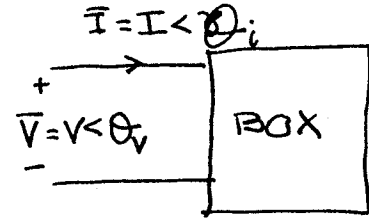


LECTURE 3

REVIEW: POWER NOTIONS

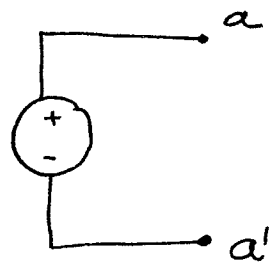
- THREE-PHASE SYSTEMS
- POWER IN THREE-PHASE SYSTEMS

POWER NOTIONS REVIEW

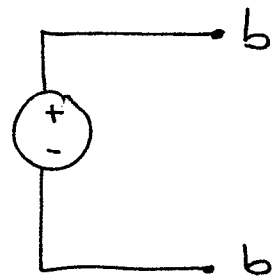


$$\begin{aligned} \bar{V} &= V e^{j\theta_v} \leftarrow v(t) = \sqrt{2} V \cdot \cos(\omega t + \theta_v) \\ \bar{I} &= I e^{j\theta_i} \leftarrow i(t) = \sqrt{2} I \cdot \cos(\omega t + \theta_i) \\ P(t) &= v(t) \cdot i(t) = VI \cos(\theta_v - \theta_i) + VI \cos(2\omega t + \theta_v + \theta_i) \\ \bar{P} &= VI \cos(\theta_v - \theta_i) \\ \bar{S} &\triangleq \bar{V} \cdot \bar{I}^* = \underbrace{VI \cos(\theta_v - \theta_i)}_P + j \underbrace{VI \sin(\theta_v - \theta_i)}_Q \end{aligned}$$

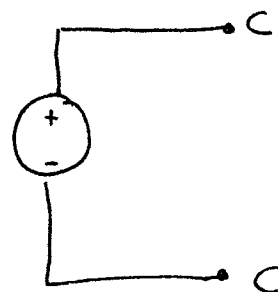
THREE-PHASE SYSTEMS



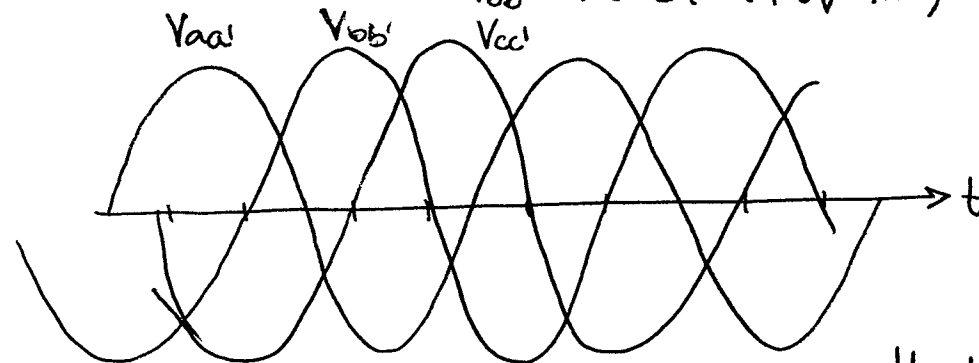
$$V_{aa'} = V \cos(\omega t + \theta_v)$$



$$V_{bb'} = V \cos(\omega t + \theta_v - 120^\circ)$$



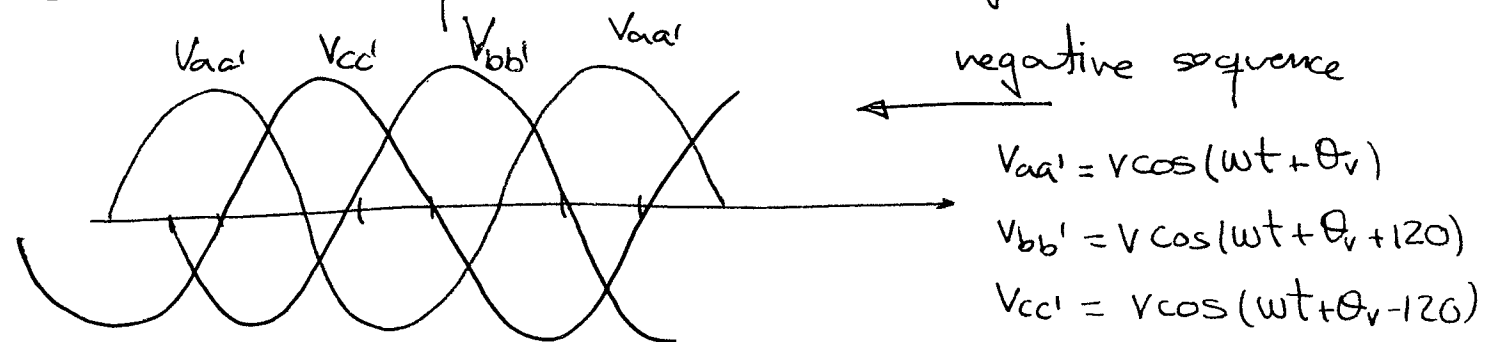
$$V_{cc'} = V \cos(\omega t + \theta_v + 120^\circ)$$



