ECE 463: Digital Communications Lab.

Lecture 9: OFDM I

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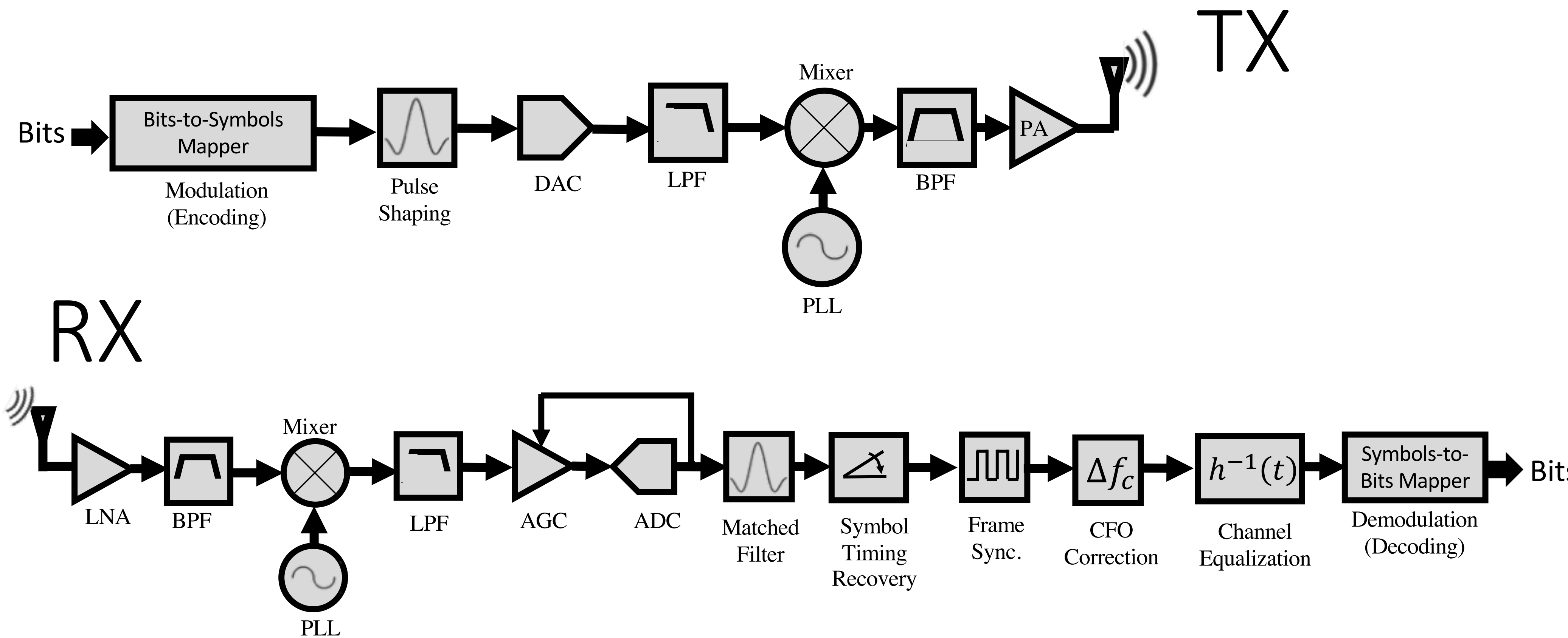
Previous Lecture:

- ✓ Maximum Likelihood Decoding
- ✓ QAM & QPSK
- ✓ BER vs. SNR
- ✓ Quantization Noise & AGCs

This Lecture:

- ❑ Multi-Carrier Modulation
- ❑ Orthogonal Frequency Division Multiplexing

Digital Communication System



Single Carrier Modulation

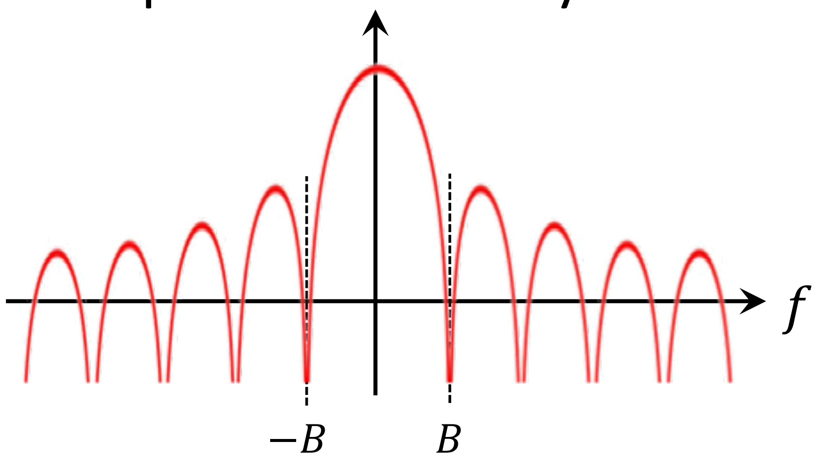
Symbols modulated on a single carrier frequency

