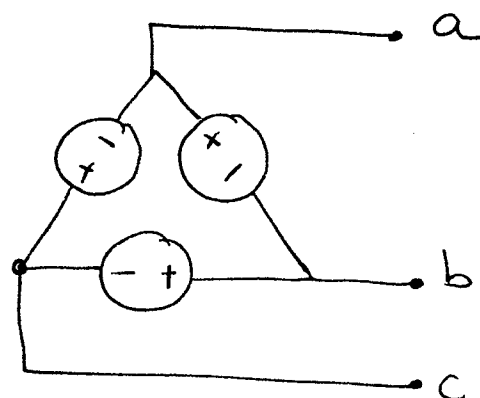
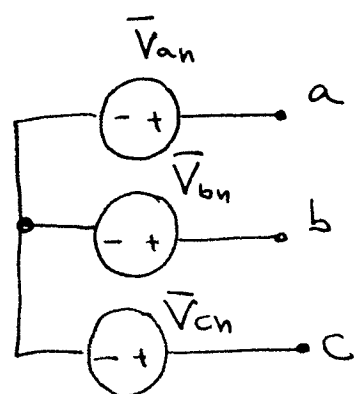


$$\bar{I}_a = \bar{I}_1 - \bar{I}_3 \quad \bar{I}_b = \bar{I}_2 - \bar{I}_1 \quad \bar{I}_c = \bar{I}_3 - \bar{I}_2$$



Δ -Y TRANSFORMATIONS

SOURCES

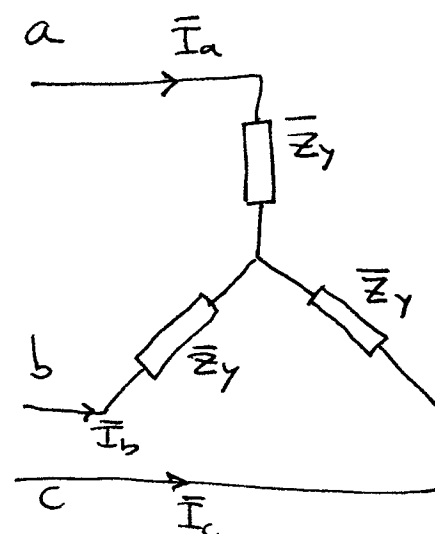
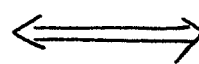
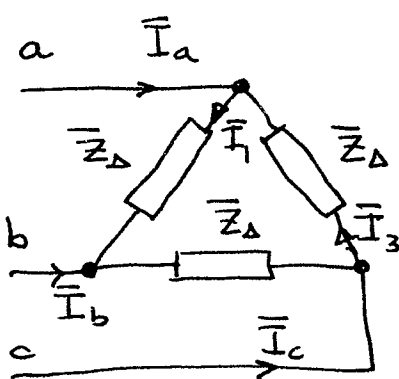


$$\bar{V}_{ab} = \bar{V}_{an} \cdot \sqrt{3} < 30$$

$$\bar{V}_{bc} = \bar{V}_{bn} \cdot \sqrt{3} < 30$$

$$\bar{V}_{ca} = \bar{V}_{cn} \cdot \sqrt{3} < 30$$

LOADS



$$\bar{I}_a = \bar{I}_1 - \bar{I}_3 = \frac{\bar{V}_{ab}}{\bar{Z}_A} - \frac{\bar{V}_{ca}}{\bar{Z}_A}$$

