ECE 463: Digital Communications Lab.

Lecture 8: Modulation III

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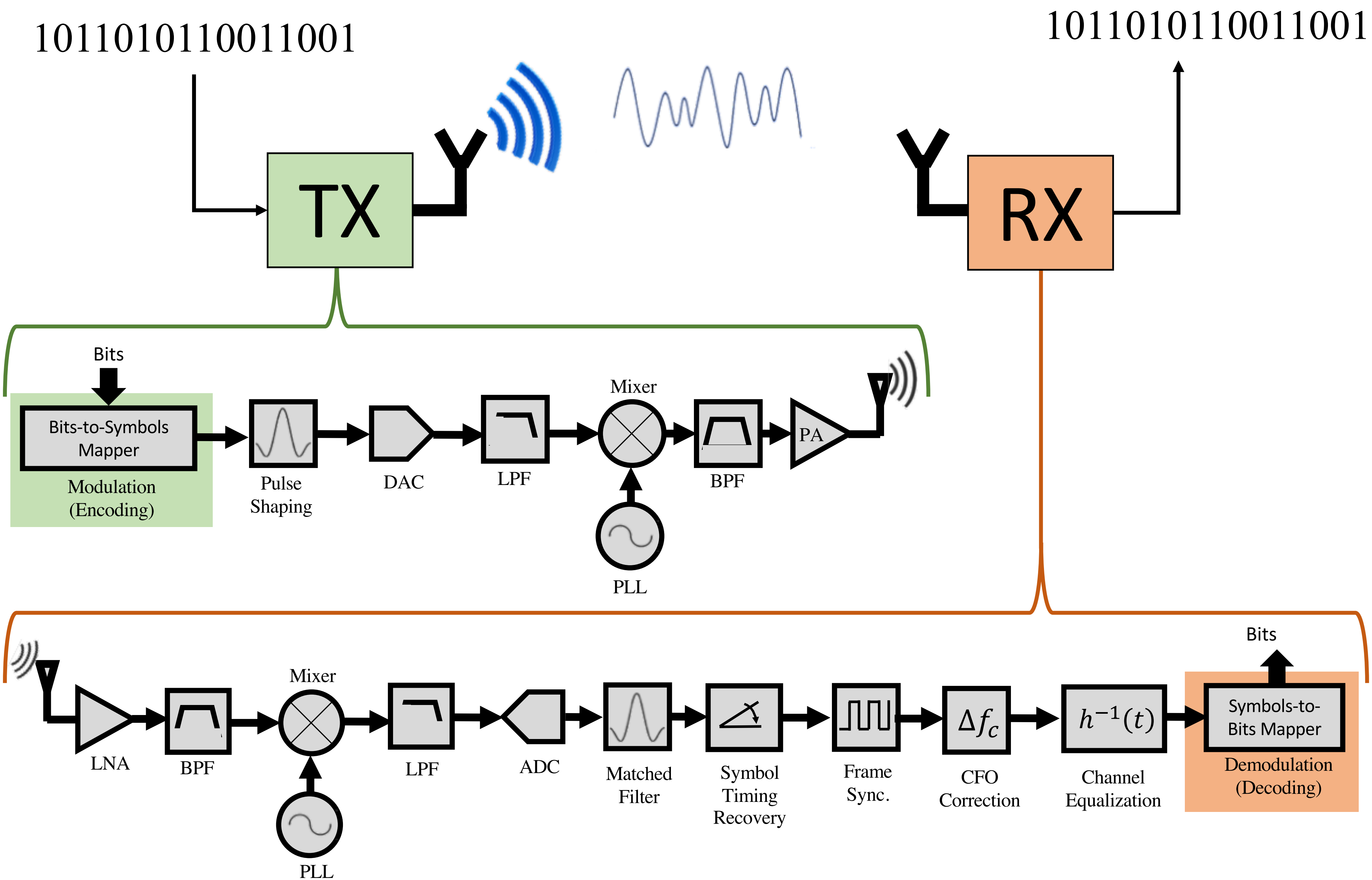
Previous Lecture:

- ✓ CFO Estimation
- ✓ CFO Correction
- ✓ Frame Synchronization
- ✓ Phase Tracking

This Lecture:

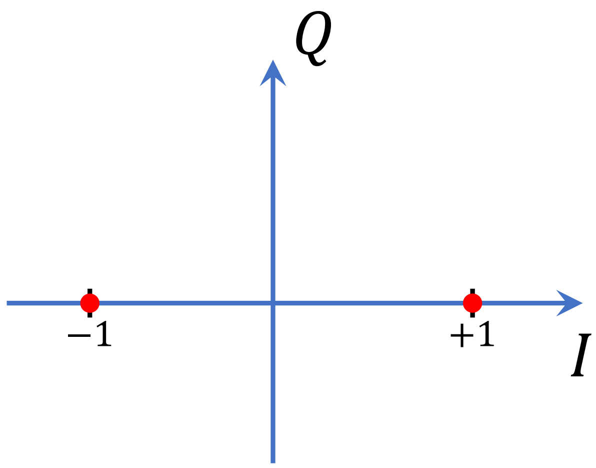
- ❑ Maximum Likelihood Decoding
- ❑ QAM & QPSK
- ❑ BER vs. SNR
- ❑ Quantization Noise & AGCs

Digital Communication System



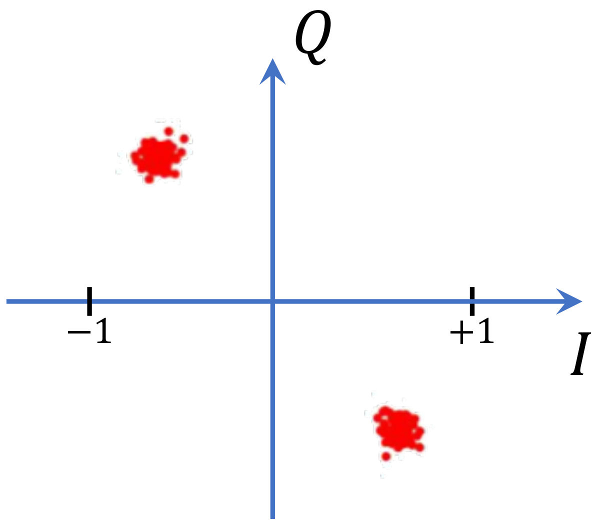
BPSK Modulation

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

Transmitted Constellation



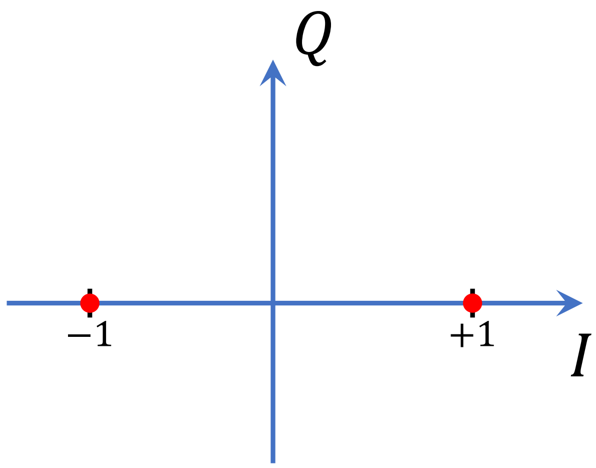
$y(t) = hx(t) + v(t)$

Received Constellation

After,
Matched Filtering
Timing Recovery
CFO Correction

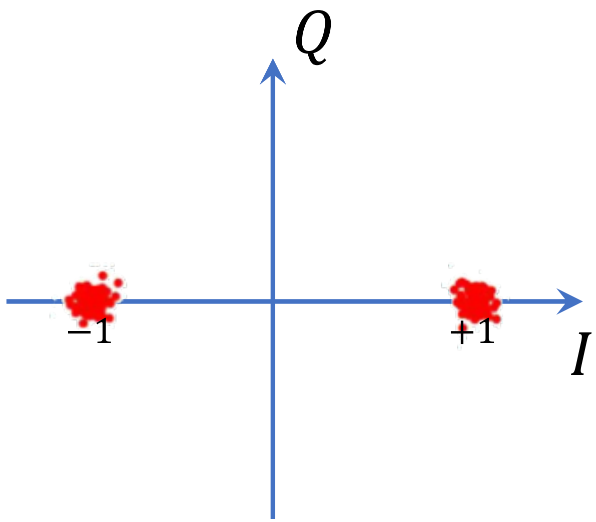
BPSK Modulation

$$0 \to -1$$
$$1 \to +1$$



$x(t)$

Transmitted Constellation



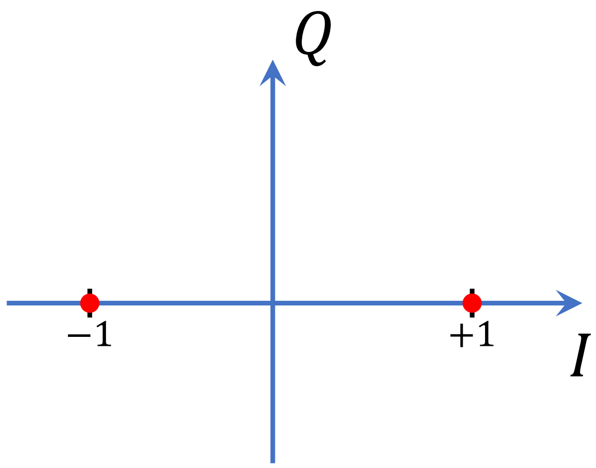
$y(t) = x(t) + v(t)/h$

Received Constellation

After,
Matched Filtering
Timing Recovery
CFO Correction
Channel Equalization

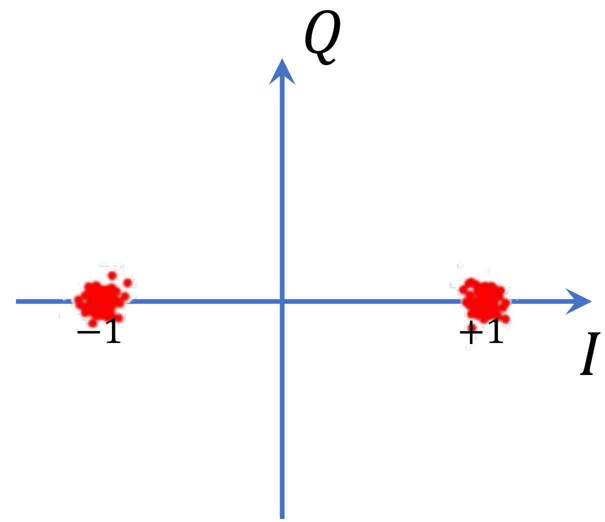
BPSK Modulation

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

Transmitted Constellation



$y(t) = x(t) + v(t)/h$

Received Constellation

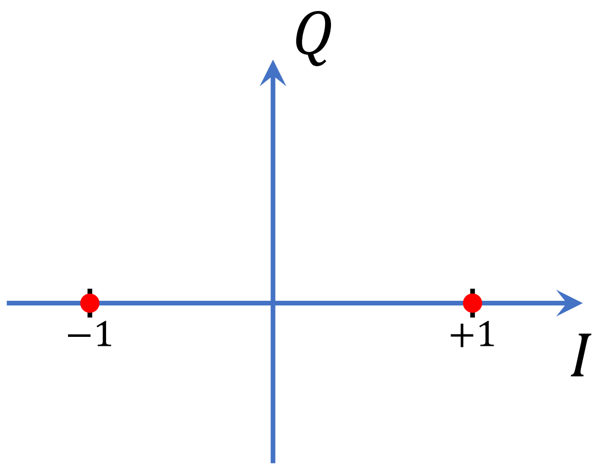
How well we can decode depends on SNR?

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E[|hx(t)|^2]}{E[|v(t)|^2]} = \frac{|h|^2 E_s}{N'_0} = \frac{E_s}{N_0}$$

Lump attenuation in noise

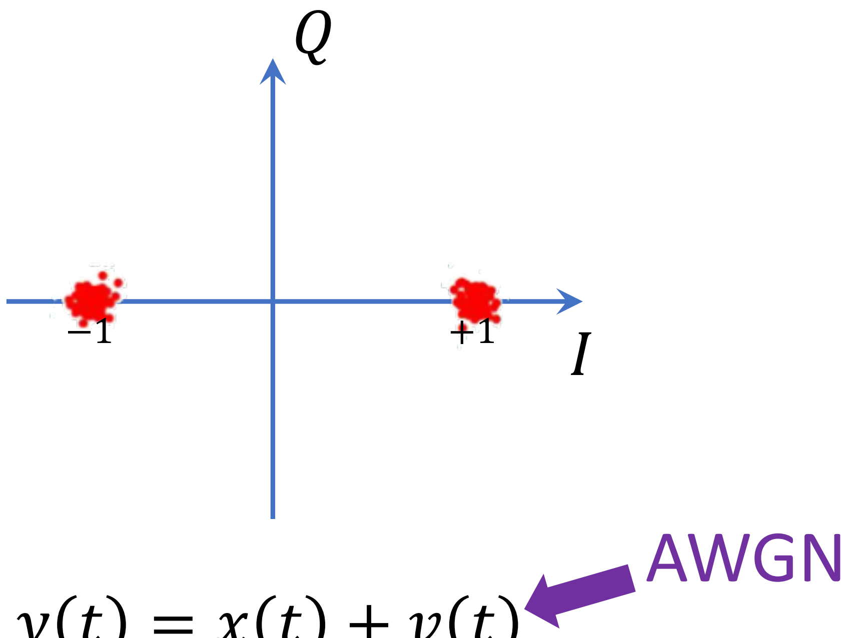
BPSK Modulation

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

Transmitted Constellation



$y(t) = x(t) + v(t)$

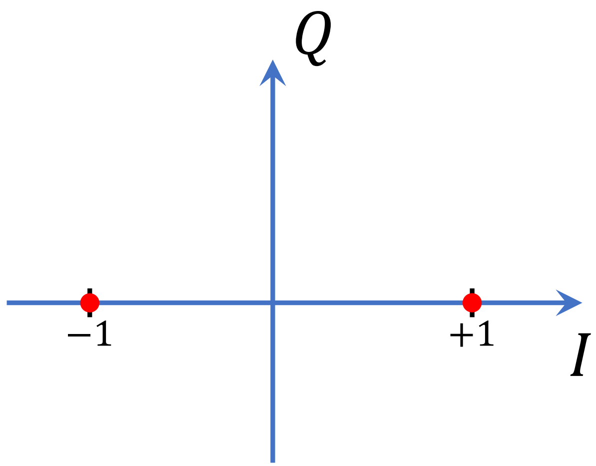
Received Constellation

How well we can decode depends on SNR?

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E_s}{N_0} = 25\ \text{dB}$$

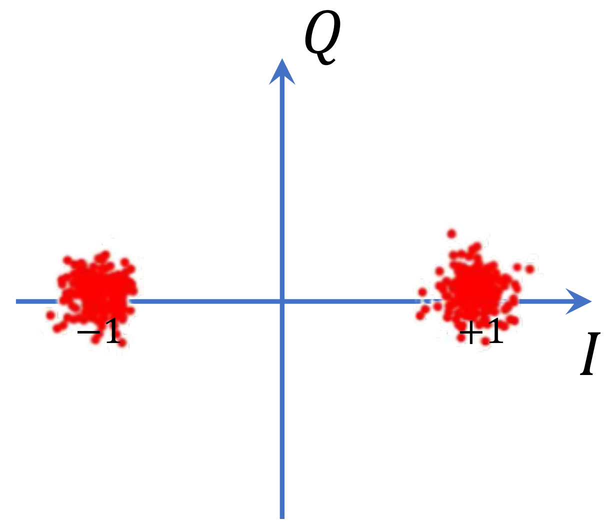
BPSK Modulation

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

Transmitted Constellation



AWGN

$y(t) = x(t) + v(t)$

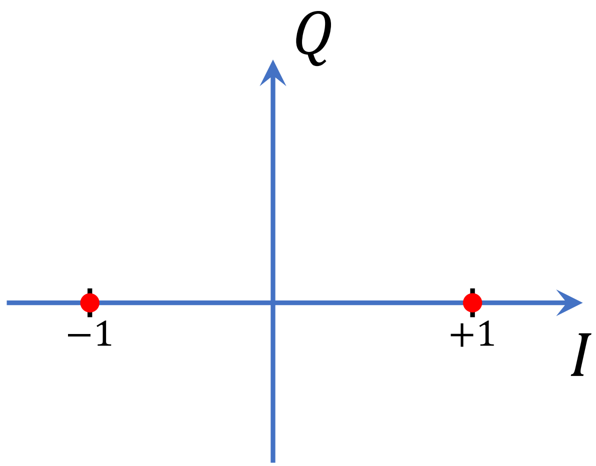
Received Constellation

How well we can decode depends on SNR?