$$s[n]\cos(2\pi f_c t)$$

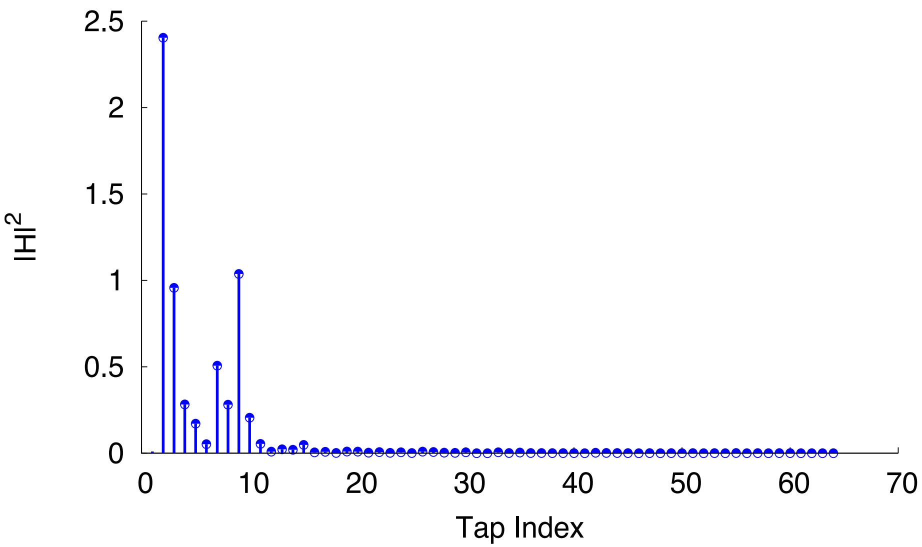
Single Carrier Modulation

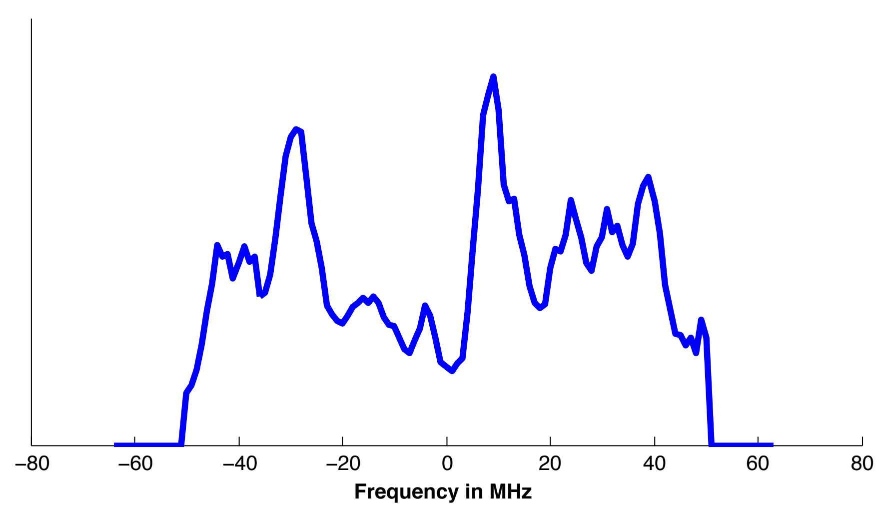
Symbols modulated on a single carrier frequency

- Low Spectral Efficiency: sinc & raised cosine leakage



- ISI: Inter-Symbol-Interference limits performance

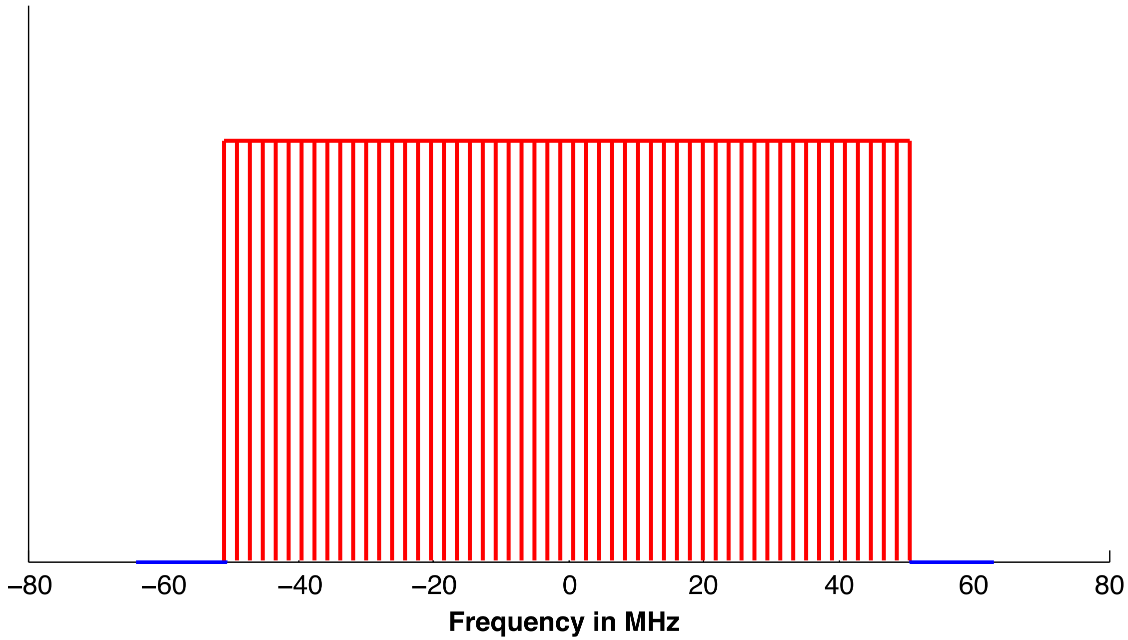




Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

- Divide spectrum into many narrow bands



$$x(t) = \sum_i s_i[n] e^{-j2\pi f_i t}$$

- Transmit symbols on different carriers in narrow bands
- Channel is Flat → No need to worry about ISI

Multi-Carrier Modulation

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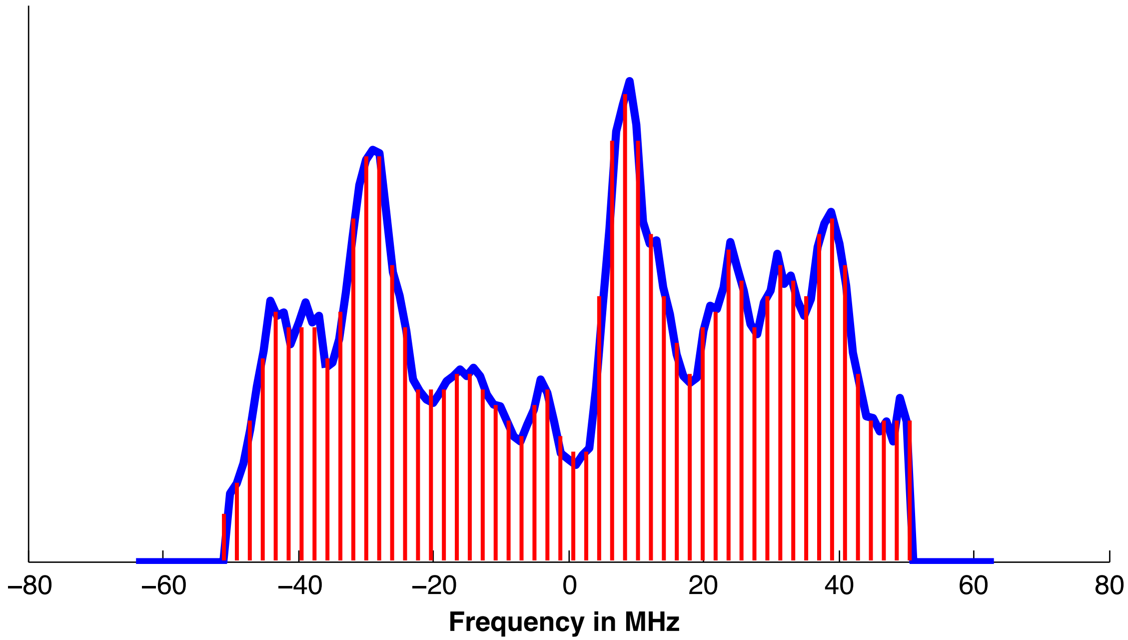