This is called positive (abc) sequence

- This has to do with the way we find the maximums as we traverse the time axis in the positive direction

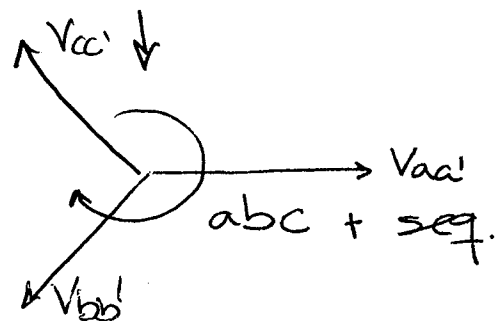
- If we exchange the phase shifting of bb and cc



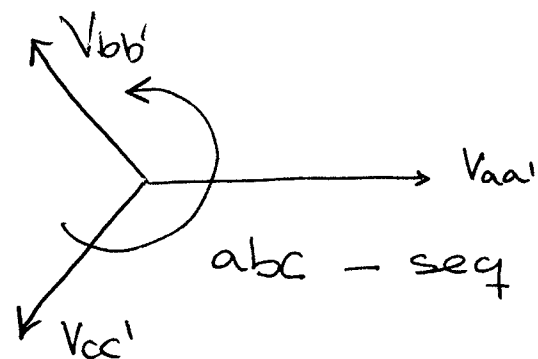
PHASOR REPRESENTATION

For simplicity, let's make $\theta_v = 0$ and assume

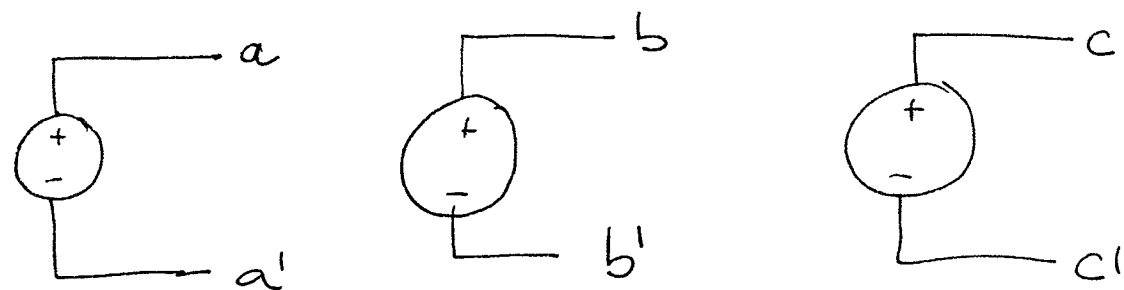
$$+ \begin{cases} \bar{V}_{aa'} = V e^{j\theta_v} \\ \bar{V}_{bb'} = V e^{j(\theta_v - 120)} \\ \bar{V}_{cc'} = V e^{j(\theta_v + 120)} \end{cases}$$



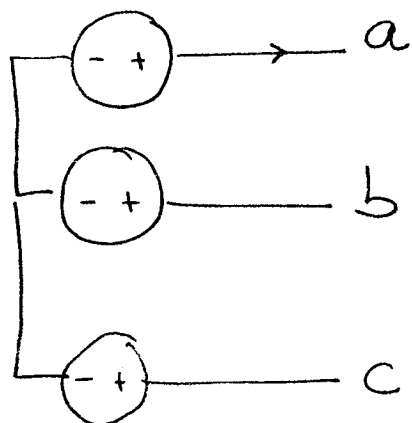
$$- \begin{cases} \bar{V}_{aa'} = V e^{j\theta_v} \\ \bar{V}_{bb'} = V e^{j(\theta_v + 120)} \\ \bar{V}_{cc'} = V e^{j(\theta_v - 120)} \end{cases}$$