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E_s}{N_0} = 19\text{ dB}$$

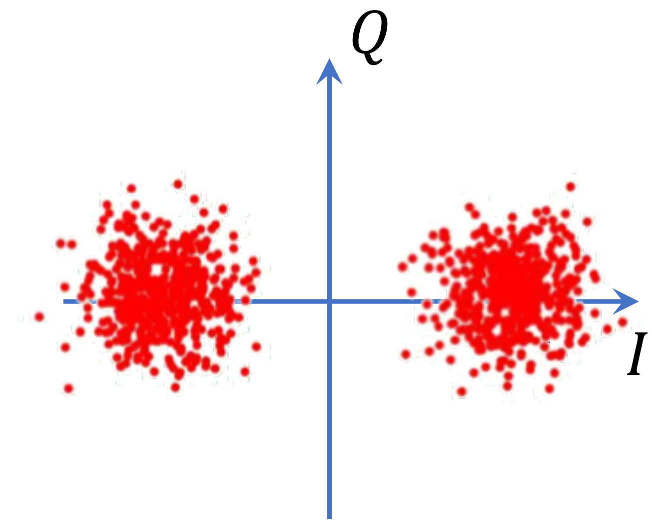
BPSK Modulation

$0 \rightarrow -1$
$1 \rightarrow +1$



$x(t)$

Transmitted Constellation



AWGN

$y(t) = x(t) + v(t)$

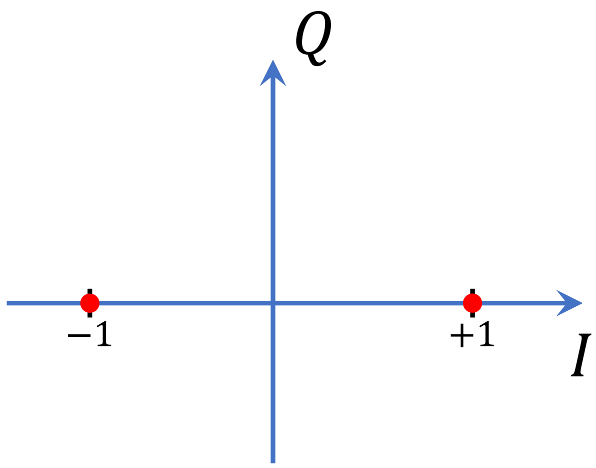
Received Constellation

How well we can decode depends on SNR?

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E_s}{N_0} = 13\ \text{dB}$$

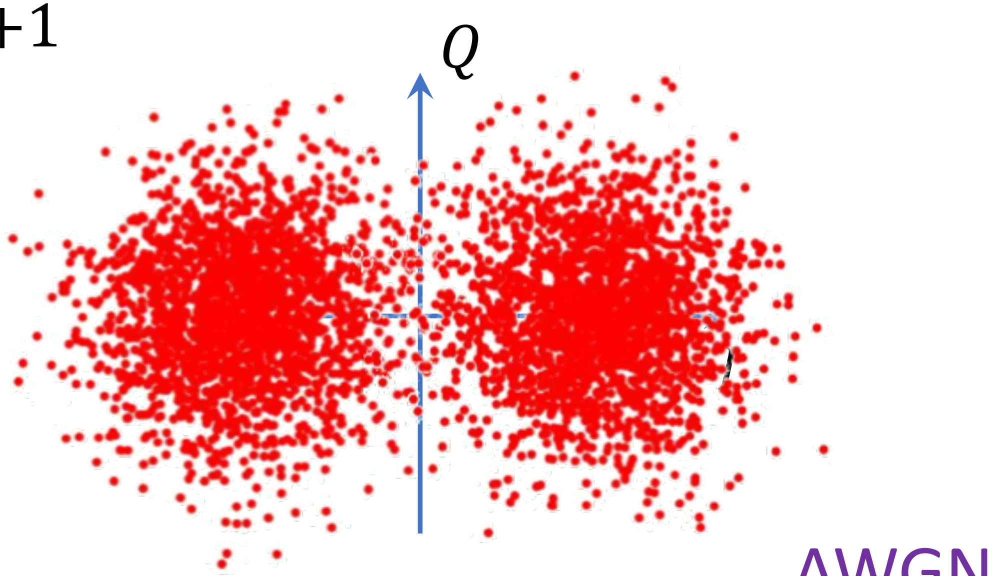
BPSK Modulation

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

Transmitted Constellation



$y(t) = x(t) + v(t)$

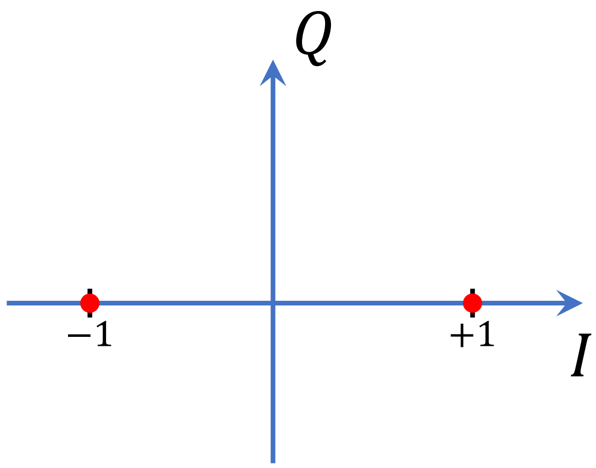
Received Constellation

How well we can decode depends on SNR?

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E_s}{N_0} = 7\ \text{dB}$$

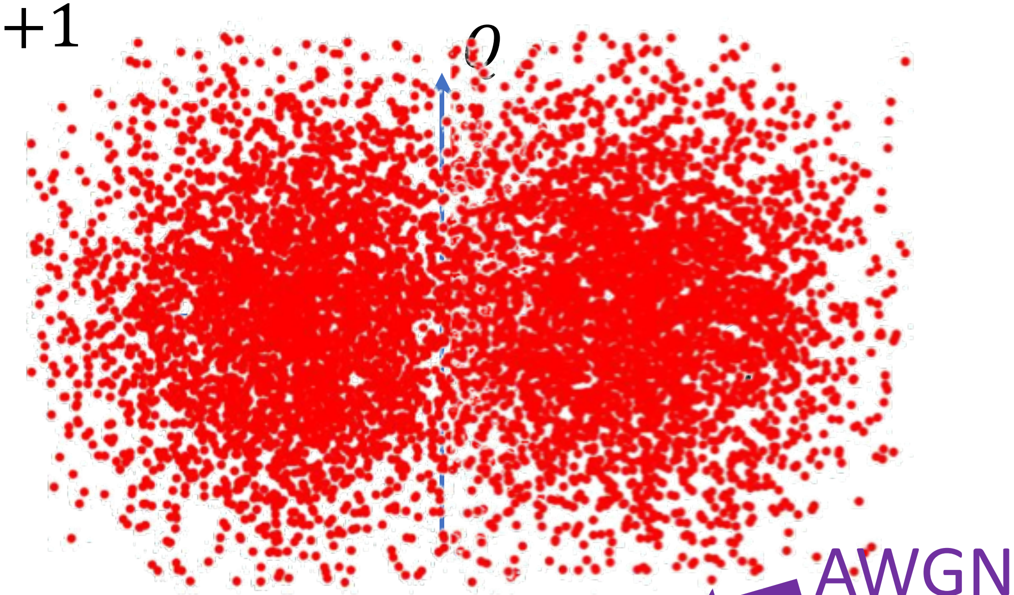
BPSK Modulation

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$

Transmitted Constellation



$y(t) = x(t) + v(t)$

Received Constellation

How well we can decode depends on SNR?

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E_s}{N_0} = 3.5\ \text{dB}$$

Maximum Likelihood Decoder

$$x = \pm 1 \longrightarrow y = x + v$$

$$P(x = +1|y) \underset{0}{\overset{1}{\gtrless}} P(x = -1|y)$$

$$\frac{P(x = +1)\times P(y|\, x = +1)}{P(y)} \underset{0}{\overset{1}{\gtrless}} \frac{P(x = -1)P(y|\, x = -1)}{P(y)}$$

$$P(x = +1)$$
$$= P(x = -1) = \frac{1}{2}$$

$$P(y|x = +1) \underset{0}{\overset{1}{\gtrless}} P(y|x = -1)$$

Maximum Likelihood Decoder

$$x = \pm 1 \longrightarrow y = x + v$$

$$P(y|x=+1) \underset{0}{\overset{1}{\gtrless}} P(y|x=-1)$$

Guassian Noise: $v \sim \mathcal{CN}(0, \sigma) \longrightarrow y|x \sim \mathcal{CN}(x\ \sigma)$

$$P(v) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|v|^2}{2\sigma^2}}$$

$$P(y|x=+1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|y-1|^2}{2\sigma^2}}$$

$$2\sigma^2 = N_0 \rightarrow \sigma = \sqrt{N_0/2}$$

$$P(y|x=-1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|y+1|^2}{2\sigma^2}}$$

Maximum Likelihood Decoder

$$x = \pm 1 \longrightarrow y = x + v$$

$$P(y|x=+1) \underset{0}{\overset{1}{\gtrless}} P(y|x=-1)$$

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|y-1|^2}{2\sigma^2}} \underset{0}{\overset{1}{\gtrless}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|y+1|^2}{2\sigma^2}}$$

$$e^{-\frac{|y-1|^2}{2\sigma^2}} \underset{0}{\overset{1}{\gtrless}} e^{-\frac{|y+1|^2}{2\sigma^2}}$$

$$-\frac{|y-1|^2}{2\sigma^2} \underset{0}{\overset{1}{\gtrless}} -\frac{|y+1|^2}{2\sigma^2}$$

Maximum Likelihood Decoder

$$x = \pm 1 \longrightarrow y = x + v$$

$$P(y|x = +1) \underset{0}{\overset{1}{\gtrless}} P(y|x = -1)$$

$$-\frac{|y - 1|^2}{2\sigma^2} \underset{0}{\overset{1}{\gtrless}} -\frac{|y + 1|^2}{2\sigma^2}$$

$$-|y - 1|^2 \underset{0}{\overset{1}{\gtrless}} -|y + 1|^2$$

$$-(Re\{y\} - 1)^2 - Im\{y\}^2 \underset{0}{\overset{1}{\gtrless}} -(Re\{y\} + 1)^2 - Im\{y\}^2$$

Maximum Likelihood Decoder

$$x = \pm 1 \longrightarrow y = x + v$$

$$P(y|x=+1) \underset{0}{\overset{1}{\gtrless}} P(y|x=-1)$$

$$-(Re\{y\}-1)^2 \underset{0}{\overset{1}{\gtrless}} -(Re\{y\}+1)^2$$

$$-Re\{y\}^2 + 2Re\{y\} - 1 \underset{0}{\overset{1}{\gtrless}} -Re\{y\}^2 - 2Re\{y\} - 1$$

$$4Re\{y\} \underset{0}{\overset{1}{\gtrless}} 0$$

Maximum Likelihood Decoder

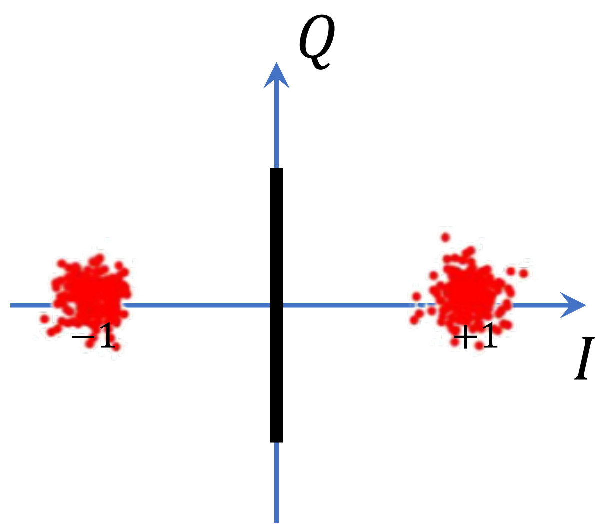
$$x = \pm 1 \longrightarrow y = x + v$$

$$P(y|x=+1) \underset{0}{\overset{1}{\gtrless}} P(y|x=-1)$$

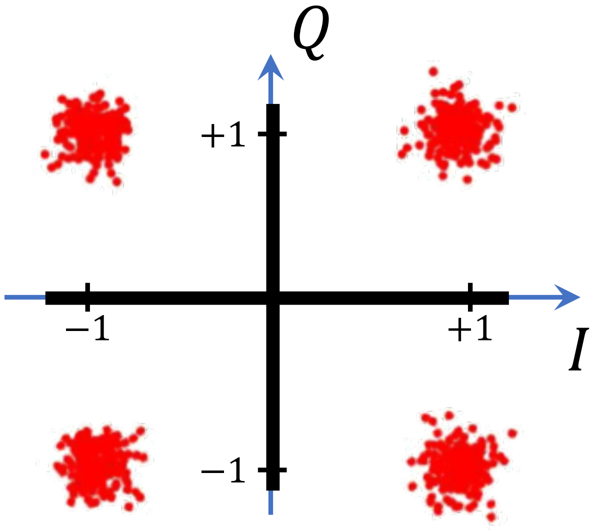
$$Re\{y\} \underset{0}{\overset{1}{\gtrless}} 0$$

