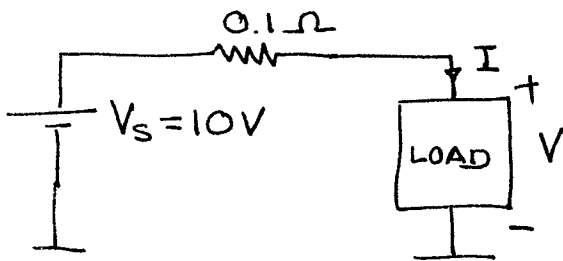


LECTURE 2

- READING: Chapters 1 & 2 of SG&O
- THE CENTRAL PROBLEM IN POWER SYSTEMS
- REVIEW OF PHASORS
- POWER: INSTANTANEOUS, AVERAGE & COMPLEX
- REACTIVE POWER COMPENSATION

THE CENTRAL PROBLEM IN POWER SYSTEMS

- It is to compute all electric variables given load consumptions. In a nutshell with a simple DC system



LOAD

$P = 10W$, what are the values of V & I ?

Power:

$$\left. \begin{array}{l} V \cdot I = P = 10 \\ V_s = 0.1 \cdot I + V \end{array} \right\} \rightarrow I = \frac{10 - V}{0.1} \quad \left\{ \begin{array}{l} 10 = V \cdot I \\ 10 = V \cdot \left(\frac{10 - V}{0.1} \right) \end{array} \right. \rightarrow$$

$$\rightarrow 1 = 10V - V^2 = 0 \rightarrow V^2 - 10V + 1 = 0$$

$$V = \frac{10 \pm \sqrt{10^2 - 4}}{2} = \begin{cases} \frac{10 + \sqrt{96}}{2} = 9.8990V \rightarrow I = 1.01A \\ \frac{10 - \sqrt{96}}{2} = 0.1010V \rightarrow I = 98.99A \end{cases}$$

Both solutions are possible in real life.

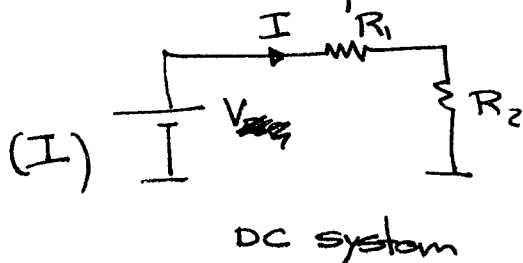
- The first one represents a case when the system is working properly (good voltage on the load, small currents)

- The second case is typical of a situation where there could be a fault (a short-circuit). We are interested in understanding this case to size properly conductors and to set protections properly.

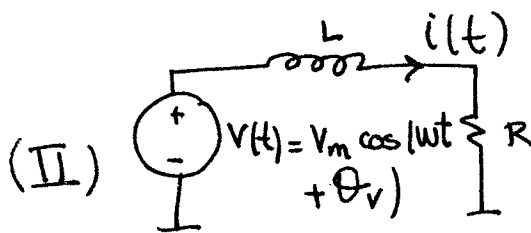
REVIEW OF PHASORS

- The systems we will study extensively in this class are AC electric power systems.
- The use of phasors is very convenient to ~~solve~~ formulate and solve problems of interest.

- Let's go back to the DC example.



$V_{eff} = (R_1 + R_2) I \rightarrow$ This is an algebraic expression, which we know how to manipulate.



Kirchoff's laws always apply, i.e., the physics holds, so we can write the following diff eq:

$$v(t) = R \cdot i(t) + L \cdot \frac{di}{dt}$$

Q: Can we use some trick to represent (II) in such a way that we can use algebraic representation instead of differential equation? Hopefully, this will make our lives easier

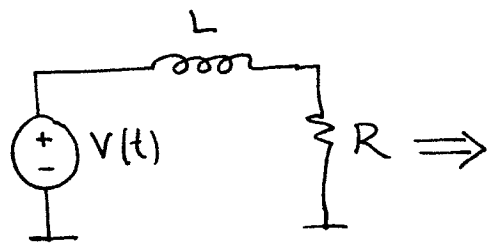
A: YES! PHASORS!!