$$x(t) = \sum_i s_i[n]\mathrm{e}^{-j2\pi f_i t}$$

$$y(t) = \sum_i h_i s_i[n]\mathrm{e}^{-j2\pi f_i t}$$

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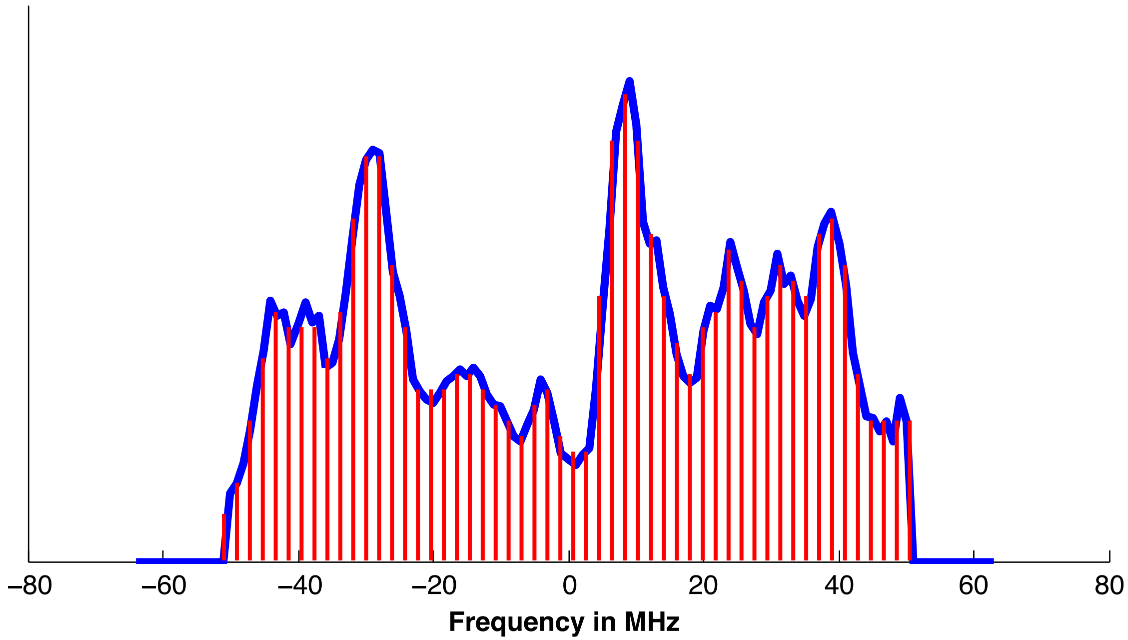

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Not That Simple!

Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

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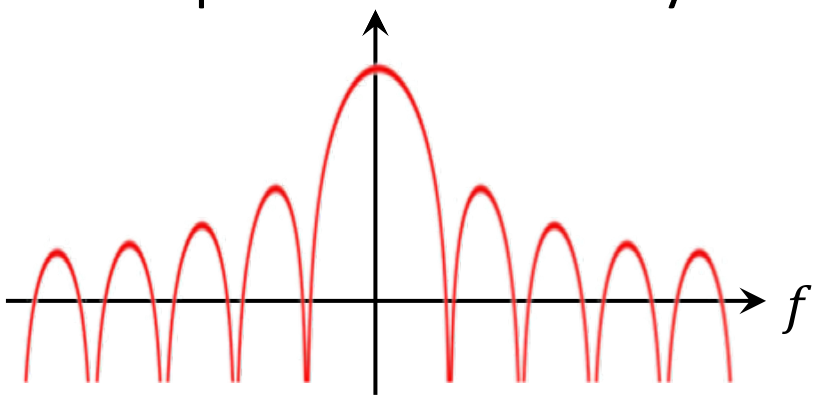
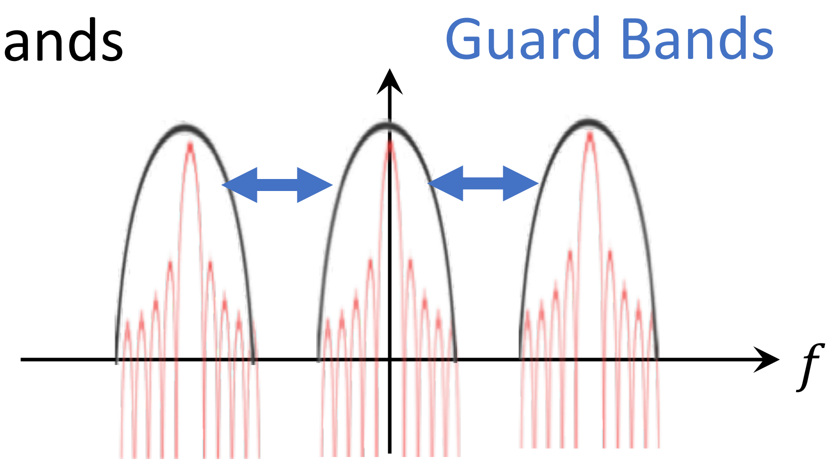




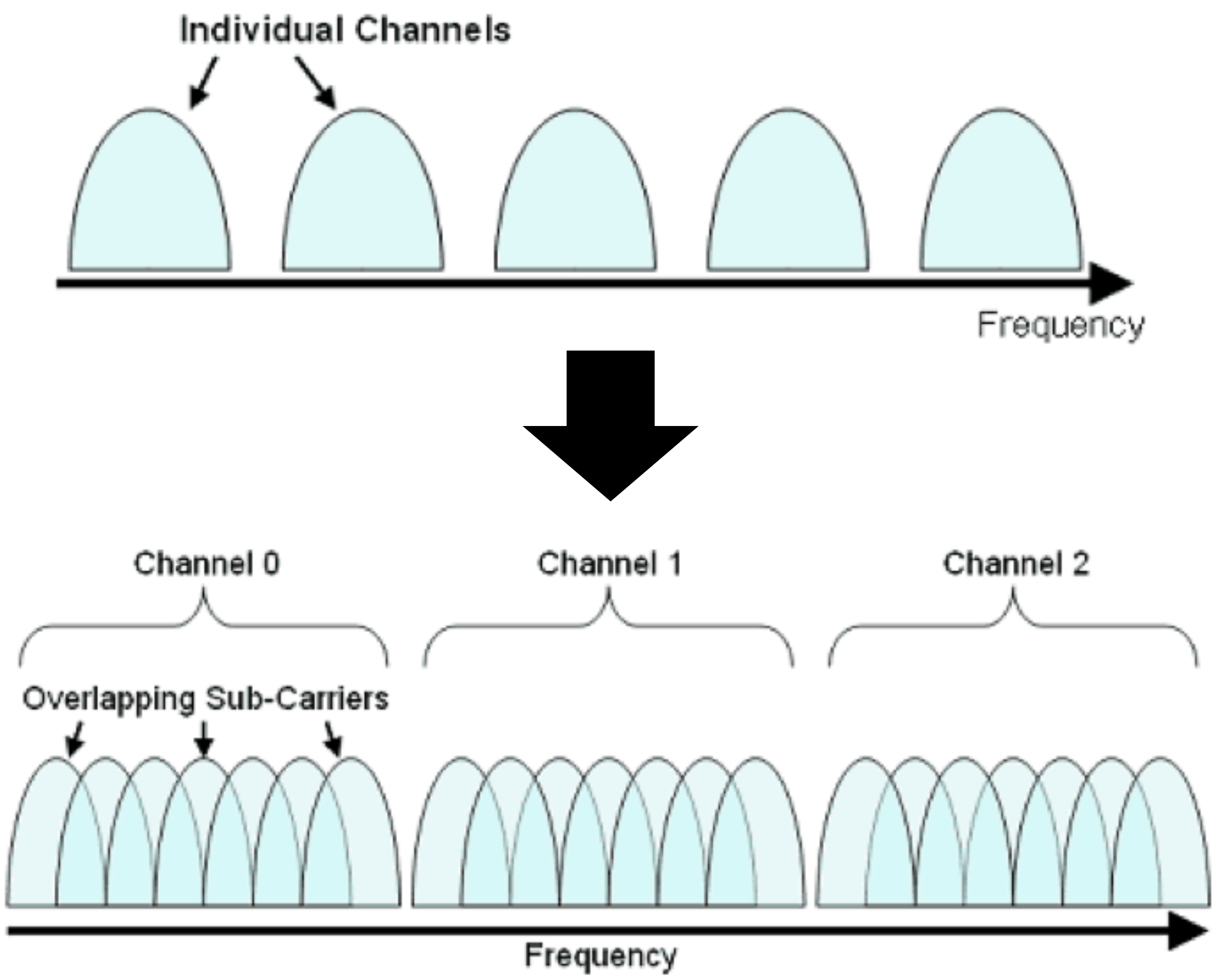
- Significant Leakage between adjacent subcarriers
- Need Guard Bands → Very inefficient!