$$\bar{Z}_A = \frac{\bar{V}_{ab} - \bar{V}_{ca}}{\bar{I}_a}$$

$$\frac{\bar{V}_{ab} - \bar{V}_{ca}}{\bar{I}_a} = 3\bar{Z}_Y$$

$$\boxed{\bar{Z}_\Delta = 3\bar{Z}_Y}$$

$$\bar{V}_{ab} = \bar{Z}_Y(\bar{I}_a - \bar{I}_b)$$

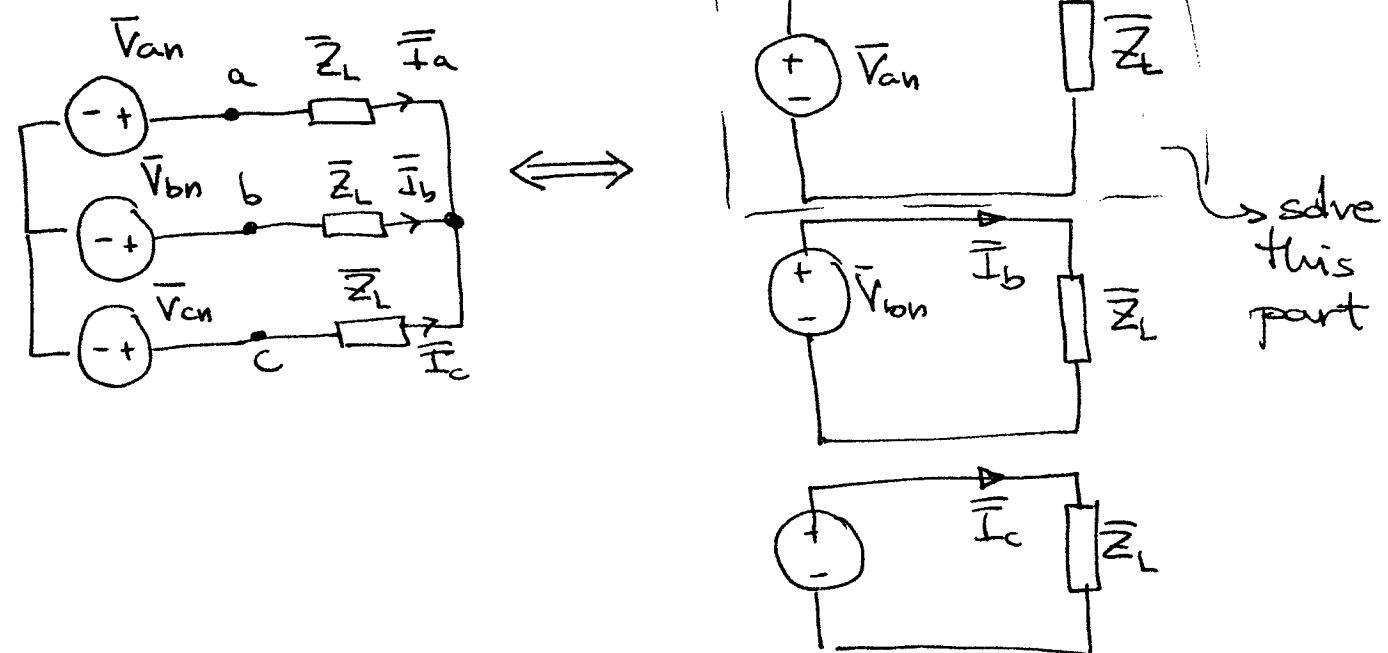
$$\bar{V}_{ca} = \bar{Z}_Y(\bar{I}_c - \bar{I}_a)$$

$$\frac{\bar{V}_{ab} - \bar{V}_{ca}}{\bar{Z}_Y} = 2\bar{I}_a - \underbrace{\bar{I}_b - \bar{I}_c}_{+\bar{I}_a}$$

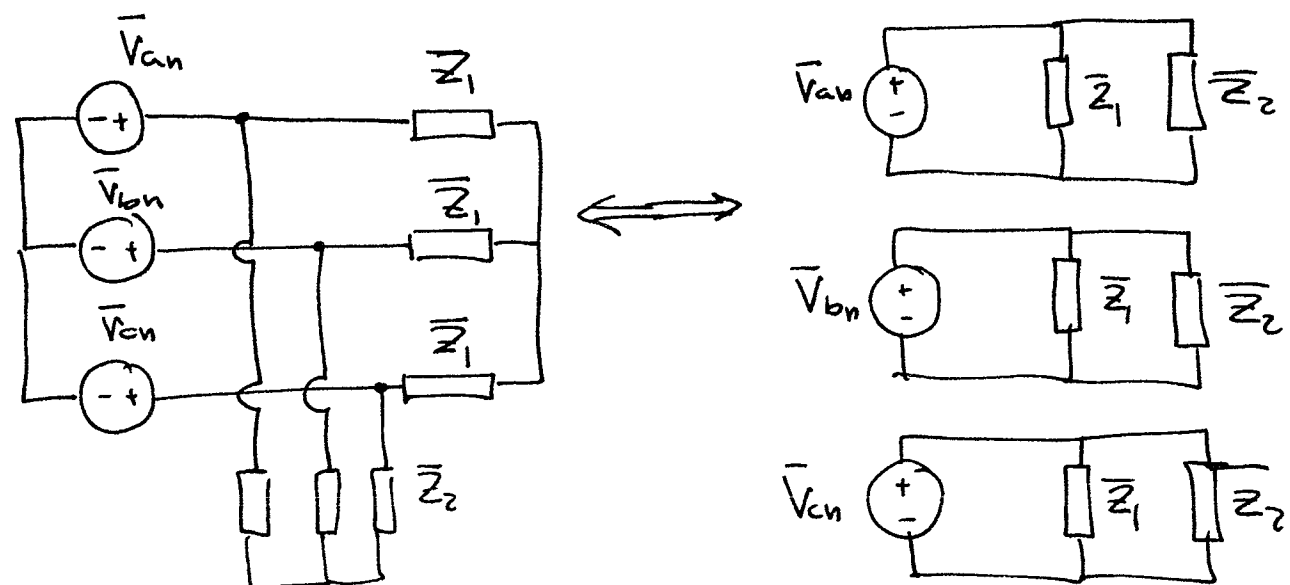
$$\frac{\bar{V}_{ab} - \bar{V}_{ca}}{\bar{Z}_Y} = 3\bar{I}_a$$