THREE-PHASE CONNECTIONS

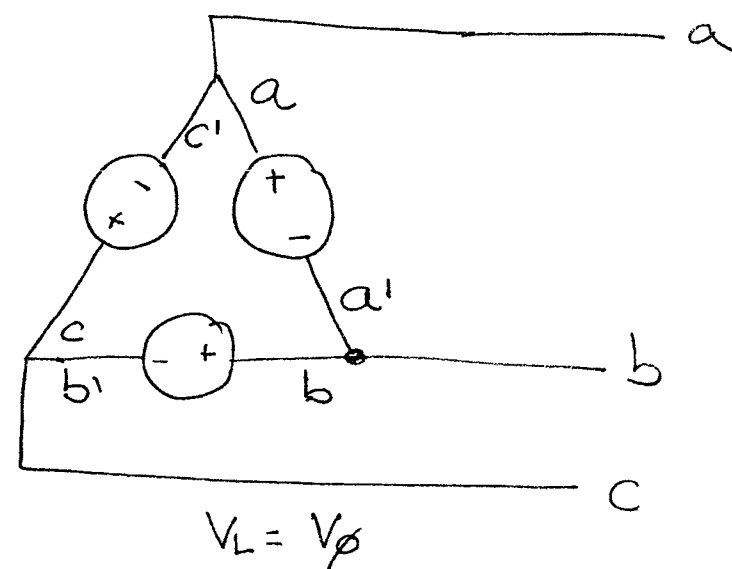


WYE CONNECTION



$$i_L = i_\phi$$

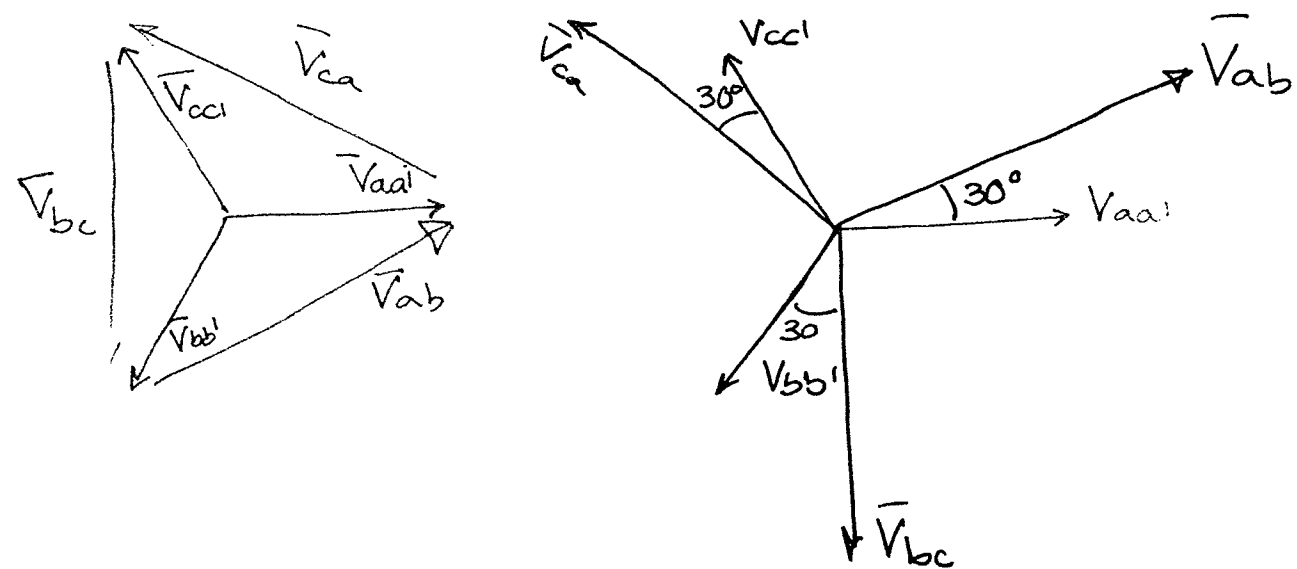
DELTA CONNECTION



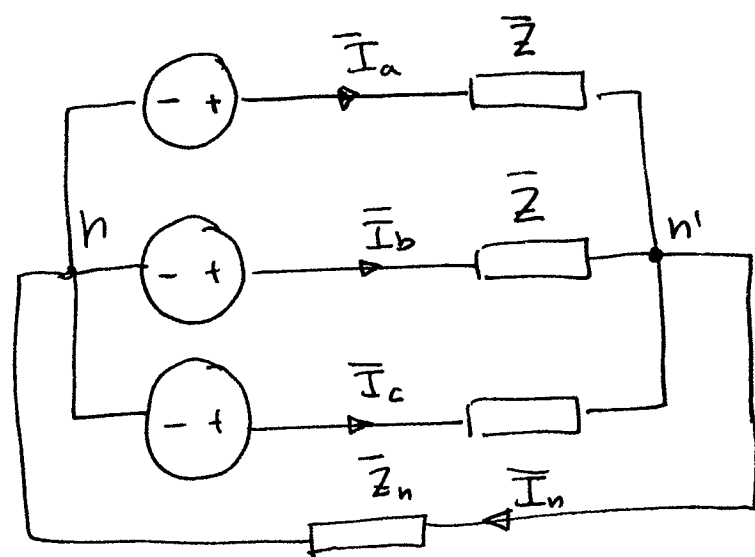
DEFINITIONS

- Line voltage: Voltage across any two lines (V_L)
- Phase voltage: Voltage across any individual source or load (V_ϕ)
- Line current: Current flowing in and out of each line i_L
- Phase current: The current flowing through each individual source (or load) (i_ϕ)

RELATIONS BETWEEN LINE & PHASE VOLTAGES FOR Y-C



Y-Y CONNECTION WITH BALANCED LOAD



$$\begin{aligned}\bar{V}_{an} &= V e^{j0} \\ \bar{V}_{bn} &= V e^{-j120} \\ \bar{V}_{cn} &= V e^{j120}\end{aligned}$$

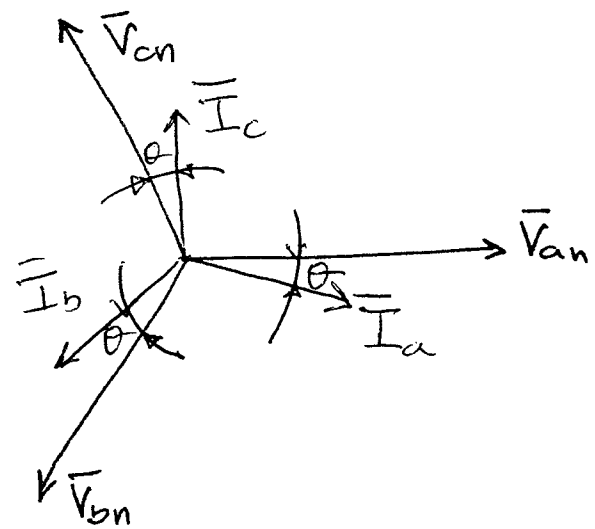