Solution: Make the Sub-Carriers Orthogonal

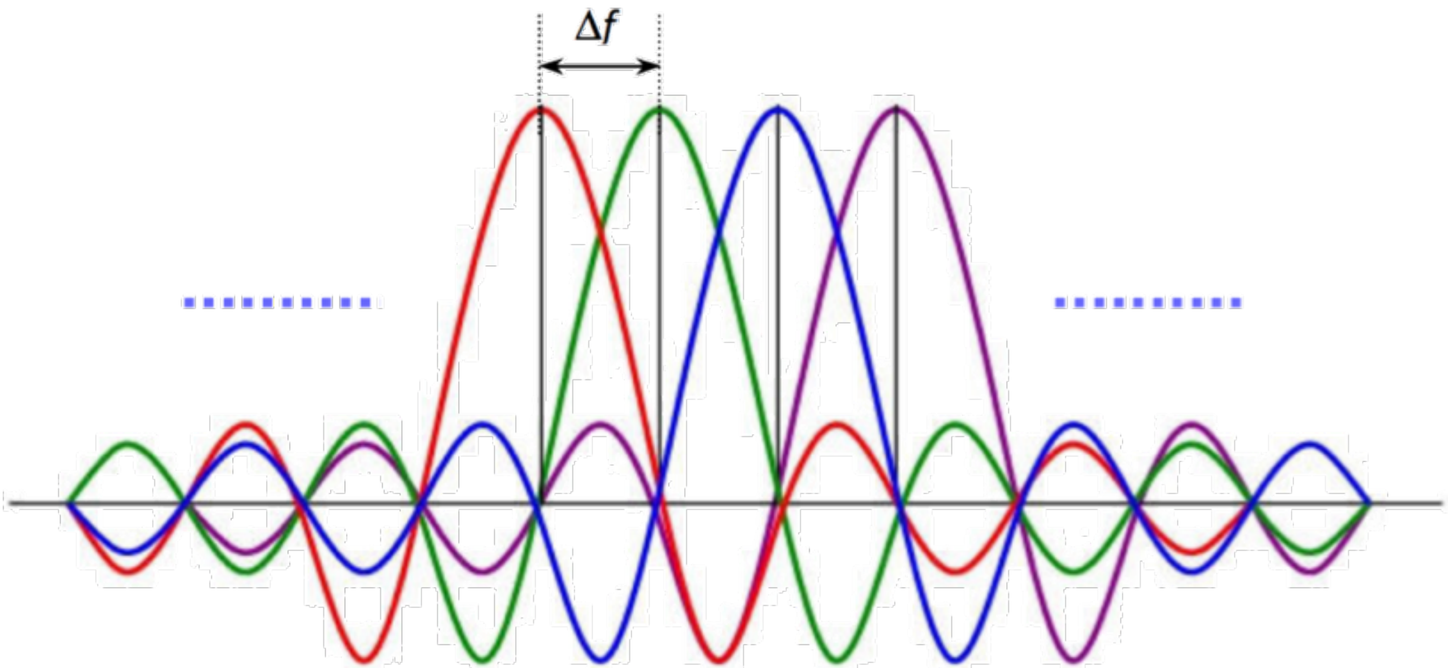
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