$0 \rightarrow -1$

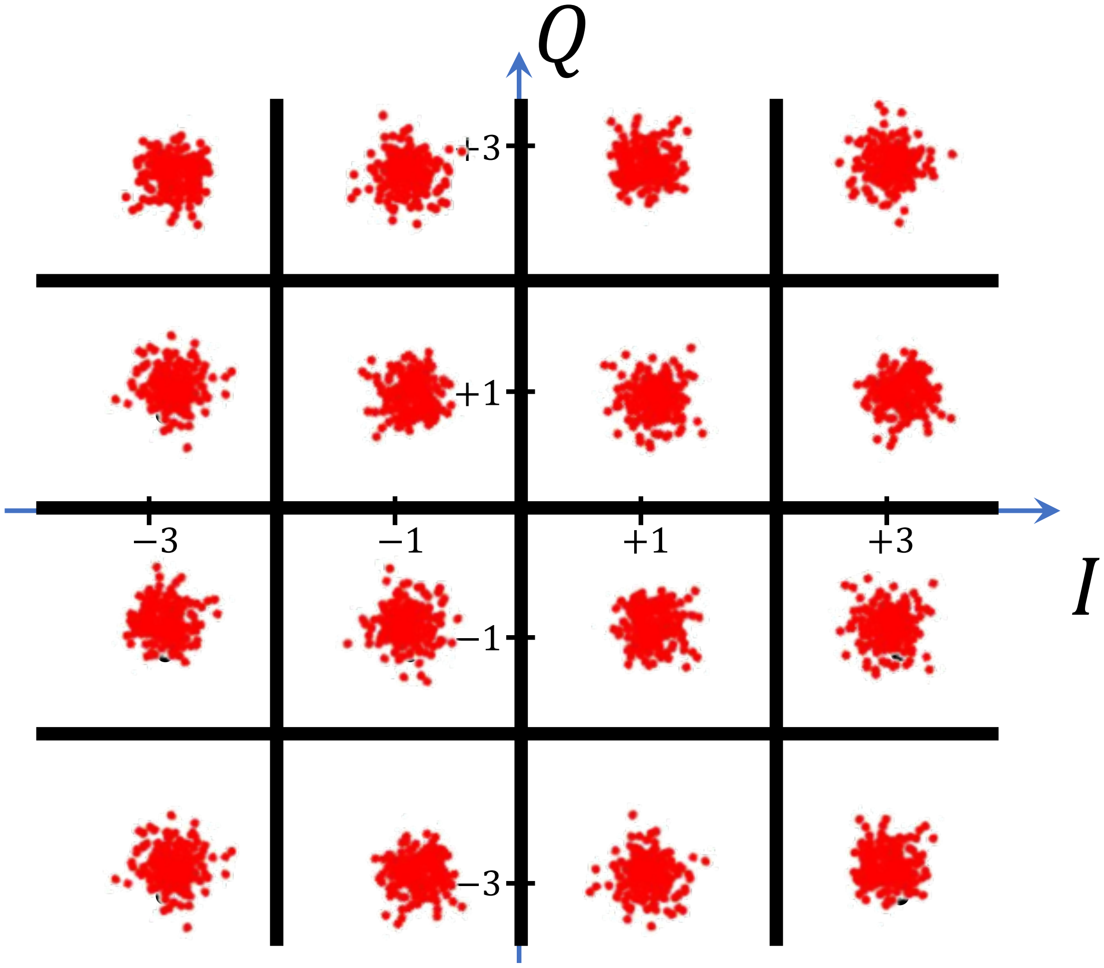
$1 \rightarrow +1$



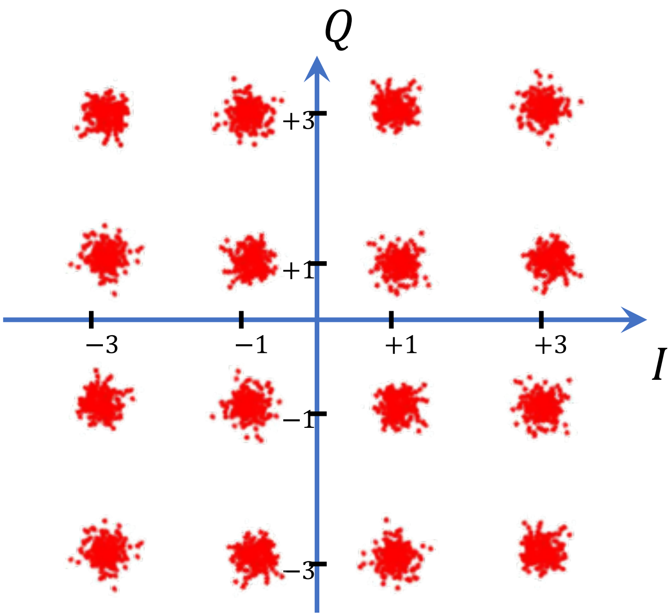
4-QAM



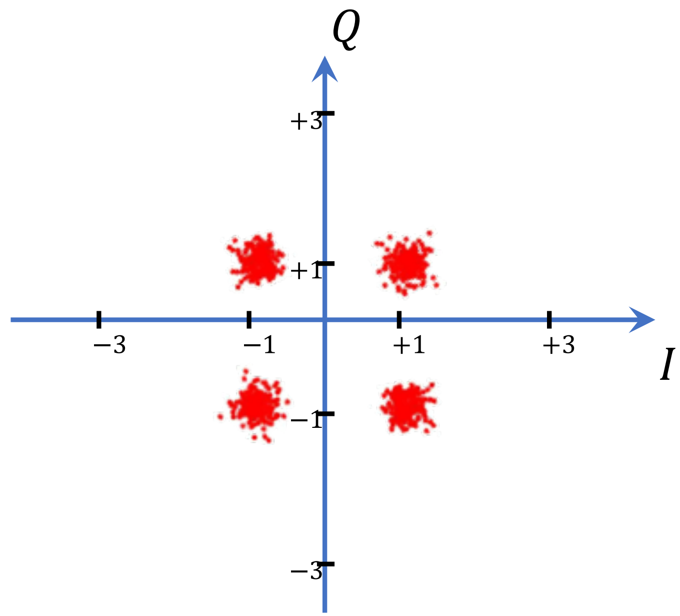
16-QAM



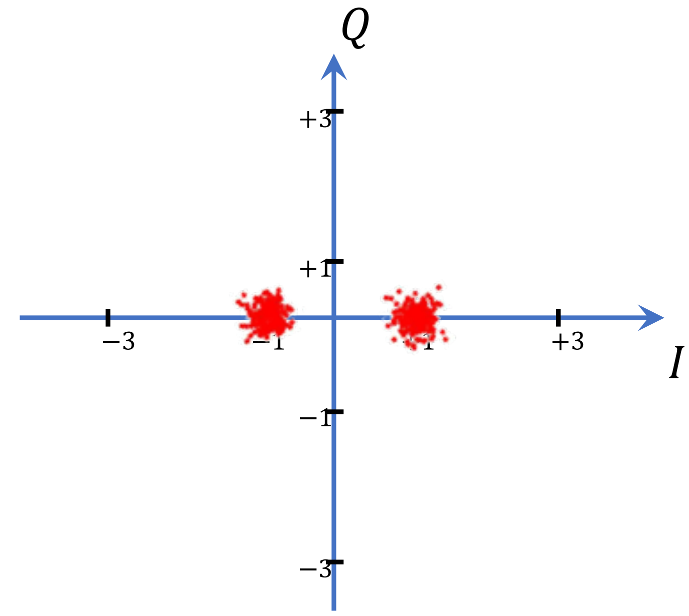
16-QAM



4-QAM



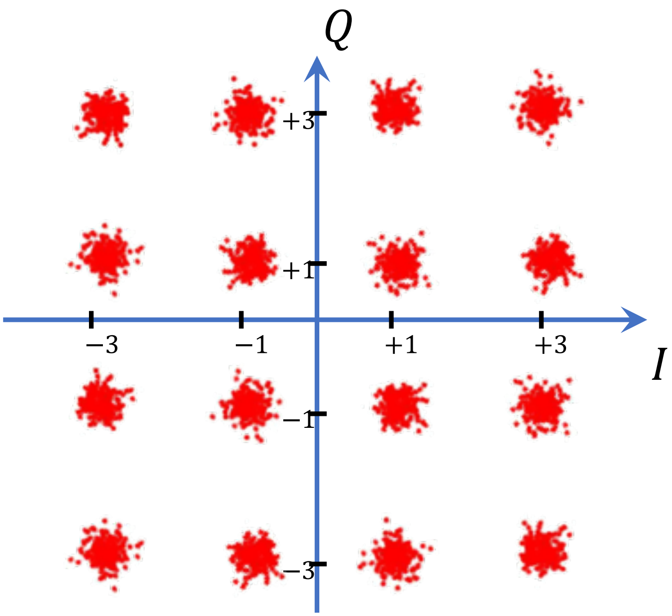
BPSK



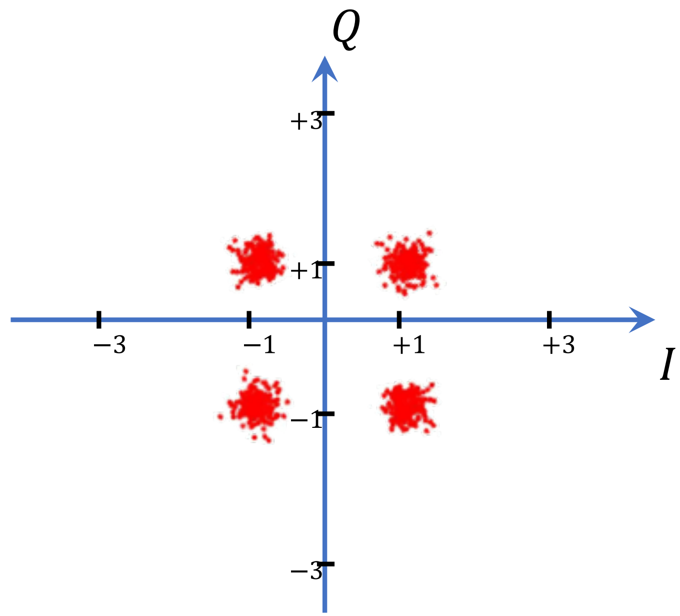
Is the SNR the same in these 3 constellations?

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E[|x(t)|^2]}{E[|v(t)|^2]} = \frac{E_s}{N_0}$$

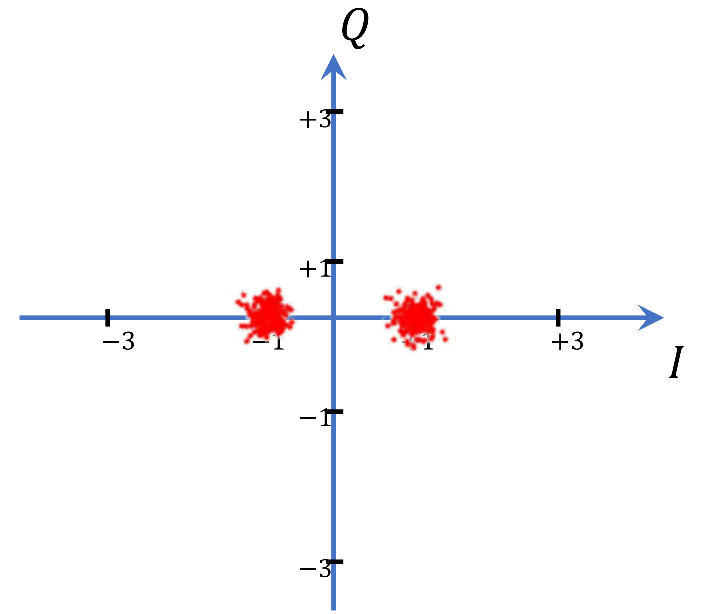
16-QAM



4-QAM



BPSK



$$SNR = \frac{E[|x(t)|^2]}{N_0}$$

$$SNR = \frac{(4\times 2 + 8\times 10 + 4\times 18)/16}{N_0} = \frac{10}{N_0}$$

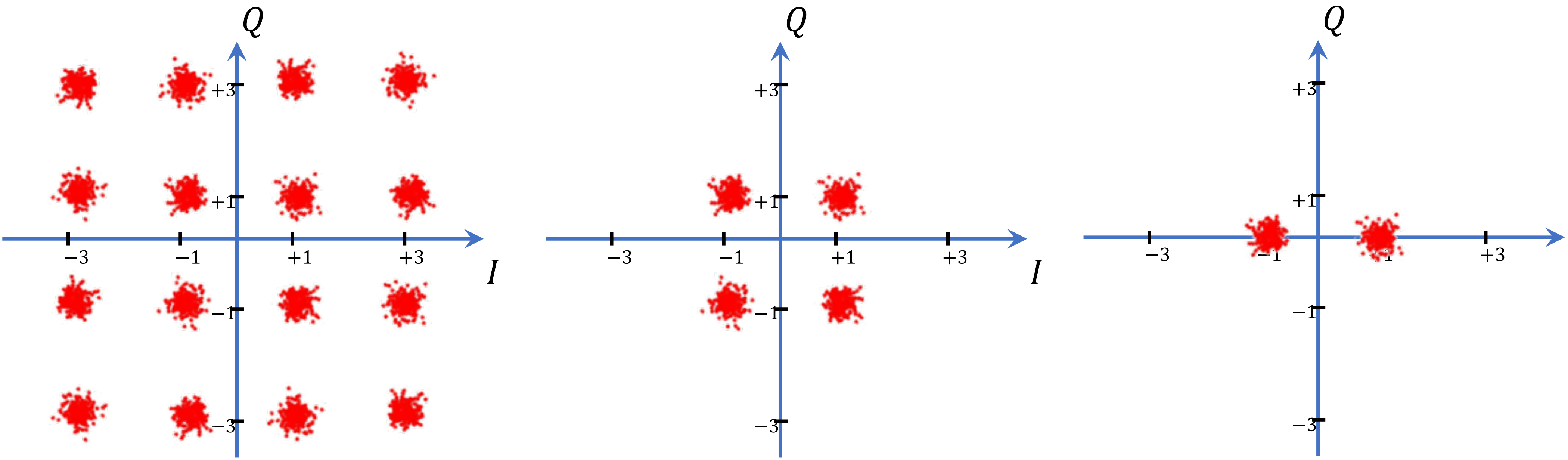
$>$

$$SNR = \frac{(2+2+2+2)/4}{N_0} = \frac{2}{N_0}$$

$>$

$$SNR = \frac{(1+1)/2}{N_0} = \frac{1}{N_0}$$

16-QAM 4-QAM BPSK



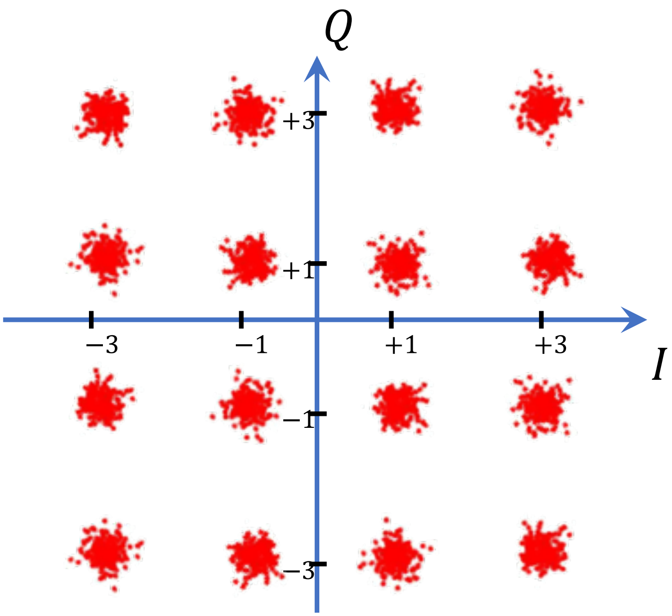
Transmit power is not the same!

$$SNR = \frac{(4\times 2 + 8\times 10 \;\; +4\times \ldots}{\ldots} \quad \ldots \quad \frac{\ldots 1 + 1)/2}{N_0}$$

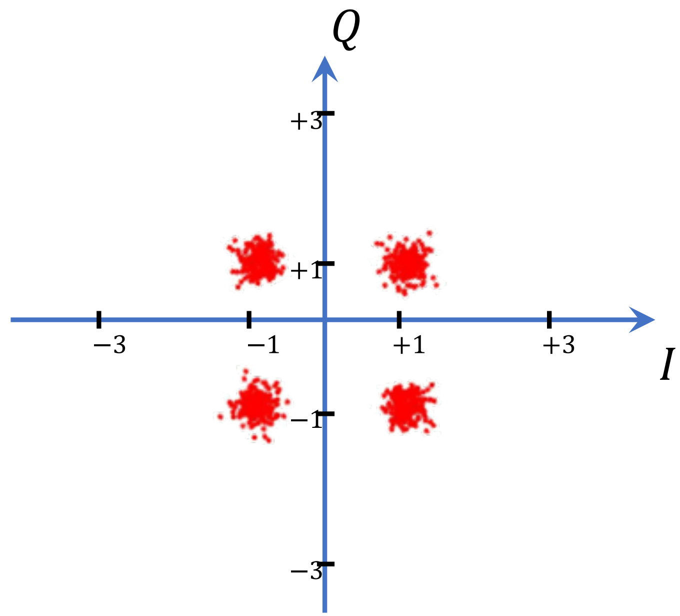
$$= \frac{10}{N_0} \quad \ldots \quad \frac{1}{N_0}$$

Must not increase transmit power to use higher order modulation!

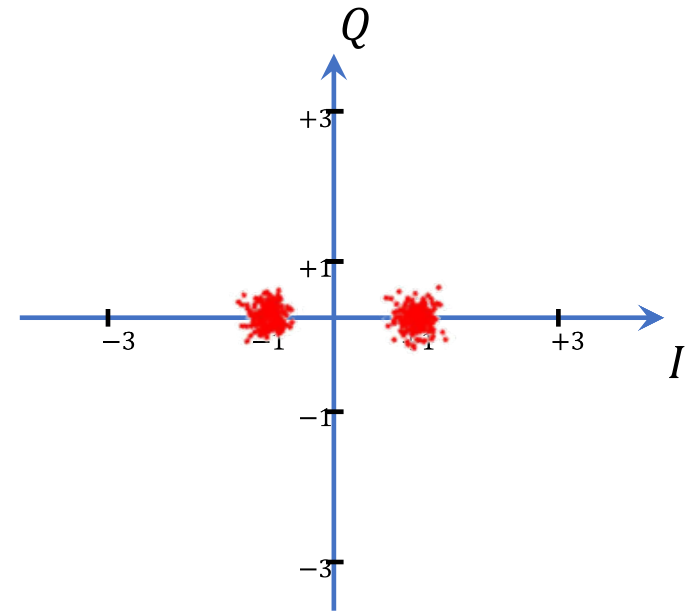
16-QAM



4-QAM



BPSK



Normalize to maintain constant transmit power.