$$\left. \begin{aligned}\bar{V}_{an} &= \bar{Z} \cdot \bar{I}_a + \bar{Z}_n \bar{I}_n \\ \bar{V}_{bn} &= \bar{Z} \bar{I}_b + \bar{Z}_n \bar{I}_n \\ \bar{V}_{cn} &= \bar{Z} \bar{I}_c + \bar{Z}_n \bar{I}_n\end{aligned} \right\} \rightarrow \begin{aligned}\bar{V}_{an} + \bar{V}_{bn} + \bar{V}_{cn} &= \bar{Z} (\bar{I}_a + \bar{I}_b + \bar{I}_c) + \\ &+ 3 \bar{Z}_n \bar{I}_n\end{aligned}$$

$$0 = (\bar{Z} + 3 \bar{Z}_n) \bar{I}_n$$

$$\boxed{\bar{I}_n = 0}$$

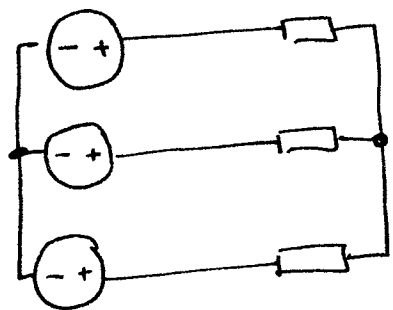
Now let's assume that $\bar{Z} = Z \angle \theta$

$$\bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}} \quad \bar{I}_b = \frac{\bar{V}_{bn}}{\bar{Z}} \quad \bar{I}_c = \frac{\bar{V}_{cn}}{\bar{Z}}$$

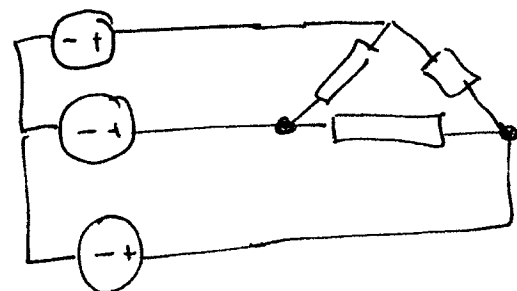


This result is basically telling us that we can analyze each phase separately as single-phase circuits and then assemble the result.