Make the Sub-Carriers Orthogonal



OFDM: Orthogonal Frequency Division Multiplexing

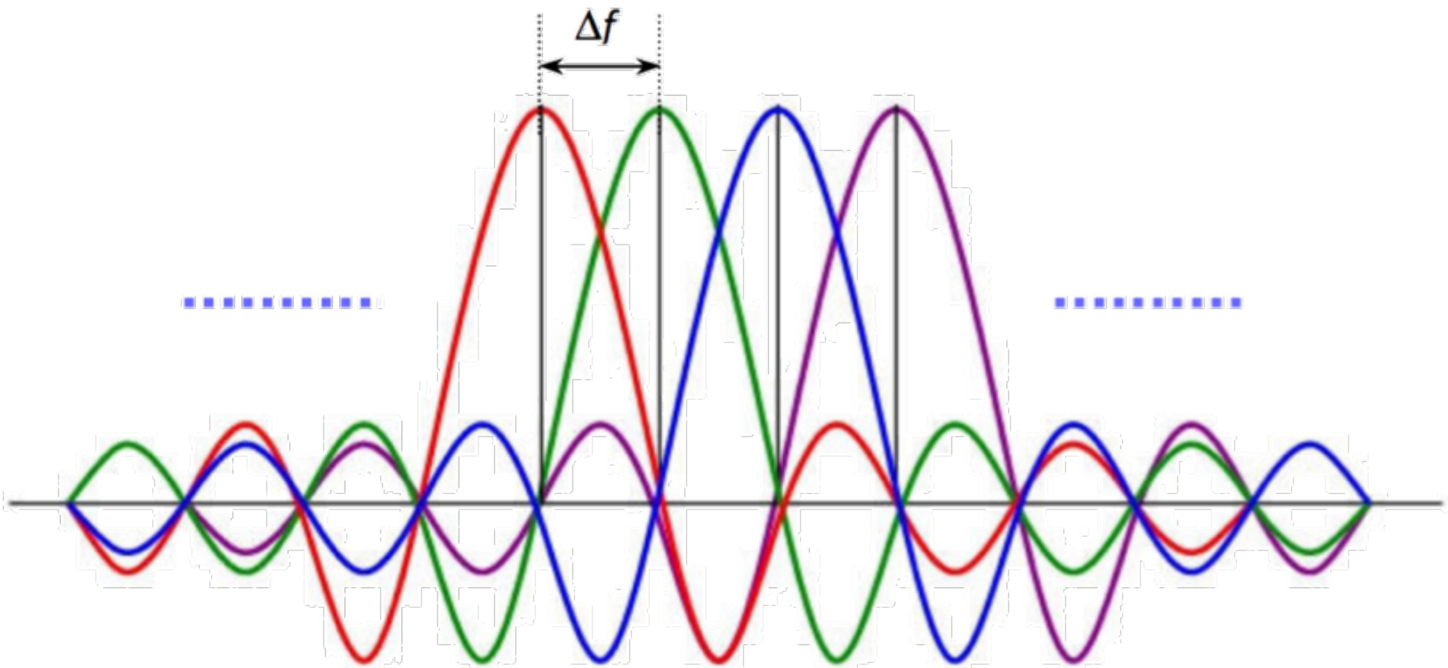


- Subcarriers are orthogonal: At the sub-carrier frequency, the sampled value has zero leakage from other subcarriers.

- Subcarrier separation can be very small, for N subcarriers and bandwidth B:

$$\Delta f = \frac{B}{N}$$

OFDM: Orthogonal Frequency Division Multiplexing



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How to Achieve This?

OFDM: Orthogonal Frequency Division Multiplexing

Use DFT: Discrete Fourier Transform

N-Point DFT: $X(f_i) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)e^{-j\frac{2\pi f_i t}{N}}$

N-Point IDFT: $x(t) = \sum_{f_i=0}^{N-1} X(f_i)e^{j\frac{2\pi f_i t}{N}}$

Send symbols in Frequency Domain

$X(f_i) = s[n] \rightarrow$ Compute and transmit $x(t)$ using IDFT

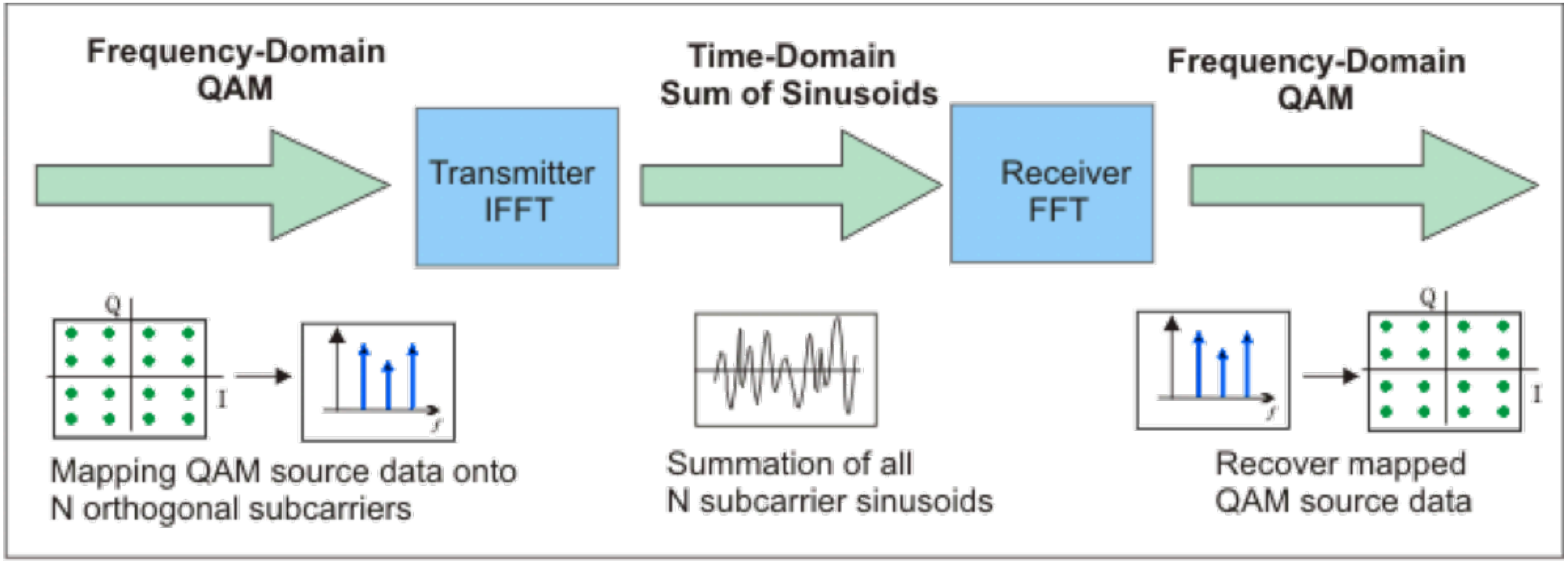
OFDM: Orthogonal Frequency Division Multiplexing

Send symbols in Frequency Domain

$X(f_i) = s[n] \rightarrow$ Compute and transmit $x(t)$ using IDFT

- N subcarrier → IDFT of length N
- Symbols $s[n]$ can come from any modulation: BPSK, QPSK, QAM…
- $x(t)$ is complex → need I & Q → No point using PAM or ASK …
- OFDM Symbol: N samples of $x(t)$ generated from the same modulated symbols using IDFT.
- OFDM Symbol Time: $T = N/B$ where B is the bandwidth.
- OFDM Frequency Bin Width: $\Delta f = 1/T = B/N$