$$SNR = \frac{(4\times 2 + 8\times 10 + 4\times 18)/16}{N_0} = \frac{10}{N_0}$$

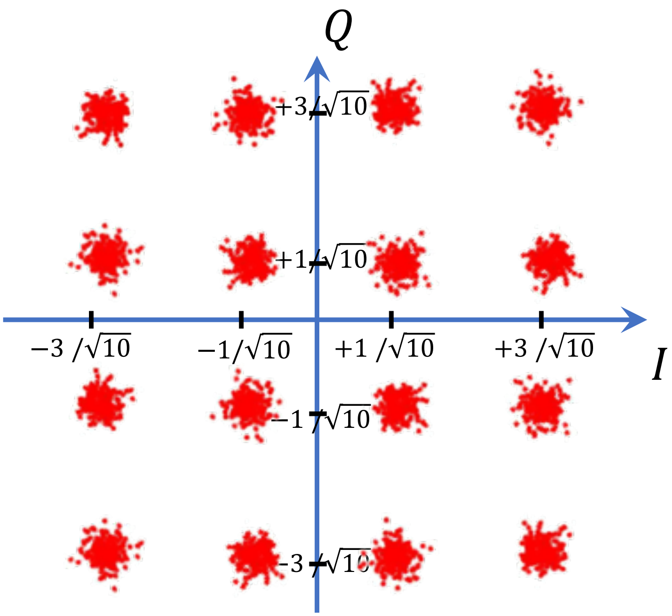
$>$

$$SNR = \frac{(2+2+2+2)/4}{N_0} = \frac{2}{N_0}$$

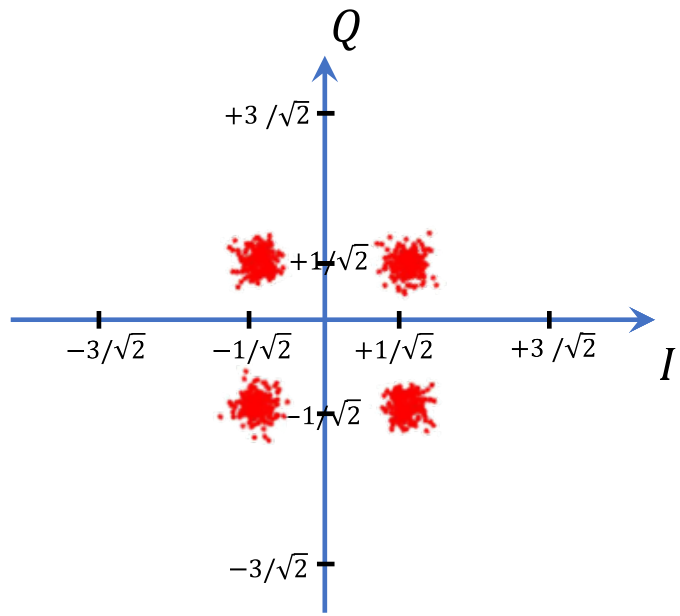
$>$

$$SNR = \frac{(1+1)/2}{N_0} = \frac{1}{N_0}$$

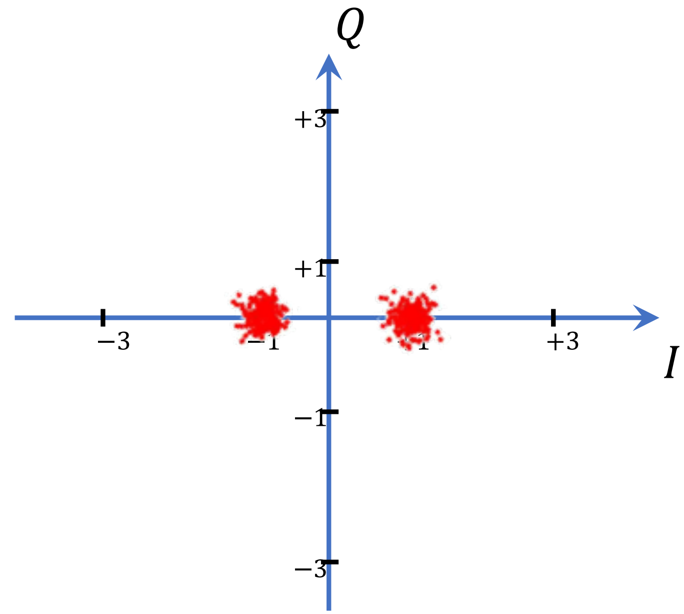
16-QAM



4-QAM



BPSK



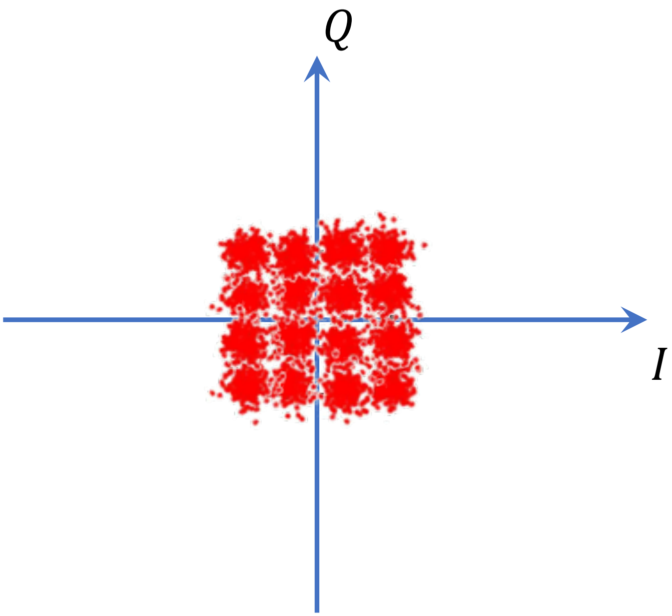
Normalize to maintain constant transmit power.

$$SNR = \frac{(4\times 2/10 + 8\times 1 + 4\times 18/10)/16}{N_0} = \frac{1}{N_0}$$

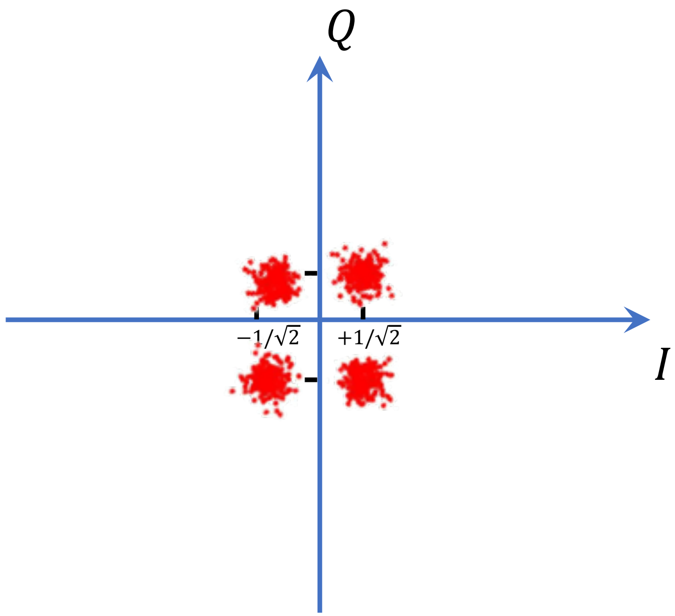
$$SNR = \frac{(1+1+1+1)/4}{N_0} = \frac{1}{N_0}$$

$$SNR = \frac{(1+1)/2}{N_0} = \frac{1}{N_0}$$

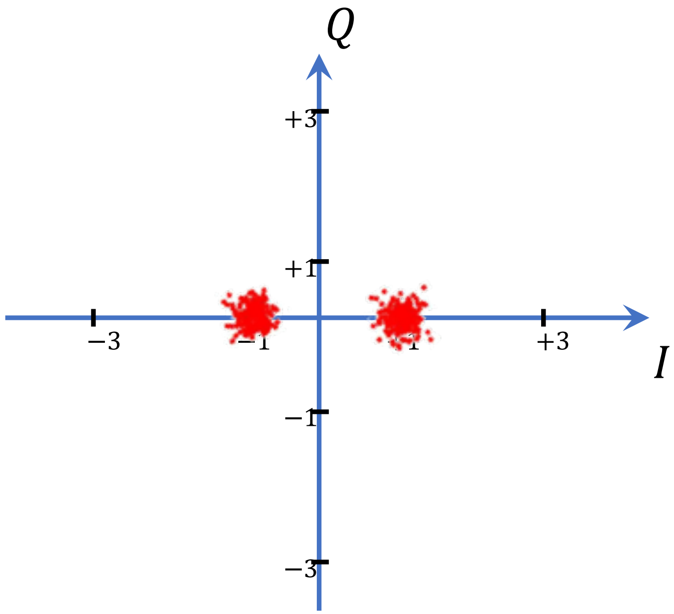
16-QAM



4-QAM



BPSK

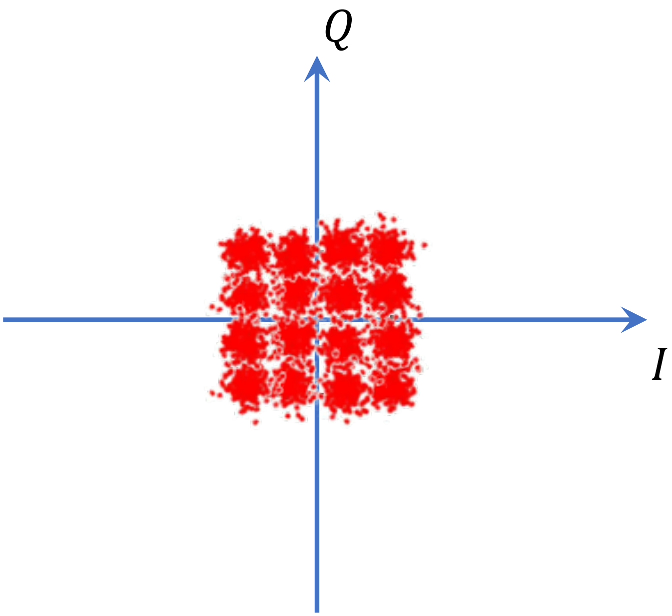


Normalize to maintain constant transmit power.

$SNR = \frac{1}{N_0}$ $SNR = \frac{1}{N_0}$ $SNR = \frac{1}{N_0}$

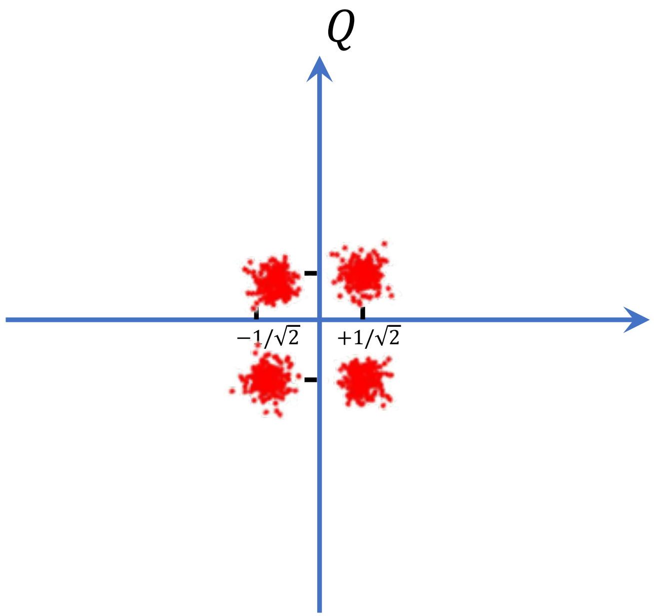
Need Higher SNR to Decode
Higher Order Modulation.

16-QAM



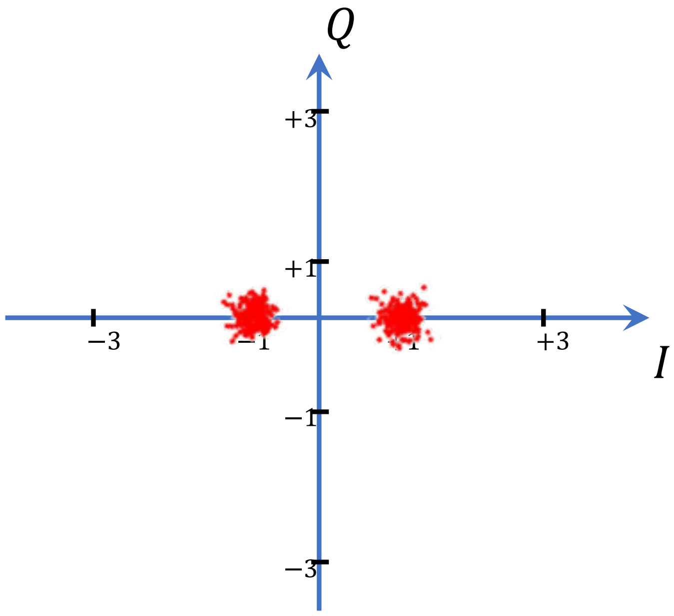
4 bits/symbol

4-QAM



2 bits/symbol

BPSK



1 bit/symbol

Higher Order Modulation:

- Needs higher SNR to decode correctly.
- Achieves higher Bite Rate

Given an SNR, choose highest order modulation that guarantees minimal Bite Error Rate (BER)

Bit Error Rate (BER) of BPSK

Encoding:

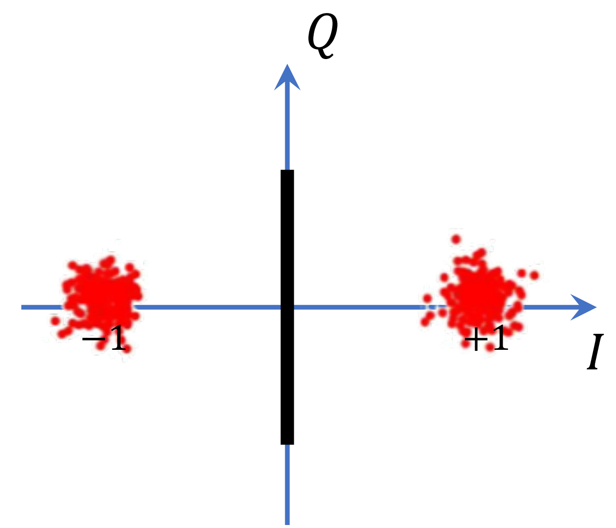
$$b = 0 \rightarrow x = -1$$
$$b = 1 \rightarrow x = +1$$

$$y = x + v$$

Decoding:

$$P(y|x = +1) \underset{0}{\overset{1}{\gtrless}} P(y|x = -1)$$

$$Re\{y\} \underset{0}{\overset{1}{\gtrless}} 0$$



$$BER = P(\hat{b} = 0 \cap b = 1) + P(\hat{b} = 1 \cap b = 0)$$

$$= P(b = 1)P(\hat{b} = 0|b = 1) + P(b = 0)P(\hat{b} = 1|b = 0)$$

$$= \frac{1}{2}P(\hat{b} = 0|b = 1) + \frac{1}{2}P(\hat{b} = 1|b = 0)$$

$$= \frac{1}{2}P(y < 0|b = 1) + \frac{1}{2}P(y > 0|b = 0)$$

$$= \frac{1}{2}P(y < 0|x = +1) + \frac{1}{2}P(y > 0|x = -1)$$

Bit Error Rate (BER) of BPSK

Encoding:

$$b = 0 \rightarrow x = -1$$
$$b = 1 \rightarrow x = +1$$

$$y = x + v$$

Decoding:

$$P(y|x=+1) \underset{0}{\overset{1}{\gtrless}} P(y|x=-1)$$

$$Re\{y\} \underset{0}{\overset{1}{\gtrless}} 0$$

$$BER = \frac{1}{2}P(y<0|x=+1) + \frac{1}{2}P(y>0|x=-1)$$

$$= \frac{1}{2}\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-1)^2}{2\sigma^2}}\, dy \qquad + \frac{1}{2}\int_{0}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y+1)^2}{2\sigma^2}}\, dy$$