$$v(t) \rightarrow \bar{V}$$

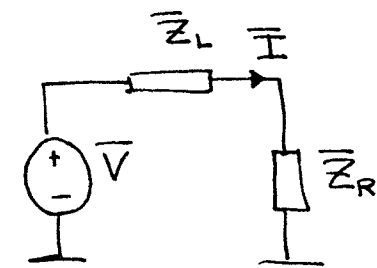
$$L \rightarrow \bar{Z}_L$$

$$R \rightarrow \bar{Z}_R$$

$$i \rightarrow \bar{I}$$

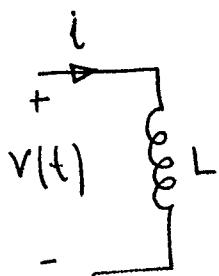


$$v(t) = L \cdot \frac{di}{dt} + R \cdot i$$



$$\bar{V} = (\bar{Z}_L + \bar{Z}_R) \bar{I}$$

Q: How do we go from $v(t)$, L , R to $\bar{V}$, $\bar{Z}_L$, $\bar{Z}_R$?



$$v(t) = L \cdot \frac{di}{dt}$$

$$v(t) = V_m \cos(\omega t + \theta_v) = L \cdot \frac{di}{dt}$$

In steady-state: $i(t) = I_m \cdot \cos(\omega t + \theta_i)$

$$V_m \cos(\omega t + \theta_v) = -L \cdot I_m \omega \sin(\omega t + \theta_i)$$

$$\text{But } -\sin(\omega t + \theta_i) = \cos\left(\theta_i + \frac{\pi}{2} + \omega t\right)$$

$$V_m \cos(\omega t + \theta_v) = L \cdot I_m \omega \cdot \cos\left(\omega t + \theta_i + \frac{\pi}{2}\right)$$

$$\boxed{I_m = \frac{V_m}{\omega L} \quad \theta_i = \theta_v - \frac{\pi}{2}}$$

Now let's add and subtract $j V_m \sin(\omega t + \theta_v)$ and $j L I_m \omega \cos(\omega t + \theta_i + \frac{\pi}{2})$:

$$V_m \cdot \cos(\omega t + \theta_v) + j \sin(\omega t + \theta_v) = L \cdot I_m \cdot \omega \cos\left(\omega t + \theta_i + \frac{\pi}{2}\right) + j \cancel{L} I_m \omega \sin(\omega t + \theta_i + \pi/2)$$

Euler's expansion:

$$e^{j\theta} = \cos \theta + j \sin \theta$$

Then:

$$V_m e^{j(\theta_v + \omega t)} = L \cdot \omega I_m e^{j(\theta_i + \omega t + \pi/2)}$$

$$V_m e^{j\theta_v} \cdot \cancel{e^{j\omega t}} = L \omega I_m e^{j(\theta_i + \pi/2)} \cdot \cancel{e^{j\omega t}} = L \cdot \omega I_m e^{j\pi/2} \cdot e^{j\theta_i} = L \omega \cdot e^{j\pi/2} I_m e^{j\theta_i}$$

Let's define: $\bar{V} \triangleq \frac{V_m}{\sqrt{2}} e^{j\theta_v}$

$$\bar{I}_m \triangleq \frac{I_m}{\sqrt{2}} e^{j\theta_i}$$

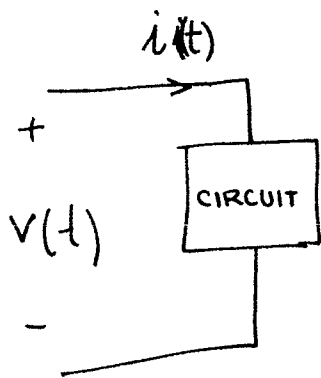
$$\bar{Z}_L \triangleq j \omega L$$

So we get: $\bar{V} = \bar{Z}_L \cdot \bar{I}$!

Similarly, we can get:

$$C \rightarrow \frac{1}{j\omega C}$$
$$R \rightarrow R$$

INSTANTANEOUS POWER



$$P(t) = V(t) \cdot i(t)$$

$$V(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$P(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

ROOT-MEAN SQUARE

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T V^2(t) dt} = \frac{V_m}{\sqrt{2}}$$

- Instead of $\bar{V} = \frac{V_m}{\sqrt{2}} < \theta_v$, we can define $\bar{V}$ as: $\bar{V} = V_{RMS} < \theta_v$.