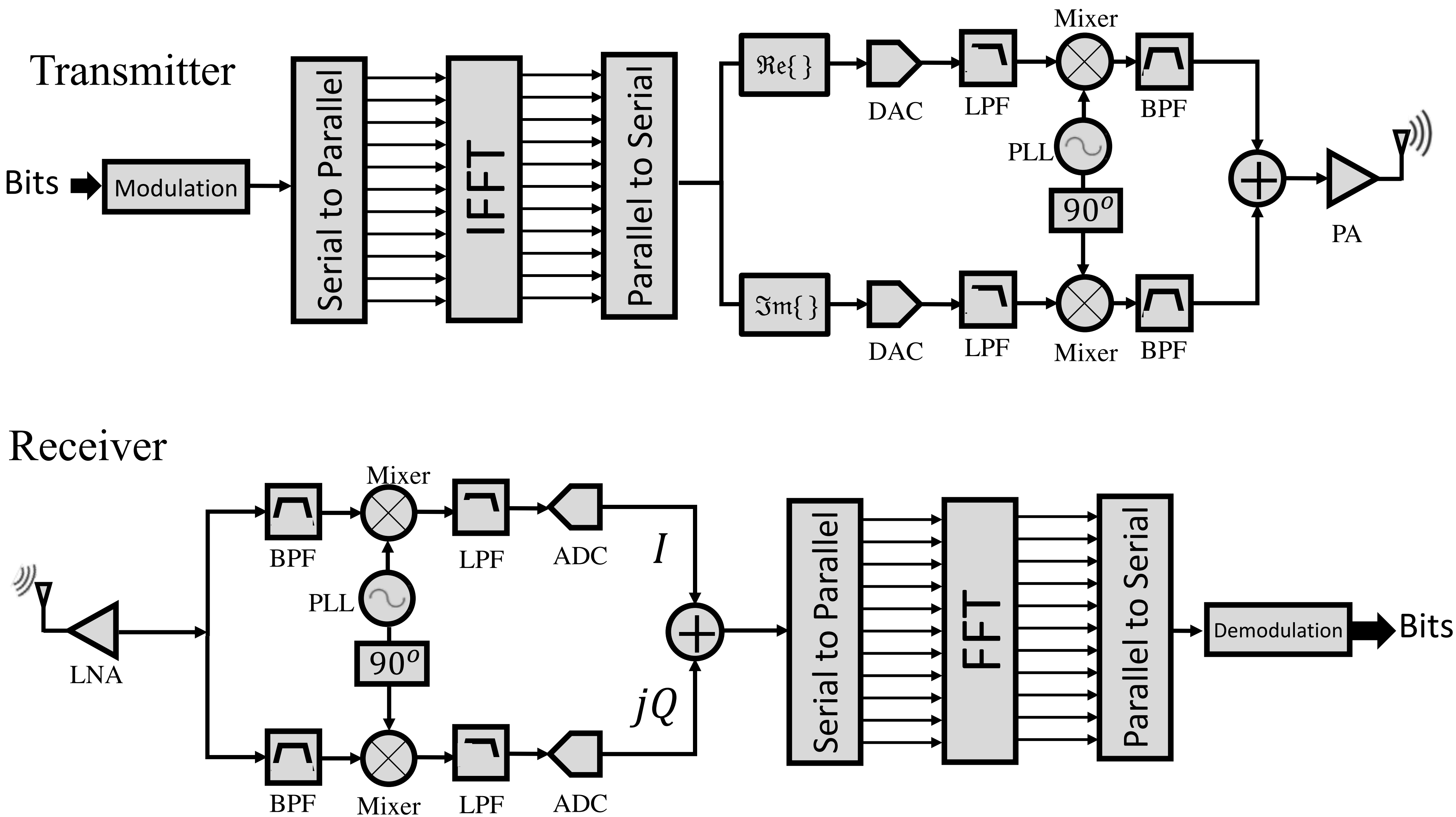
OFDM: Orthogonal Frequency Division Multiplexing



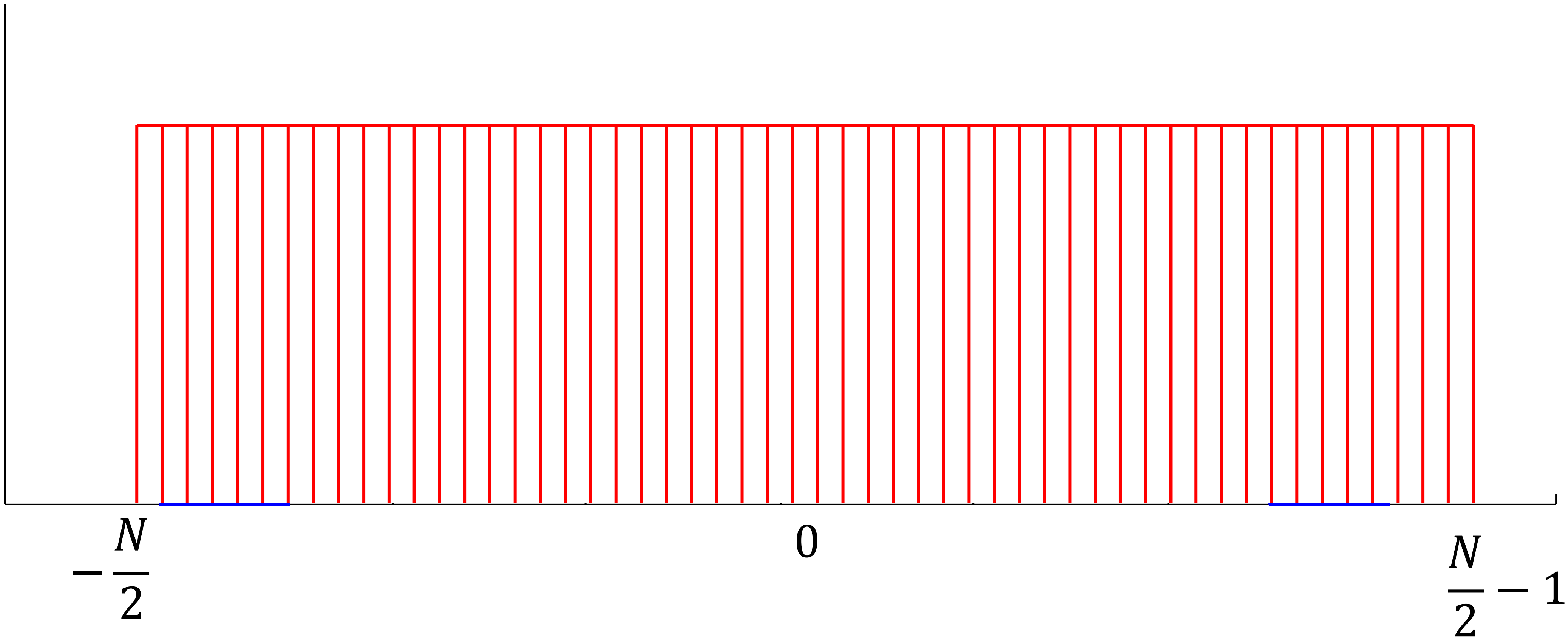
Simplified OFDM System Block Diagram

- OFDM invented long time ago but did not become viable until recently!
- DFT requires $O(N^2)$ multiplications → very power hungry.
- FFT invented in 1965 requires $O(N \log N)$ multiplications
- OFDM did not become viable until low power FFT chips came to market