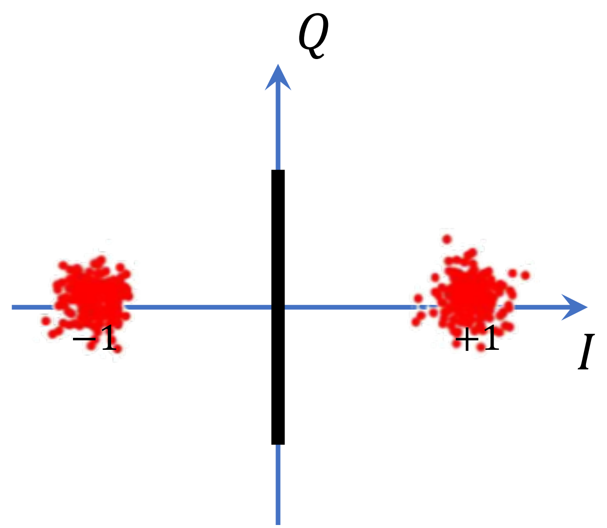
$$= \int_{0}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y+1)^2}{2\sigma^2}}\, dy \qquad u = \frac{y+1}{\sigma}$$

$$= \frac{1}{\sqrt{2\pi}}\int_{\frac{1}{\sigma}}^{\infty} e^{-\frac{u^2}{2}}\, du = \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sigma\sqrt{2}}\right) = Q(1/\sigma) = Q\left(\sqrt{2/N_0}\right) = Q\left(\sqrt{2E_s/N_0}\right)$$

$$= Q\left(\sqrt{2SNR}\right)$$

Error Function

Q Function



Bit Error Rate (BER) of BPSK

Encoding:

$$b = 0 \rightarrow x = -1$$
$$b = 1 \rightarrow x = +1$$

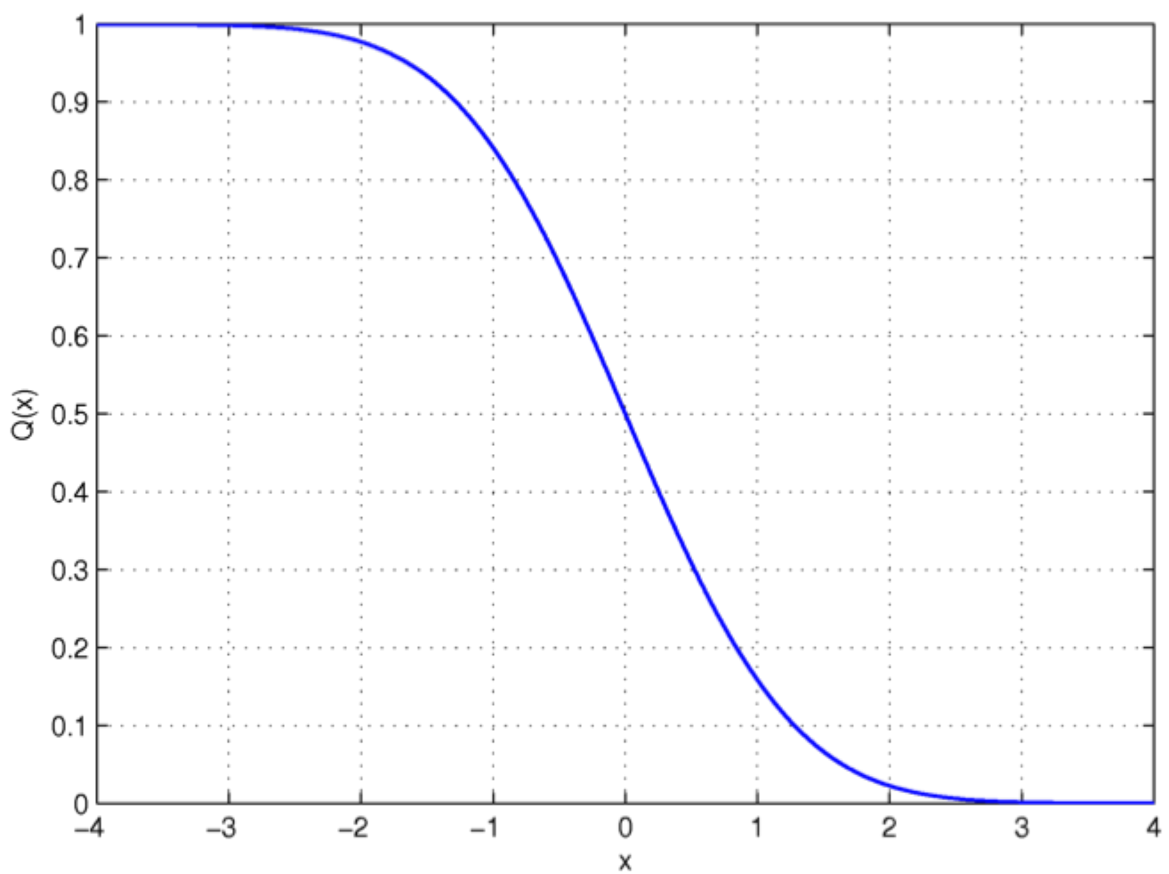
$$y = x + v$$

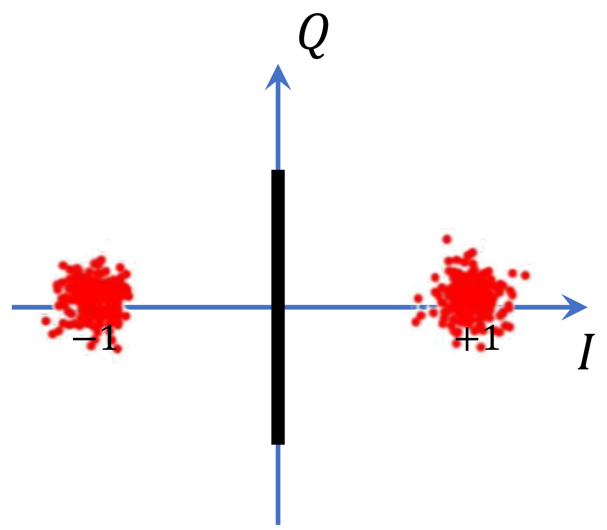
Decoding:

$$P(y|x = +1) \underset{0}{\overset{1}{\gtrless}} P(y|x = -1)$$

$$Re\{y\} \underset{0}{\overset{1}{\gtrless}} 0$$

$$BER = Q\left(\sqrt{2SNR}\right) = Q\left(\sqrt{2E_s/N_0}\right)$$





Bit Error Rate (BER) of BPSK

Encoding:

$$b = 0 \rightarrow x = -1$$
$$b = 1 \rightarrow x = +1$$

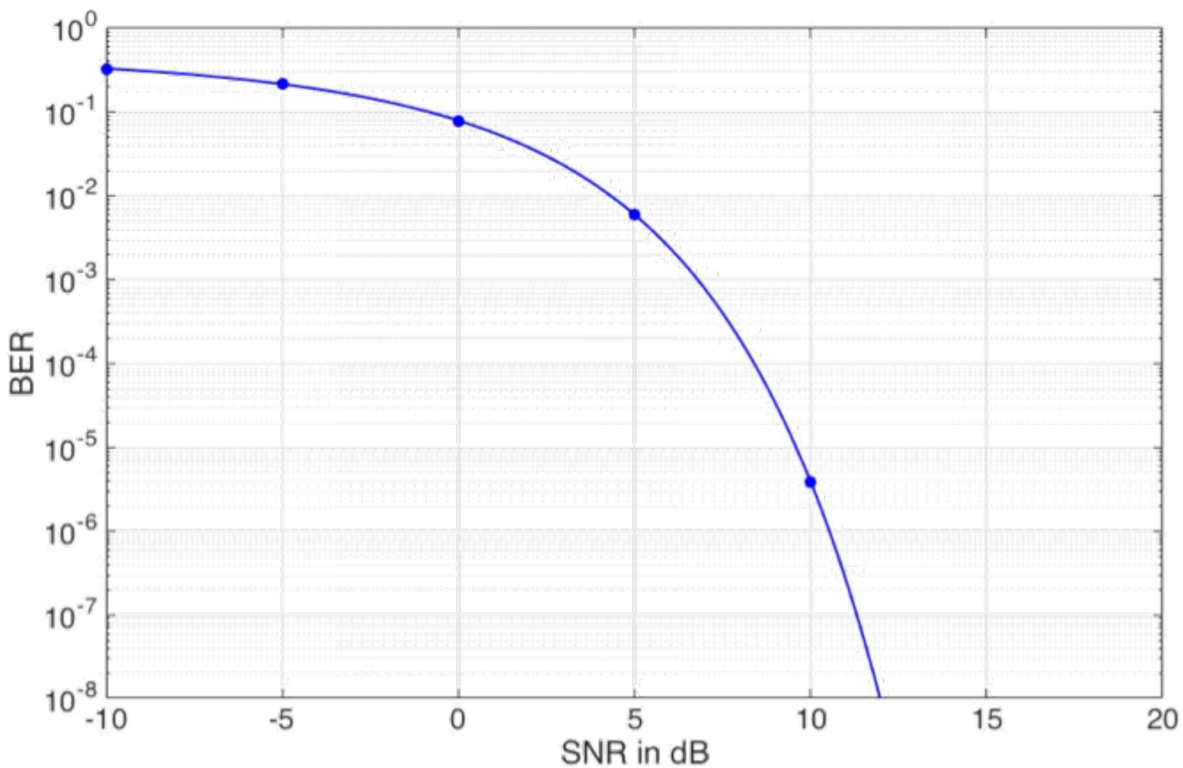
$$y = x + v$$

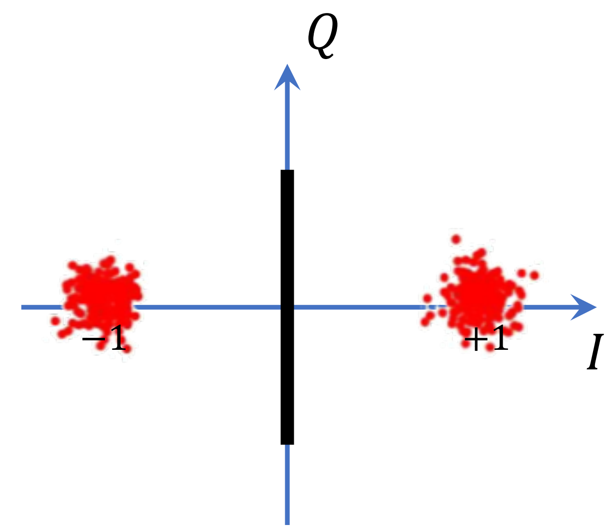
Decoding:

$$P(y|x = +1) \underset{0}{\overset{1}{\gtrless}} P(y|x = -1)$$

$$Re\{y\} \underset{0}{\overset{1}{\gtrless}} 0$$

$$BER = Q\left(\sqrt{2SNR}\right) = Q\left(\sqrt{2E_s/N_0}\right)$$





Bit Error Rate (BER)

- Number of Constellation Points: M
- Bits per symbol: $\log_2 M$
- Signal-to-Noise Ratio: SNR
- SNR per symbol: $\dfrac{E_s}{N_0}$
- SNR per bit: $\dfrac{E_b}{N_0} = \dfrac{1}{\log_2 M}\dfrac{E_s}{N_0}$
- Symbol Error Rate: $SER = P_s$
- Bit Error Rate: $BER = P_b$
- Relation between P_s and P_b?

A. $P_b \leq P_s$ B. $P_b = P_s$ C. $P_b > P_s$

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- Number of Constellation Points: M
- Bits per symbol: $\log_2 M$
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Bit Error Rate (BER)

- Number of Constellation Points: M
- Bits per symbol: $\log_2 M$
- $P_b \leq P_s$?

1 symbol error can at most generate $\log_2 M$ bit errors.

$$P_b = \frac{Number\ of\ Error\ Bits}{Total\ Number\ of\ Bits} \leq \frac{Number\ of\ Error\ Symbols \times \log_2 M}{Total\ Number\ of\ Symbols \times \log_2 M} = P_s$$

How much is P_b less than P_s?

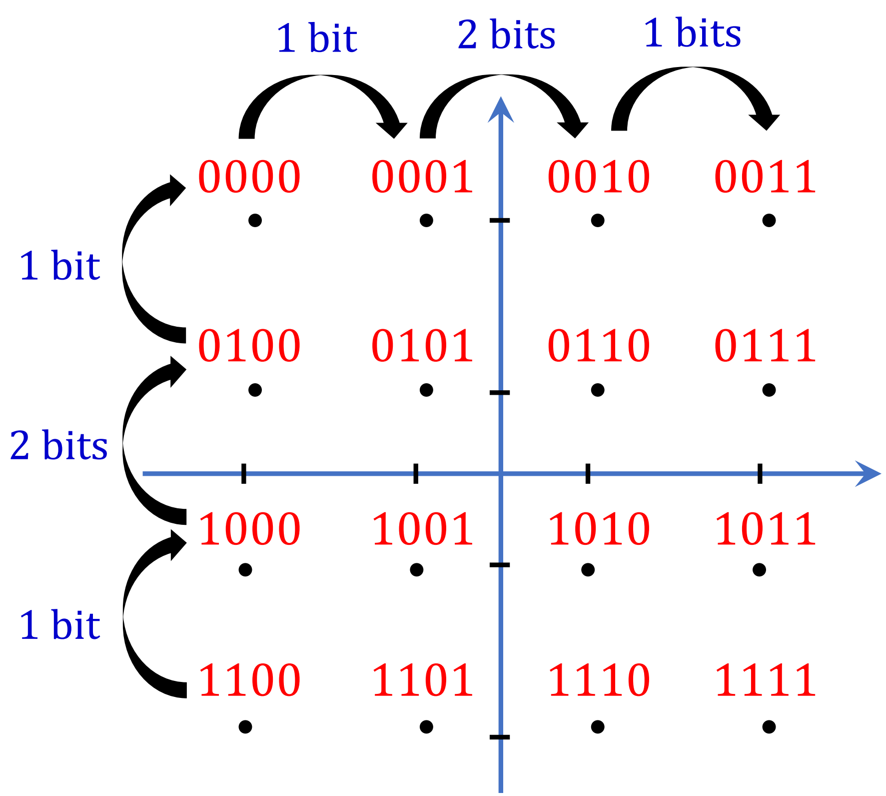
Depends on the code used: How we assign bits to symbol

How much is P_b less than P_s?

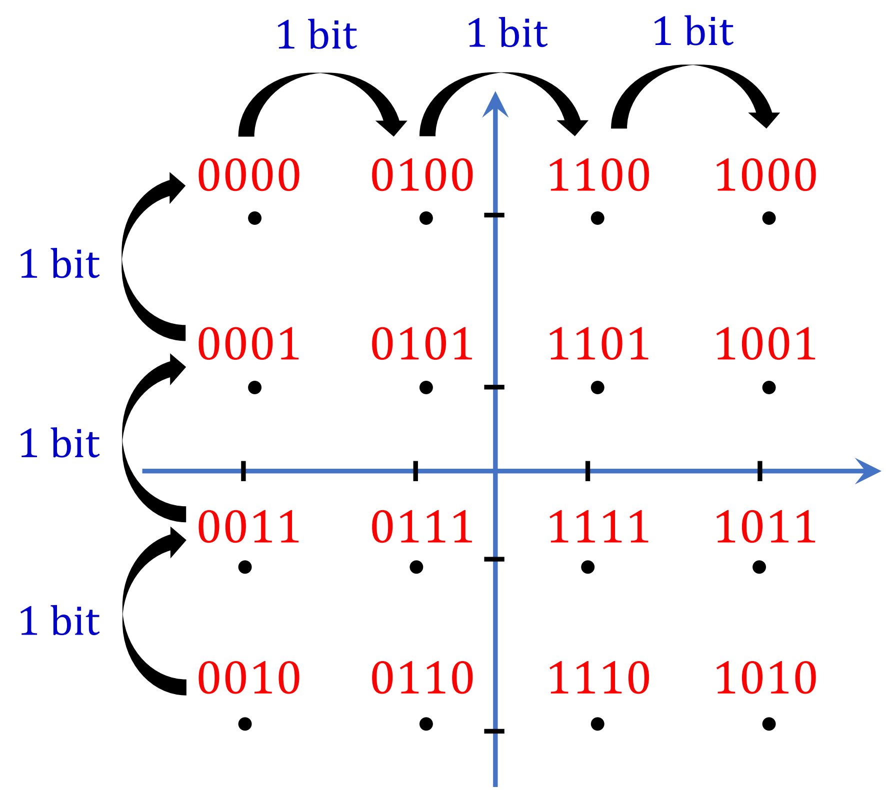
Depends on the code used: How we assign bits to symbol

16-QAM

$P_b \approx \frac{1}{3} P_s$



$P_b \approx \frac{1}{4} P_s$



Most likely error occurs between nearest neighbor constellation points!

Minimize bit flips between nearest neighbors.

How much is P_b less than P_s?