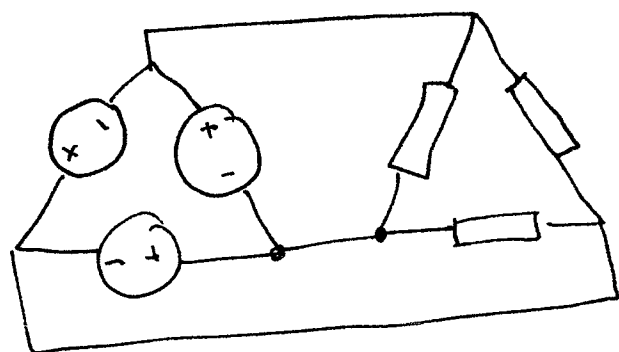
OTHER THREE-PHASE CONNECTIONS



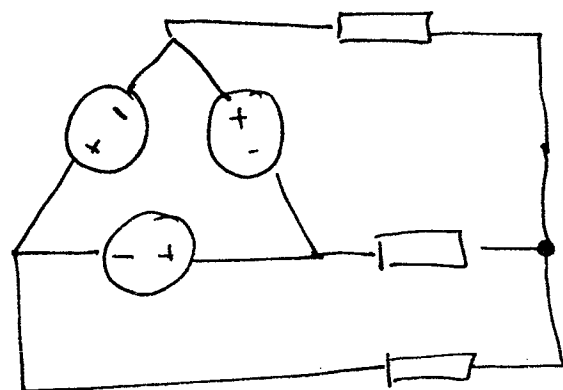
Y-Y CONNECTION



Y-Δ CONNECTION



Δ-Δ CONNECTION



Y-Y CONNECTION

THREE-PHASE POWER

$$\begin{aligned} V_{an} &= \frac{V}{\sqrt{2}} \cos(\omega t) & i_a &= \frac{I}{\sqrt{2}} \cos(\omega t - \theta) \\ V_{bn} &= \frac{V}{\sqrt{2}} \cos(\omega t - 120) & i_b &= \frac{I}{\sqrt{2}} \cos(\omega t - 120 - \theta) \\ V_{cn} &= \frac{V}{\sqrt{2}} \cos(\omega t + 120) & i_c &= \frac{I}{\sqrt{2}} \cos(\omega t + 120 - \theta) \end{aligned}$$

$$P_a(t) = VI [\cos \theta + \cos(2\omega t - \theta)]$$

$$P_b(t) = VI [\cos \theta + \cos(2\omega t - 120 - \theta)]$$

$$P_c(t) = VI [\cos \theta + \cos(2\omega t + 120 - \theta)]$$

$$P(t) = P_a(t) + P_b(t) + P_c(t) = 3VI \cos \theta + 0$$

$$\boxed{P(t) = 3VI \cos \theta} \rightarrow \text{CONSTANT}$$

$$V = V_\phi \quad I = I_\phi$$

$$\text{but } V_\phi = \frac{V_L}{\sqrt{3}} \quad I_\phi = I_L \quad (\text{Y-CONNECTION})$$

$$V_\phi = V_L \quad I_\phi = \frac{I_L}{\sqrt{3}} \quad (\text{Δ-CONNECTION})$$

$$P = \sqrt{3} V_L \cdot I_L \cos \theta$$

$$\bar{S} = P + jQ = \sqrt{3} V_L I_L (\cos \theta + j \sin \theta)$$

↑
phase shift between
voltage V_ϕ and current I_ϕ

$$\begin{aligned} \bar{S} &= 3 \cdot \overline{V_\phi} \cdot \overline{I_\phi} \cdot \cos \theta + j \dots \\ &= 3 \overline{V_{an}} \overline{I_a}^* = 3 \overline{V_{bn}} \overline{I_b}^* = 3 \overline{V_{cn}} \overline{I_c}^* = \\ &= 3 V_\phi I_\phi (\cos \theta + j \sin \theta) = \sqrt{3} V_L I_L (\cos \theta + j \sin \theta) \\ &\neq \sqrt{3} \overline{V_L} \cdot \overline{I_L}^* \\ &\quad \rightarrow \text{huge mistake.} \end{aligned}$$