OFDM: Orthogonal Frequency Division Multiplexing



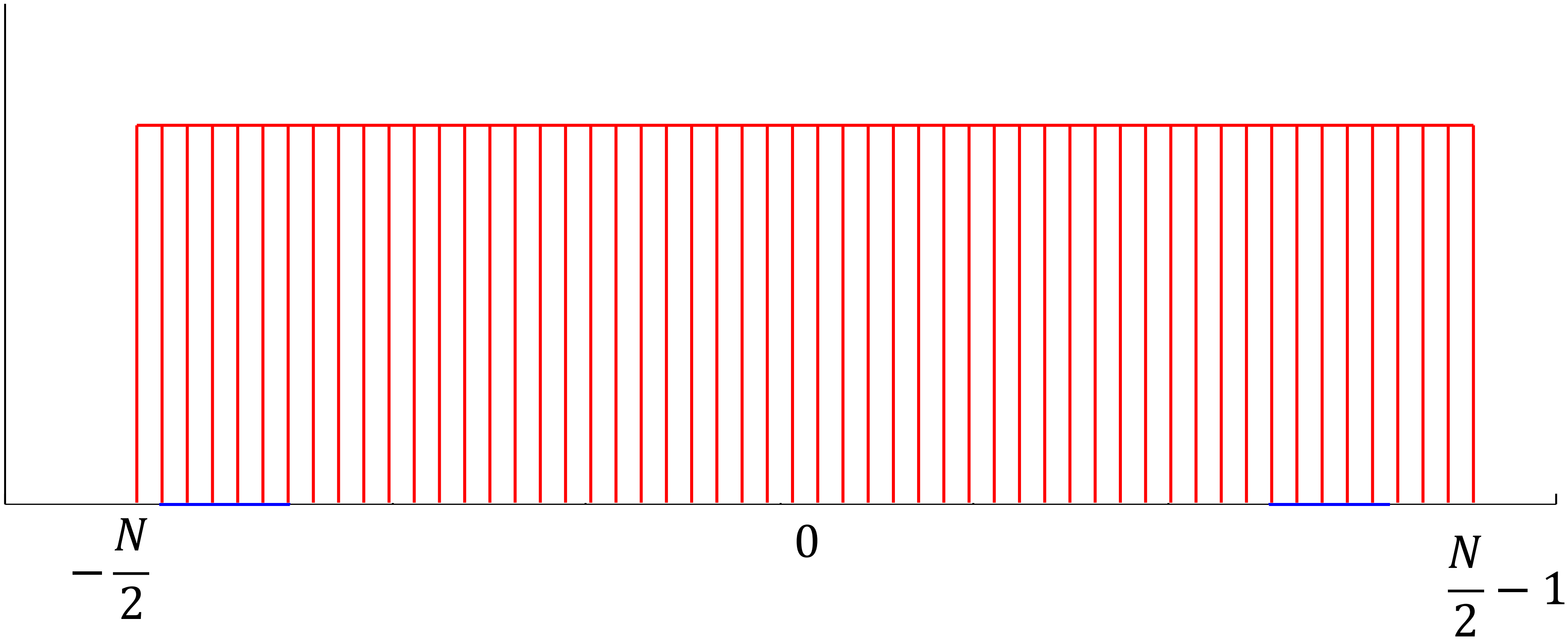
OFDM Symbol in Frequency Domain



- FFT can be represented 0 to $N-1$ or $N/2$ to $N/2\ -1$.
- OFDM Symbol created in digital baseband → 0 bin corresponds to DC

$$X(0)=\frac{1}{N}\sum_{t=0}^{N-1}x(t)e^{-j\frac{2\pi 0t}{N}}\ =\frac{1}{N}\sum_{t=0}^{N-1}x(t)\ =DC$$

OFDM Symbol in Frequency Domain



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- DC of the circuits corrupts bits sent on the 0 bin → Do not use 0 bin

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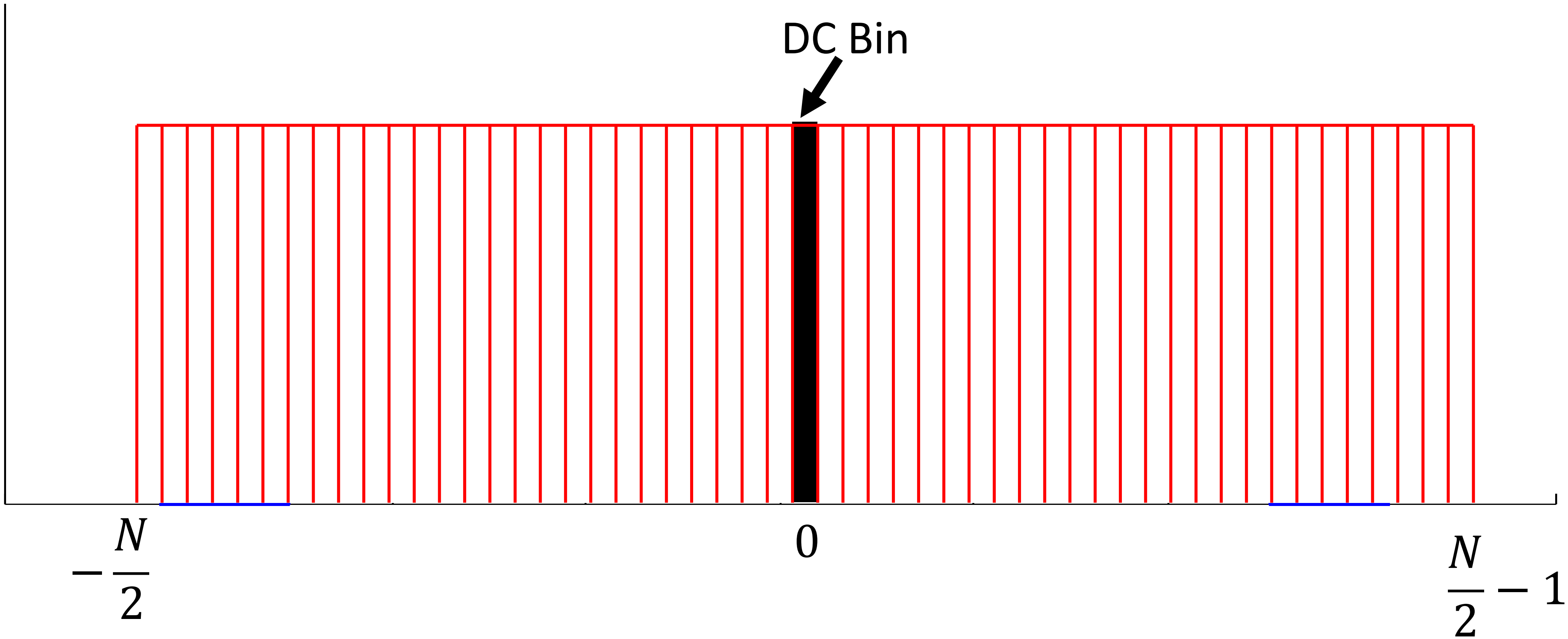