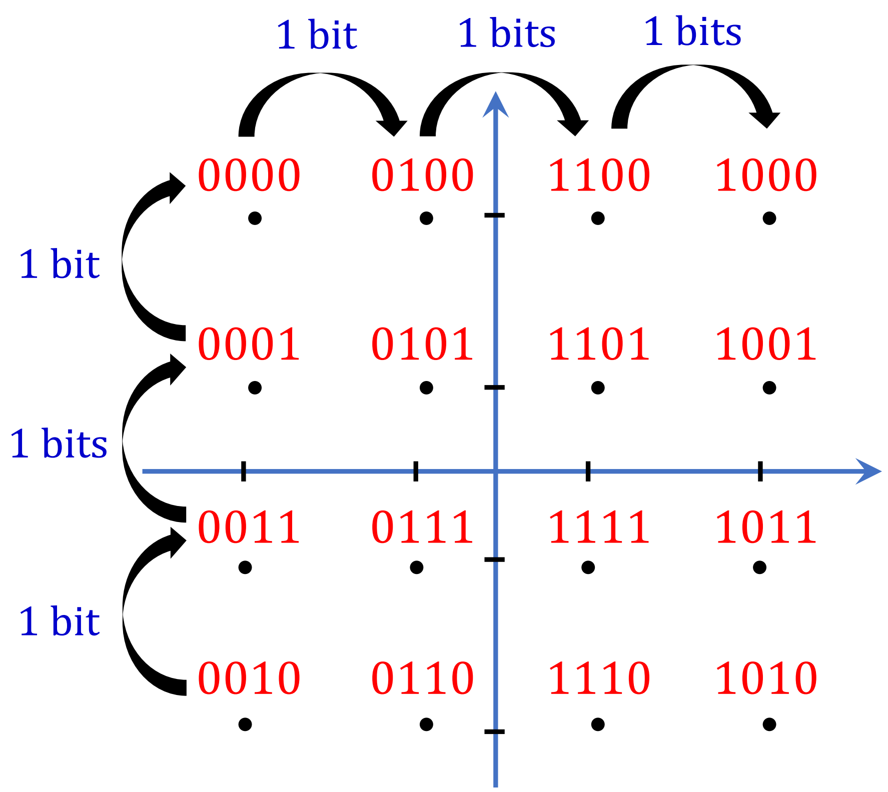
Depends on the code used: How we assign bits to symbol

16-QAM $\quad P_b \approx \frac{1}{4} P_s$

Gray Codes

Bit flips between nearest neighbors = 1

$$P_b \approx \frac{1}{\log_2 M} P_s$$



Bit Error Rate (BER)

- Number of Constellation Points: M
- Bits per symbol: $\log_2 M$
- Assuming Gray Code is used:

$$BER\text{: } P_b = \frac{1}{\log_2 M} P_s$$

- Nearest Neighbor Approximation:

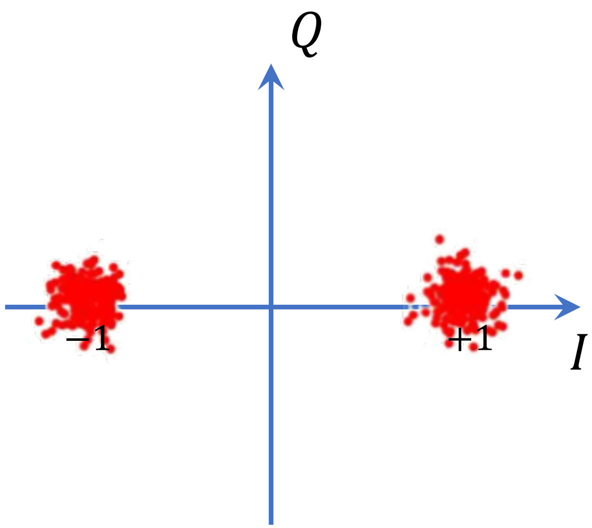
$$SER\text{: } P_s \approx \#\, nearest\ neighbors \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{2\sqrt{E_s}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{2E_s/N_0}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = Q\left(\sqrt{2E_b/N_0}\right)$$

BPSK:



$$d_{min} = 2\sqrt{E_s}$$

$$\#nn = 1$$

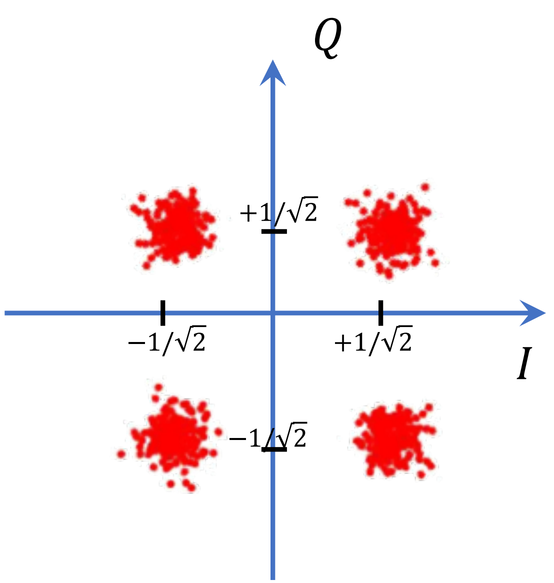
$$\log_2 M = 1$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{E_s/N_0}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = Q\left(\sqrt{2E_b/N_0}\right)$$

4-QAM/QPSK



$$d_{min} = \frac{2}{\sqrt{2}}\sqrt{E_s}$$

$$\#nn = 2$$

$$\log_2 M = 2$$

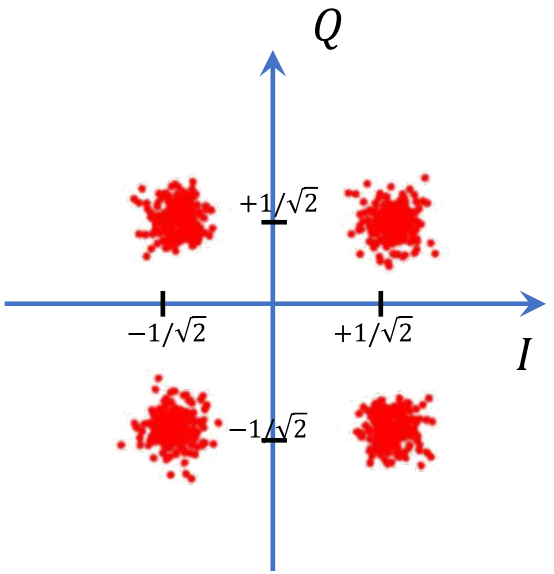
Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{E_s/N_0}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = Q\left(\sqrt{2E_b/N_0}\right)$$

Same as BPSK with $2\times$ Bitrate!

4-QAM/QPSK



$$d_{min} = \frac{2}{\sqrt{2}}\sqrt{E_s}$$

$$\#nn = 2$$

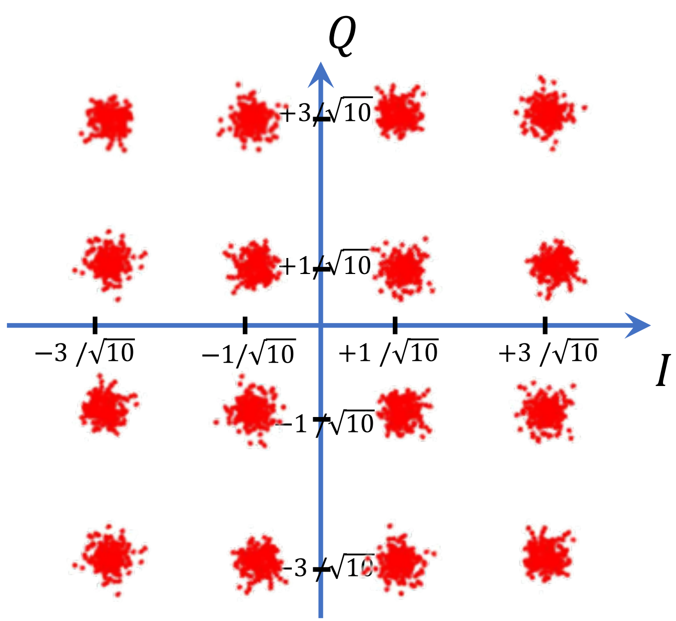
$$\log_2 M = 2$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 4Q\left(\sqrt{E_s/5N_0}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = Q\left(\sqrt{4E_b/5N_0}\right)$$

16-QAM



$$d_{min} = \frac{2}{\sqrt{10}}\sqrt{E_s}$$

$$\#nn = 4$$

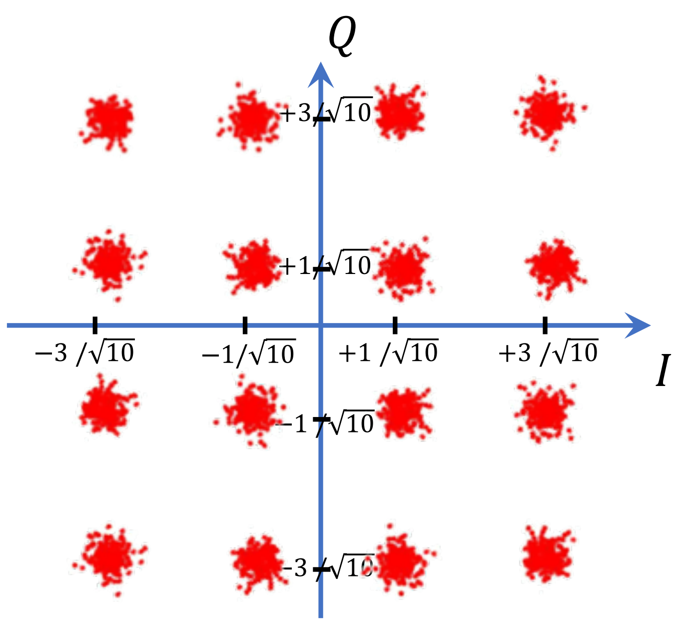
$$\log_2 M = 4$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 4Q\left(\sqrt{\frac{3E_s/N_0}{M-1}}\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s = \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\log_2 M\ E_b/N_0}{M-1}}\right)$$

M-QAM



$$d_{min} = \sqrt{\frac{6E_s}{M-1}}$$

$$\#nn = 4$$

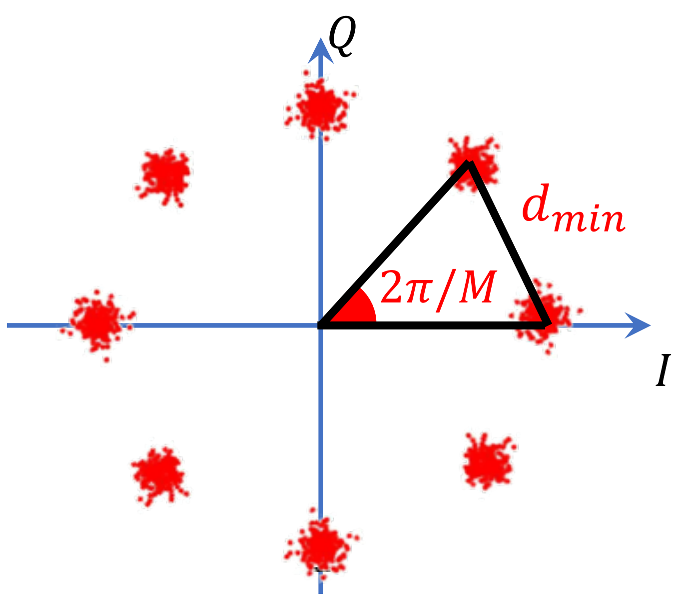
$$\log_2 M$$

Bit Error Rate (BER)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{2E_s/N_0}\sin\left(\frac{\pi}{M}\right)\right)$$

$$BER:\ P_b = \frac{1}{\log_2 M} P_s \ = \frac{2}{\log_2 M} Q\left(\sqrt{2\log_2 M\, E_b/N_0}\sin\left(\frac{\pi}{M}\right)\right)$$

M-PSK



$$d_{min} = 2\sin\left(\frac{\pi}{M}\right)\sqrt{E_s}$$

$$\#nn = 2$$

$$\log_2 M$$

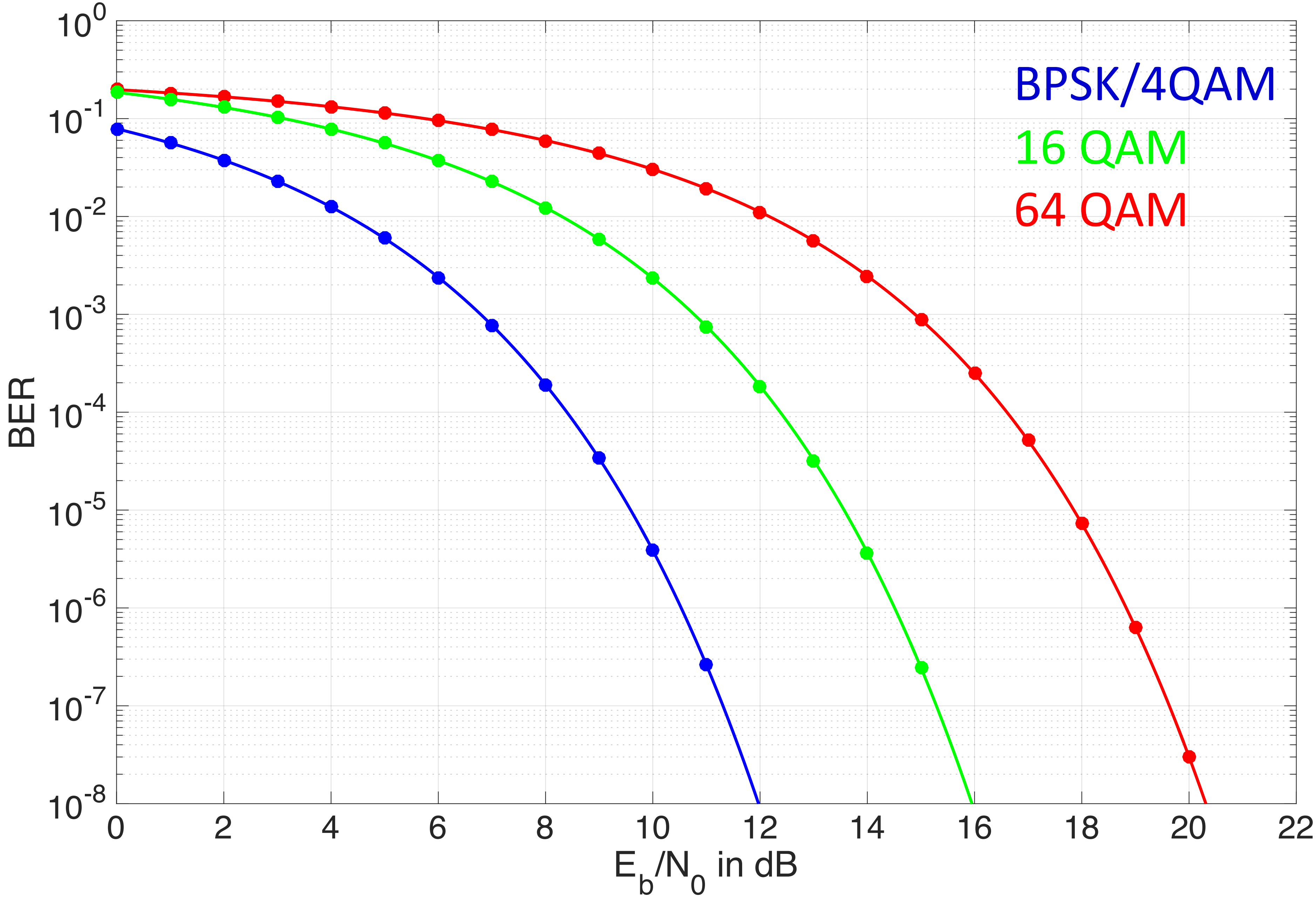
Bit Error Rate (BER)

	SER: $P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$	BER: $P_b = \frac{1}{\log_2 M} P_s$
• BPSK	$Q(\sqrt{2E_s/N_0})$	$Q(\sqrt{2E_b/N_0})$
• QPSK/4-QAM	$2\,Q(\sqrt{E_s/N_0})$	$Q(\sqrt{2E_b/N_0})$
• MPAM	$\frac{2(M-1)}{M}\,Q\left(\sqrt{\frac{6E_s/N_0}{M^2-1}}\right)$	$\frac{2(M-1)}{M\log_2 M}\,Q\left(\sqrt{\frac{6\log_2 M\,E_b/N_0}{M^2-1}}\right)$
• MPSK	$2\,Q\left(\sqrt{2E_s/N_0}\sin\left(\frac{\pi}{M}\right)\right)$	$\frac{2}{\log_2 M}Q\left(\sqrt{2\log_2 M\,E_b/N_0}\sin\left(\frac{\pi}{M}\right)\right)$
• MQAM	$4\,Q\left(\sqrt{\frac{3E_s/N_0}{M-1}}\right)$	$\frac{4}{\log_2 M}Q\left(\sqrt{\frac{3\log_2 M\,E_b/N_0}{M-1}}\right)$