- Average power then becomes:

$$P = V_{RMS} \cdot I_{RMS} \cdot \cos(\theta_v - \theta_i)$$

From now onwards, we drop RMS and just use V to represent the RMS value of $V(t) = V_m \cos(\omega t + \theta_v)$

COMPLEX POWER

Let's do a similar calculation, but instead of using $v(t)$ and $i(t)$, let's use $\bar{V}$ and $\bar{I}$

$$\bar{S} \triangleq \bar{V} \bar{I}^* = V e^{j\theta_v} \cdot I e^{-j\theta_i} = V \cdot I e^{j(\theta_v - \theta_i)}$$

$$\bar{S} = \underbrace{VI \cos(\theta_v - \theta_i)}_P + j \underbrace{VI \sin(\theta_v - \theta_i)}_Q$$

$$\boxed{Q \triangleq VI \sin(\theta_v - \theta_i)}$$

← Reactive power →

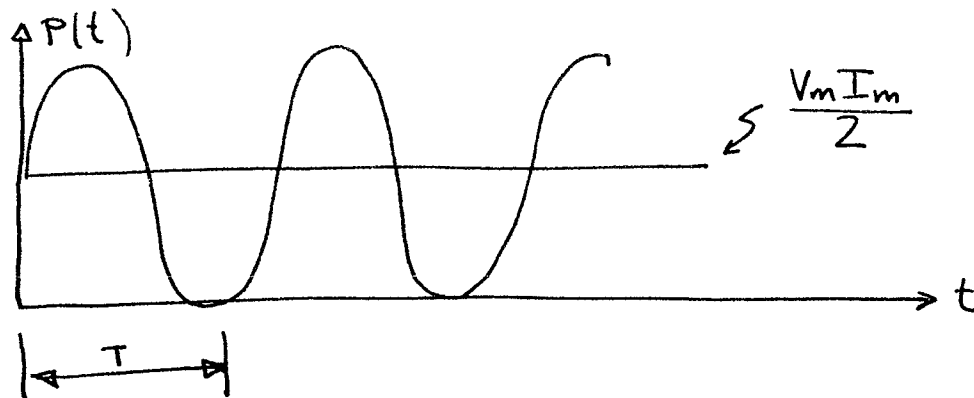
→ doesn't do anything for us but it is extremely important for maintaining voltage.

Reminder: $\cos(\theta_1) \cdot \cos(\theta_2) = \frac{1}{2} [\cos(\theta_1 - \theta_2) + \cos(\theta_1 + \theta_2)]$

Thus:

$$P(t) = \frac{1}{2} V_m I_m \cdot [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)]$$

$$P(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)$$



AVERAGE POWER

$$P = \frac{1}{T} \int_0^T \frac{VI}{2} [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] dt =$$

$$= \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{1}{T} \int_0^T \cos(2\omega t + \theta_v + \theta_i) dt =$$

$$= \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{1}{2\omega T} \sin(2\omega t + \theta_v + \theta_i) \Big|_0^T$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

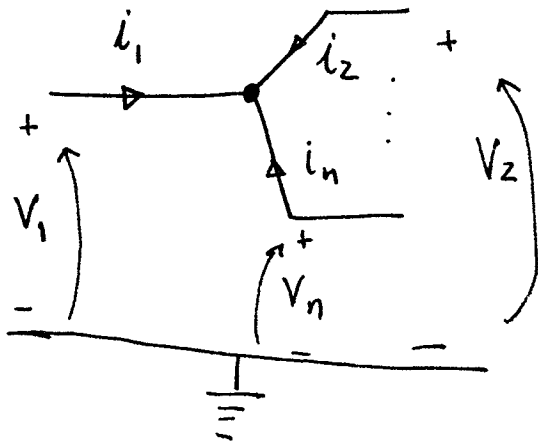
POWER FACTOR

$$PF \triangleq \cos(\theta_v - \theta_i)$$

CONSERVATION OF POWER

For any node in an electrical circuit,

- sum of real power into the node must be zero.
- sum of reactive power into the node must be zero.