PER-PHASE EQUIVALENTS

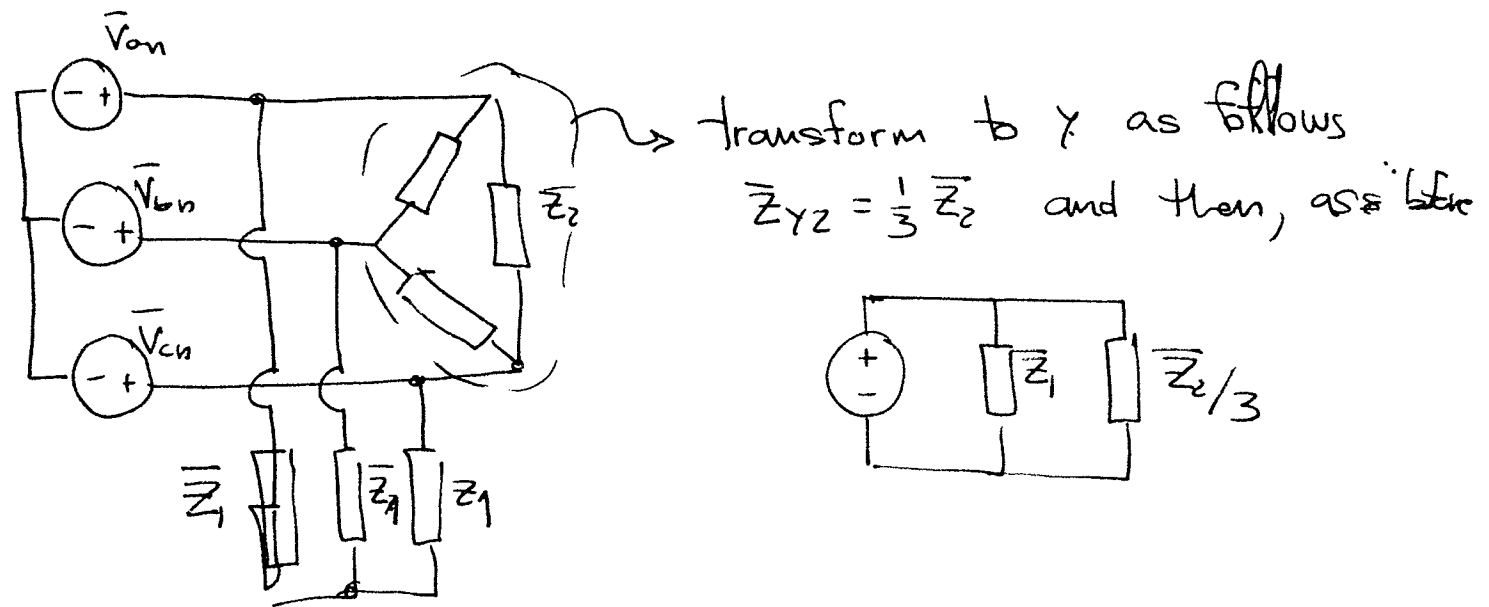
Recall:



What about?



What about?



In summary:

- For a balanced 3ϕ system with
- i) All loads and sources γ -connected
 - ii) No mutual inductances between phases:

Then:

- A) All neutrals are @ the same potential
 - B) All phases are completely decoupled.
 - C) All system values are the same sequence as others
- (+ seq: phase b lags phase a and phase c lags phase a)