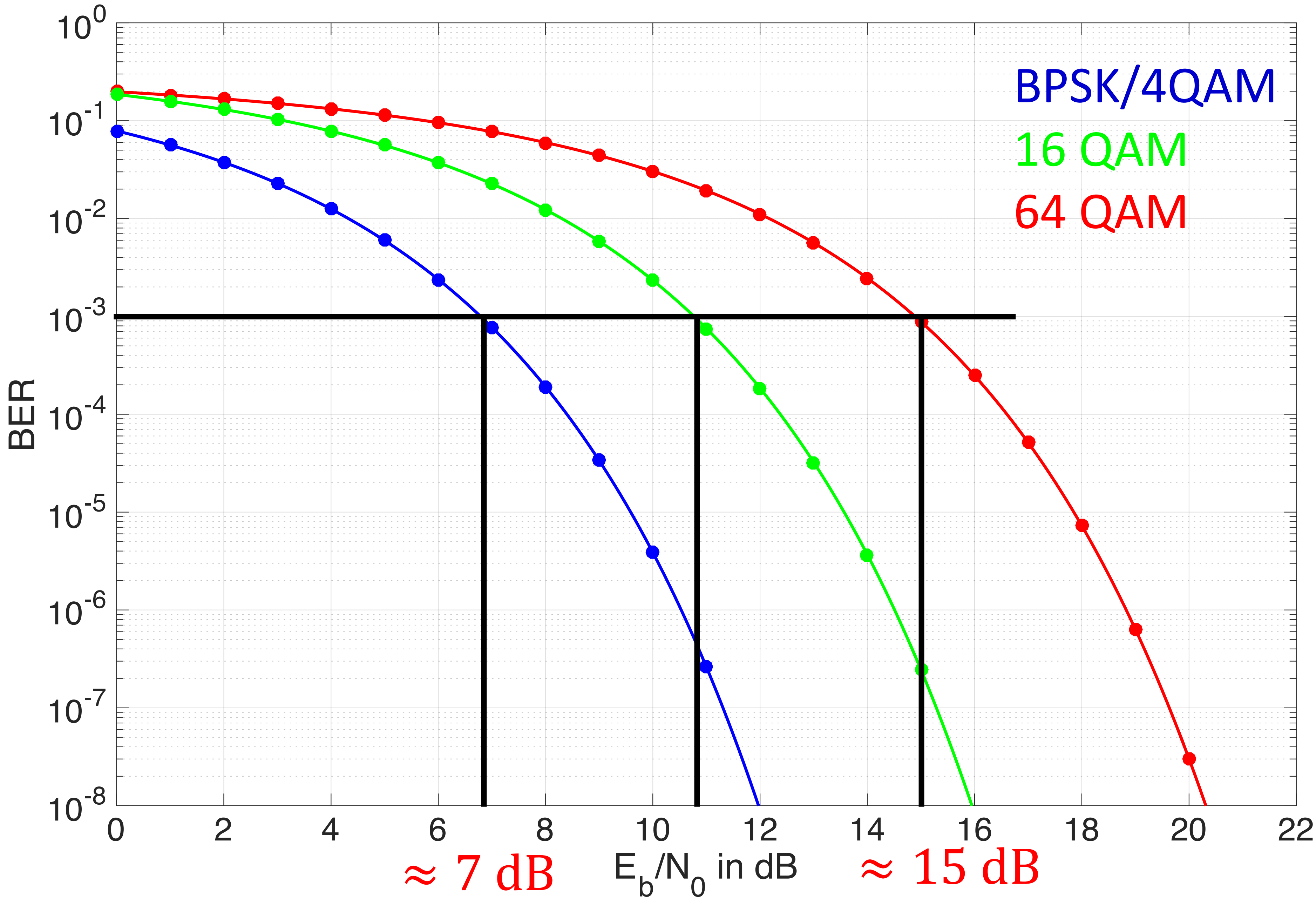
$\alpha_M Q(\sqrt{\beta_M E_s/N_0})$ $\qquad$ $\hat{\alpha}_M Q\left(\sqrt{\hat{\beta}_M E_b/N_0}\right)$

$$\hat{\alpha}_M = \frac{\alpha_M}{\log_2 M}, \qquad \hat{\beta}_M = \beta_M \log_2 M\,, \qquad \frac{E_b}{N_0} = \frac{1}{\log_2 M}\frac{E_s}{N_0}$$

Bit Error Rate (BER)

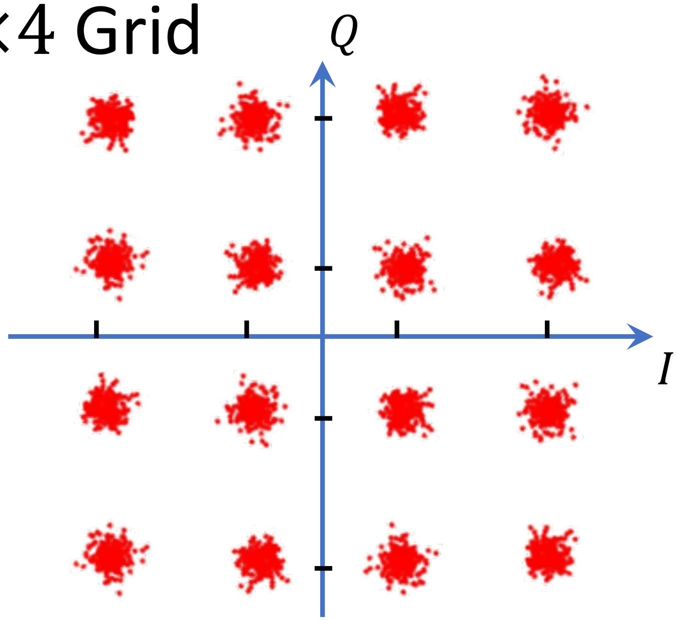


Bit Error Rate (BER)

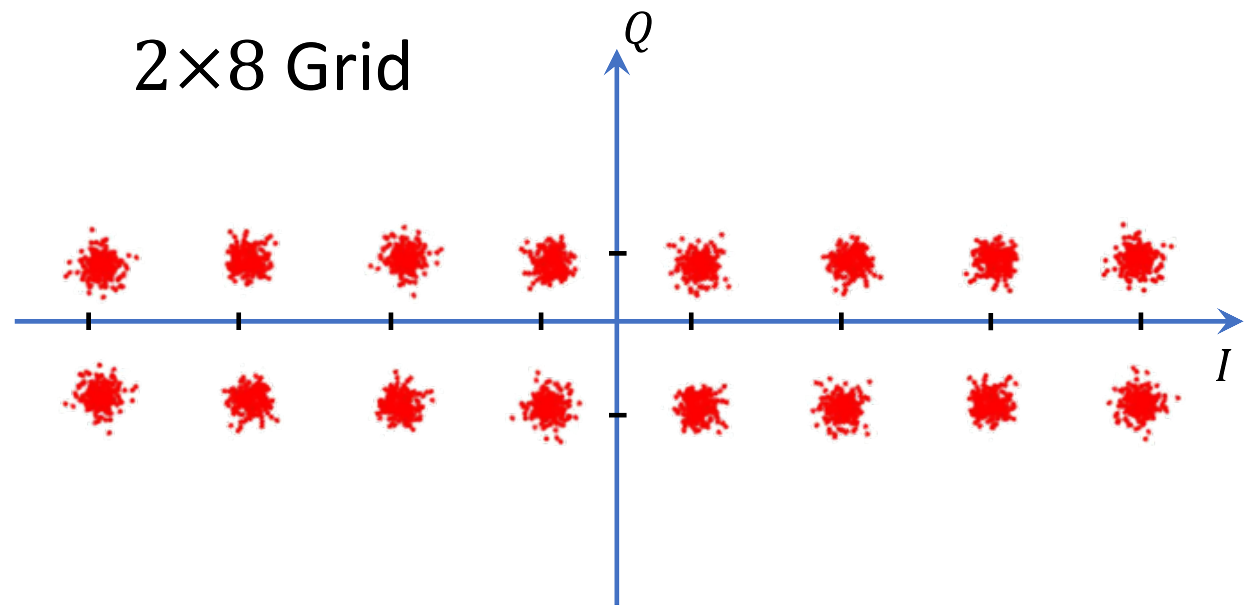


Shape of the Constellation (e.g. $M = 16$)

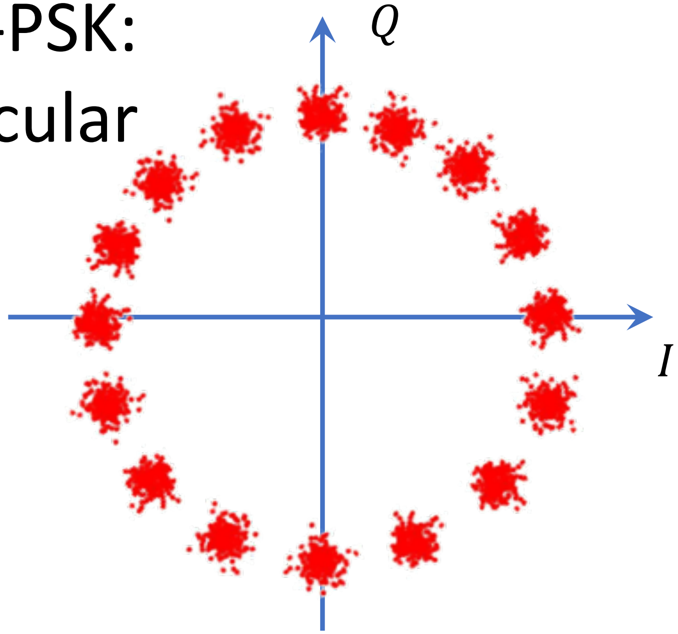
16-QAM:
4×4 Grid



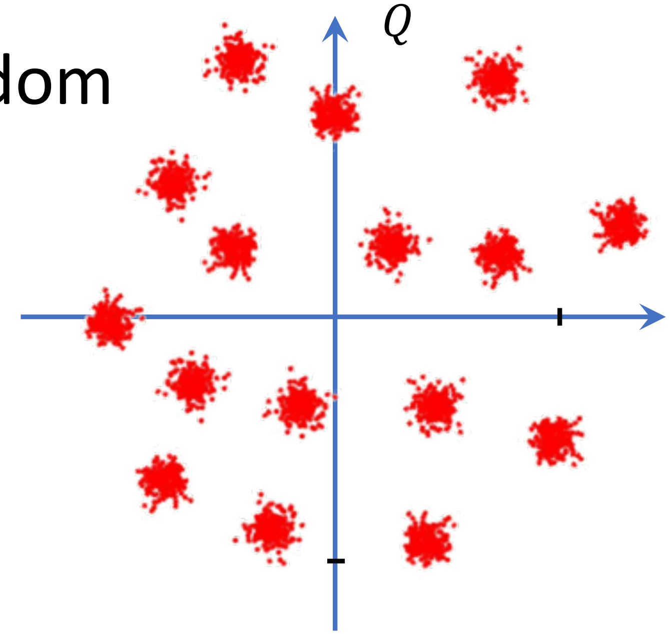
2×8 Grid



16-PSK:
Circular



Random



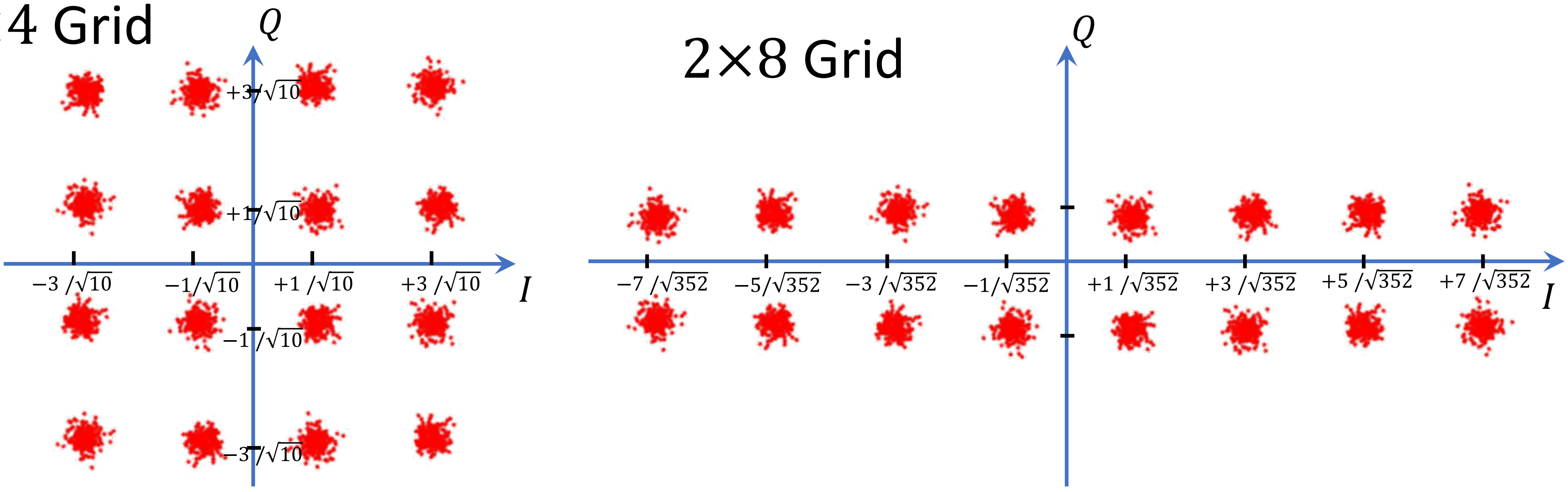
Shape of the Constellation (e.g. $M = 16$)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Goal: Maximize d_{min} for the same overall power

4×4 Grid

2×8 Grid



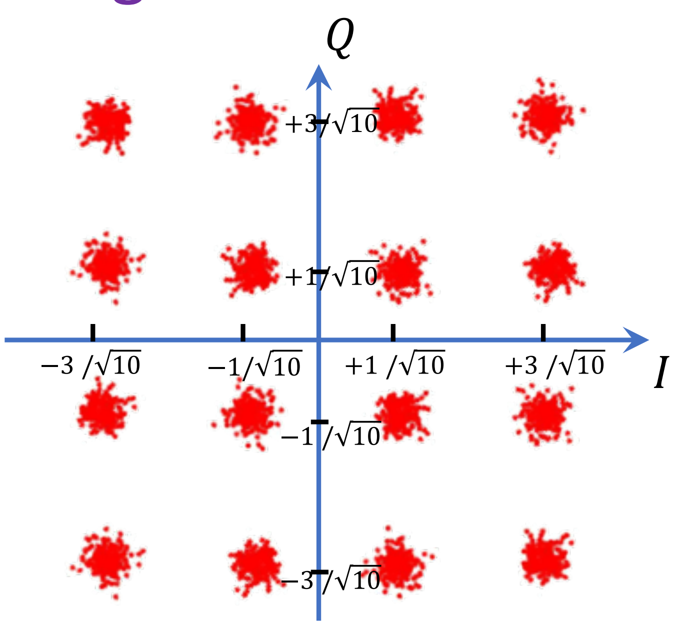
$$d_{min} = \frac{2}{\sqrt{10}}\sqrt{E_s} \gg d_{min} = \frac{2}{\sqrt{352}}\sqrt{E_s}$$

Shape of the Constellation (e.g. $M = 16$)

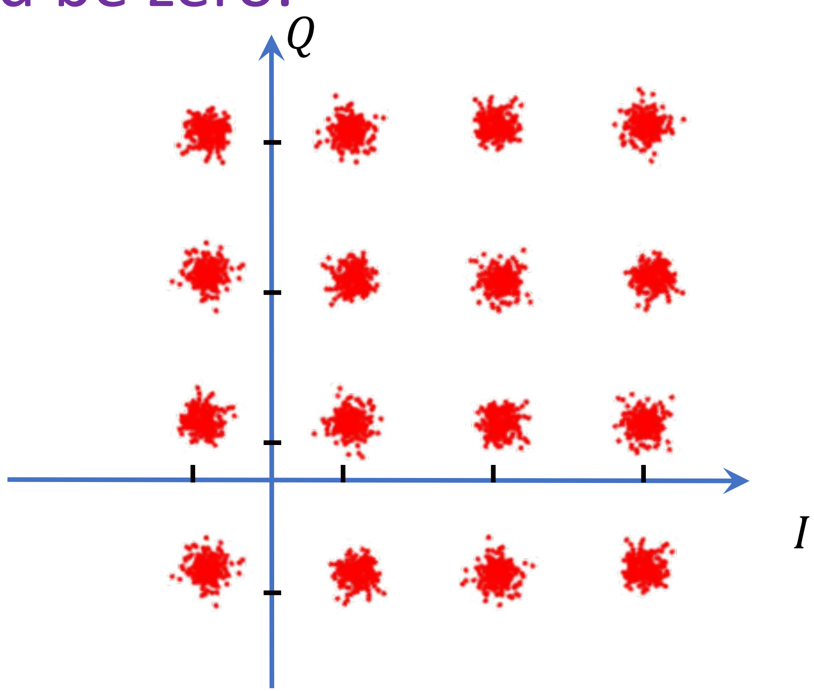
$$SER\colon\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Goal: Maximize d_{min} for the same overall power

Average of constellation should be zero.



$$E[|x(t)|^2] = 1$$



$$E[|x(t)|^2] = 1 + |E[x(t)]|^2$$

If Average is not zero, we have higher TX Power for the same d_{min} or smaller d_{min} for same TX power.