$$\sum_j i_j = 0 \quad (\text{Kirchoff's current law})$$

$$\Rightarrow \sum_j \bar{I}_j = 0$$

$$\left. \begin{aligned} \bar{S}_1 &= \bar{V}_1 \cdot \bar{I}_1^* \\ \bar{S}_2 &= \bar{V}_2 \cdot \bar{I}_2^* \\ &\vdots \\ \bar{S}_n &= \bar{V}_n \cdot \bar{I}_n^* \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \sum \bar{S}_j = \bar{S}_1 + \bar{S}_2 + \dots + \bar{S}_n = \bar{V}_1 \bar{I}_1^* + \bar{V}_2 \bar{I}_2^* + \dots + \bar{V}_n \bar{I}_n^* =$$

$$= \bar{V} (\bar{I}_1^* + \bar{I}_2^* + \dots + \bar{I}_n^*) = \bar{V} \cdot (\sum \bar{I}_j)^* = 0 \Rightarrow$$

$$\bar{V}_1 = \bar{V}_2 = \dots = \bar{V}_n \triangleq \bar{V}$$

$$\Rightarrow \boxed{\sum \bar{S}_j = 0}$$

REACTIVE POWER COMPENSATION

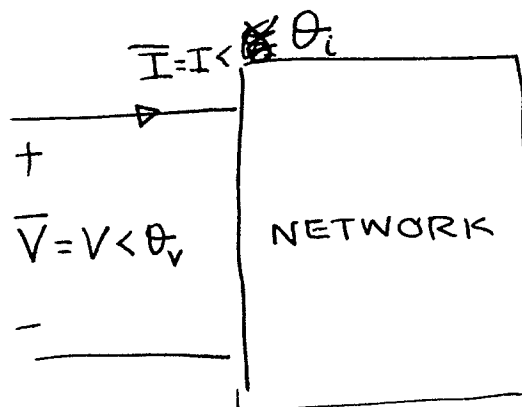
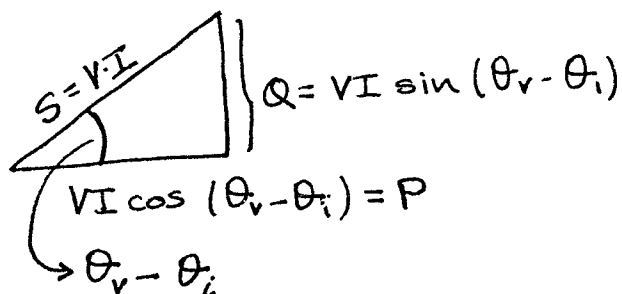
$$\bar{S} = \bar{V} \cdot \bar{I}^* = P + jQ$$

$$P = VI \cos(\theta_v - \theta_i)$$

$$Q = VI \sin(\theta_v - \theta_i)$$

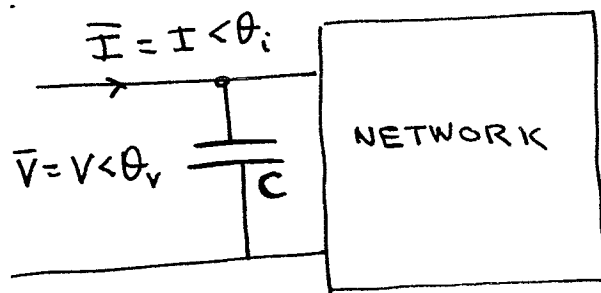
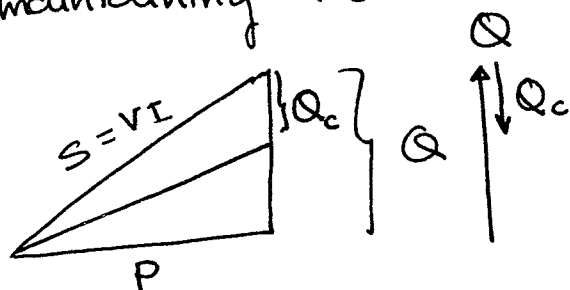
$$S = \sqrt{P^2 + Q^2}$$

We can represent S , P and Q graphically



The nature of the network, will determine $\theta_v - \theta_i$, so for a fixed V , we will have a corresponding I . If $\theta_v - \theta_i$ is too large, then I will be also large, so we need to size the conductors properly (we will also have more losses on the cables)

Is there a way we can reduce S while maintaining the same P ? → YES!



$$Q_c = + \frac{V^2}{X_c} \text{ (generates)}$$

$$S = \sqrt{P^2 + (Q - Q_c)^2} = V \cdot I_{\text{new}} \Rightarrow I_{\text{new}} < I_{\text{old}}$$