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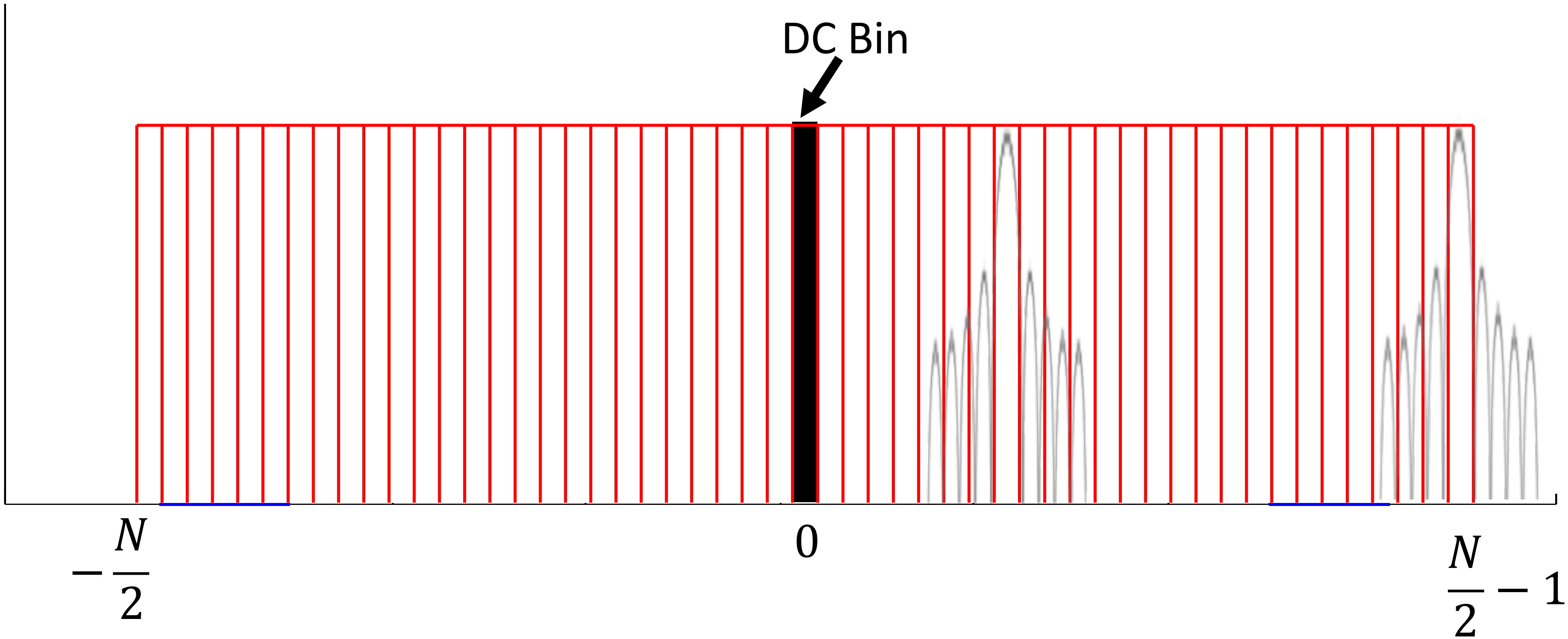
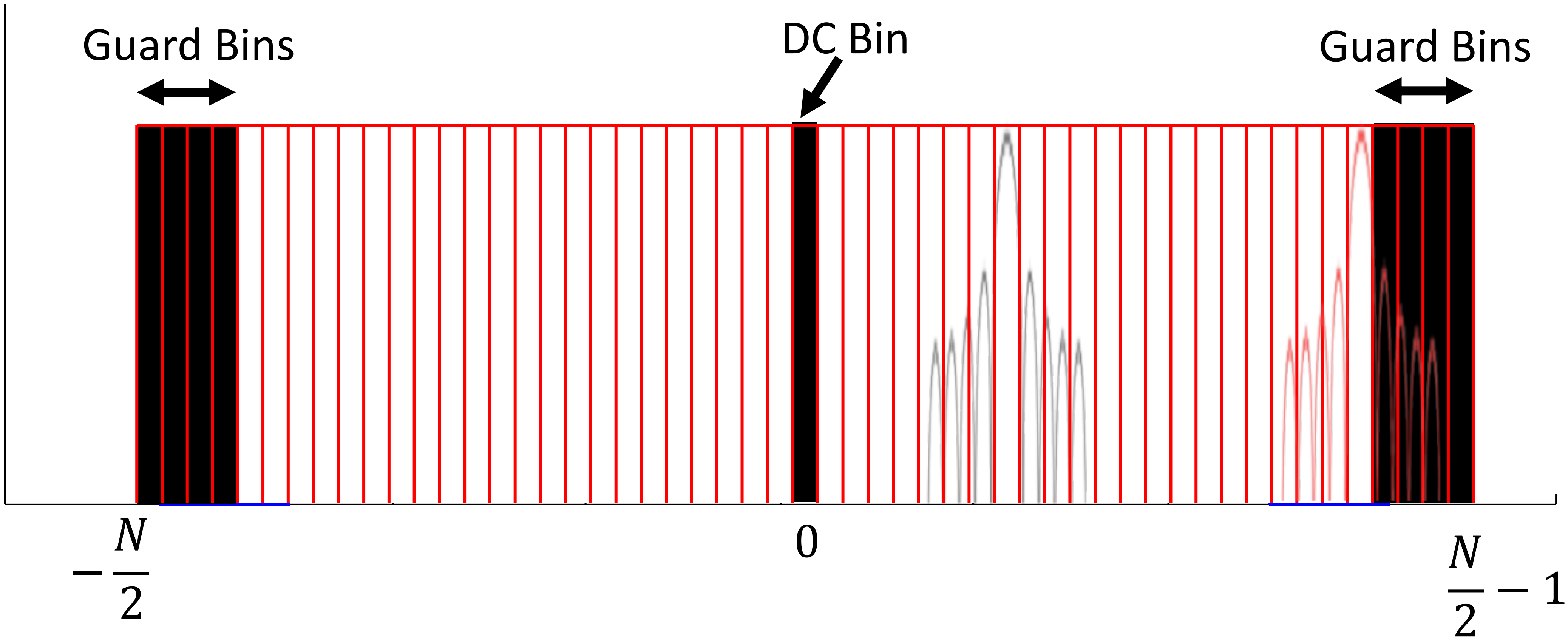


- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel → Transmit nothing there

OFDM Symbol in Frequency Domain



- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel → Transmit nothing there
- Reduce Number of Guard band from N to 2 → Very Spectrally Efficient