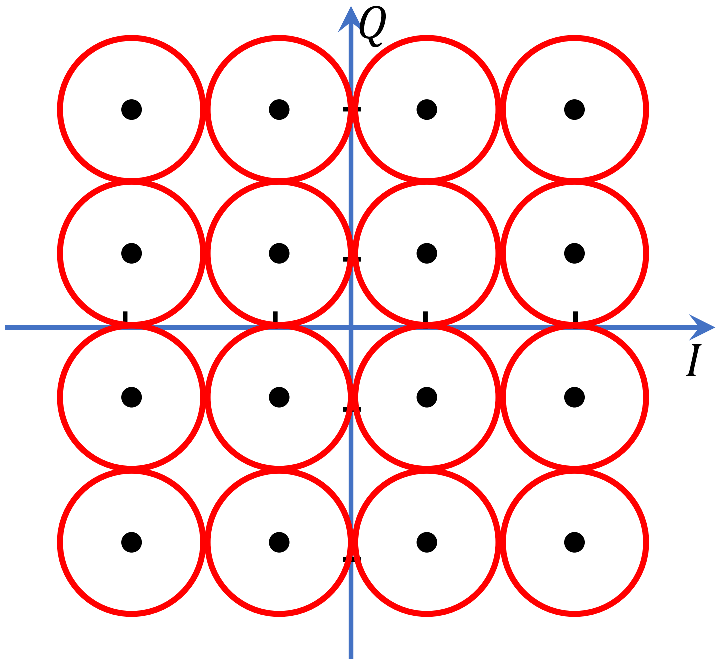
Shape of the Constellation (e.g. $M = 16$)

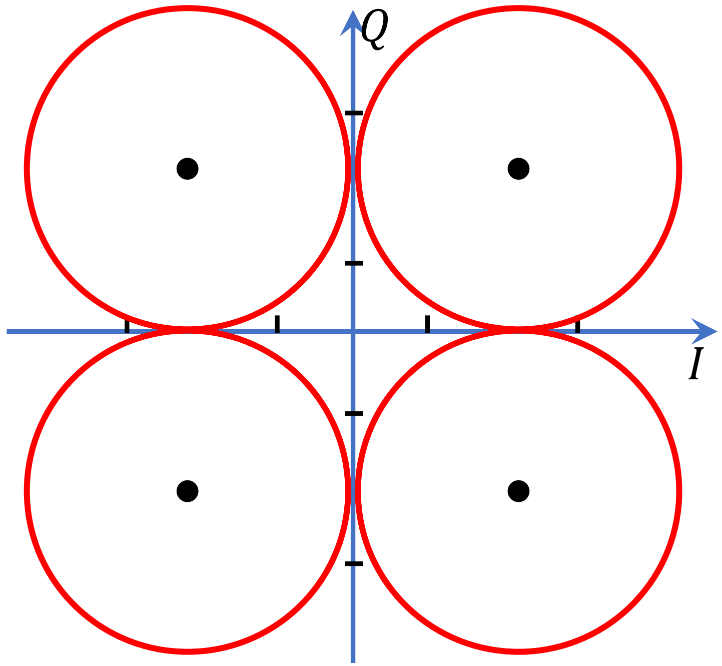
$$SER: P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

Goal: Maximize d_{min} for the same overall power

Average of constellation should be zero.

Geometric packing problem.





Shape of the Constellation (e.g. $M = 16$)

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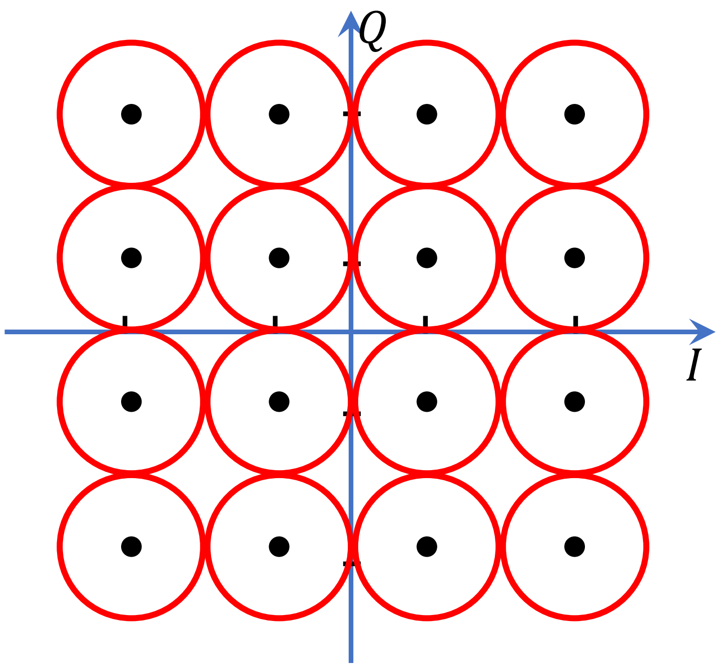

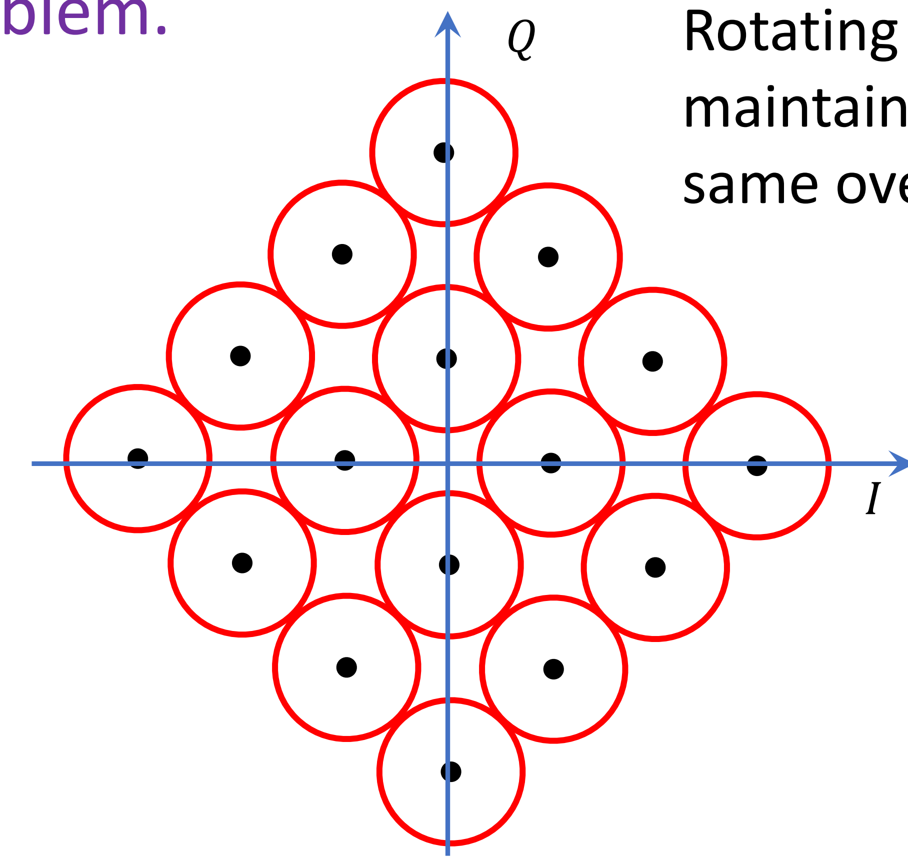

Rotating the constellation maintains same d_{min} for same overall power

Shape of the Constellation (e.g. $M = 16$)

SER: $P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

Goal: Maximize d_{min} for the same overall power

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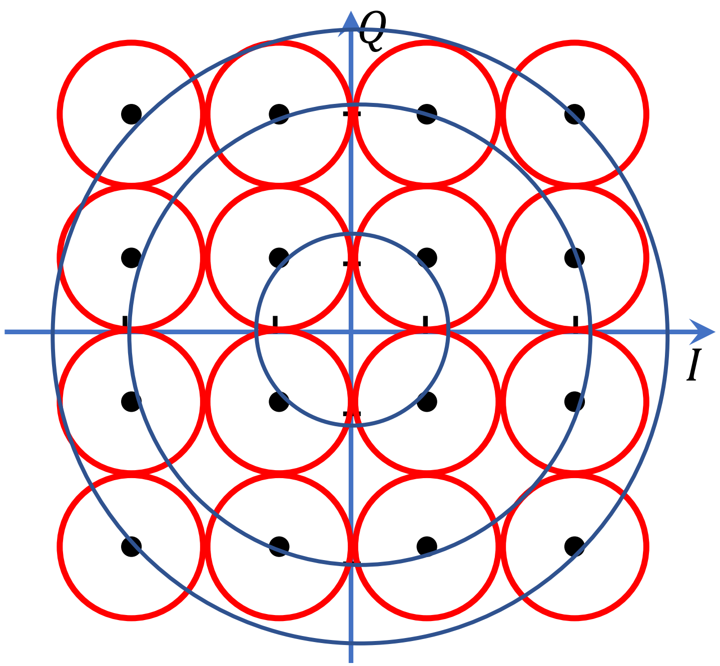

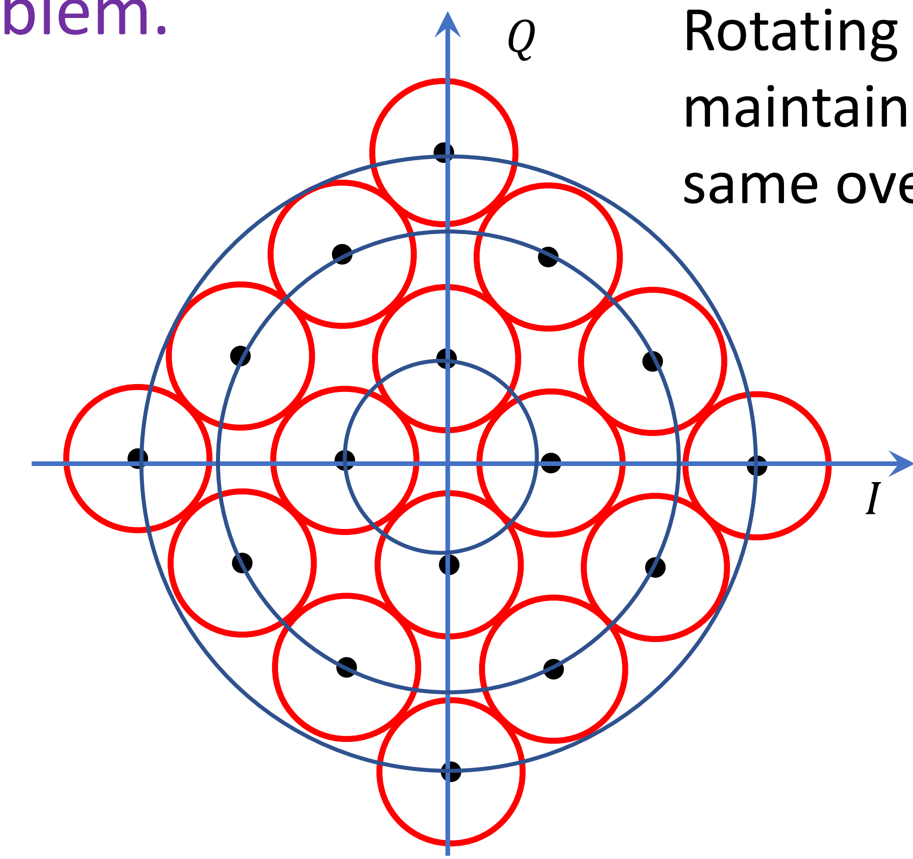

Rotating the constellation maintains same d_{min} for same overall power

Shape of the Constellation (e.g. $M = 16$)

$$SER:\ P_s \approx \#nn \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

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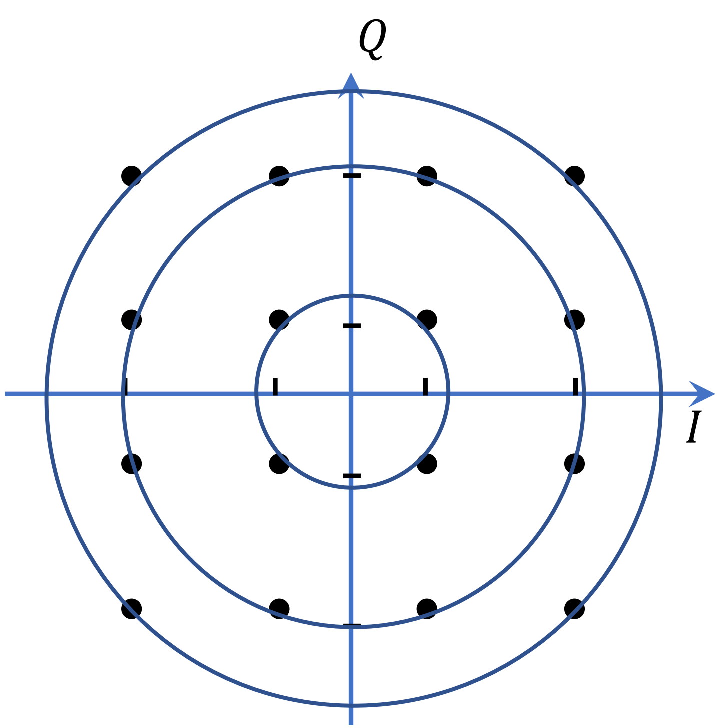
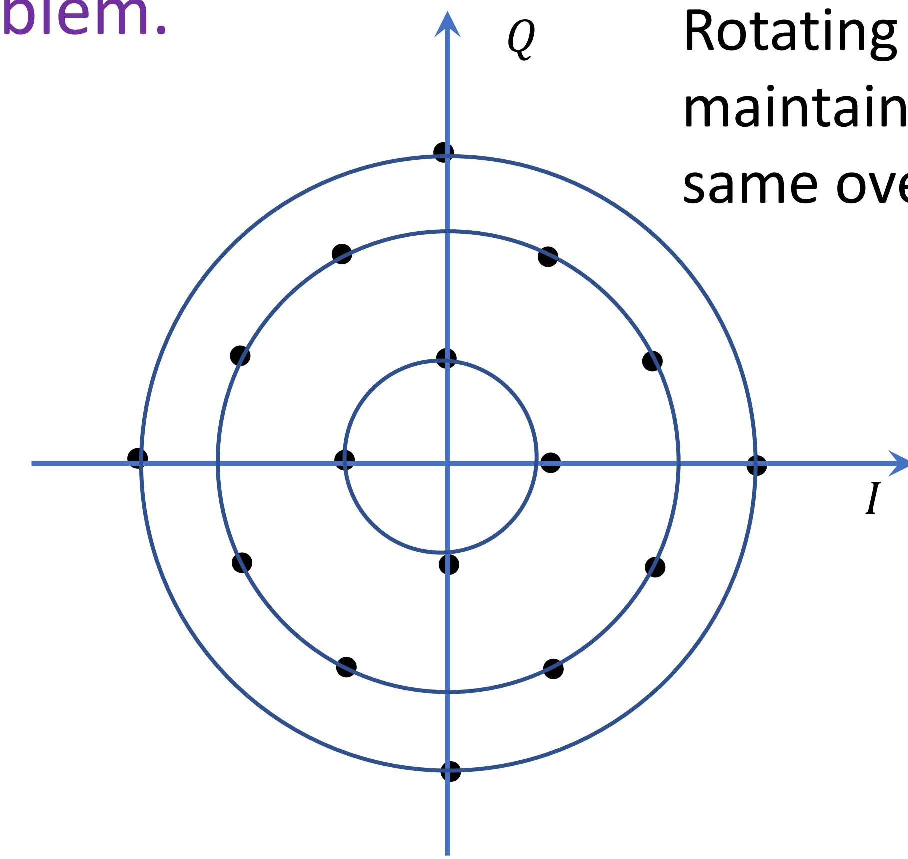
Average of constellation should be zero.

Geometric packing problem.





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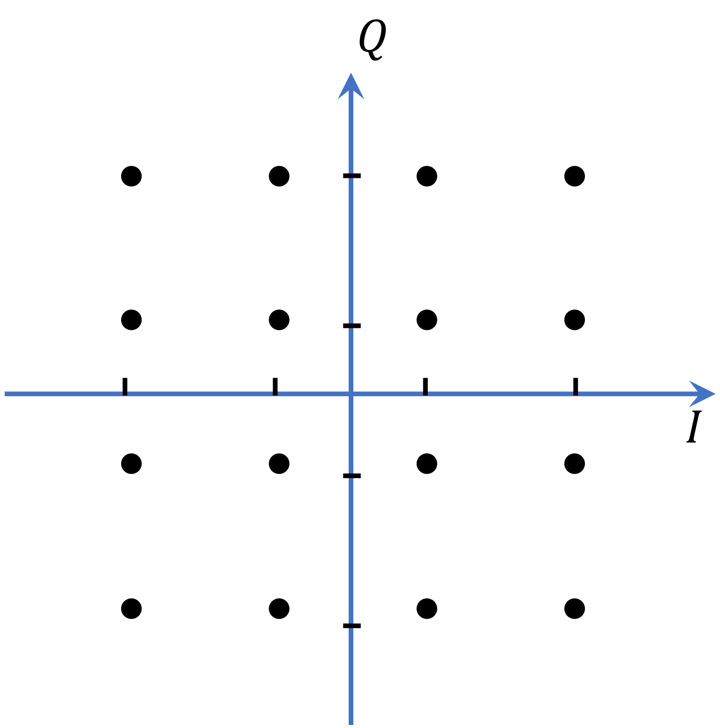
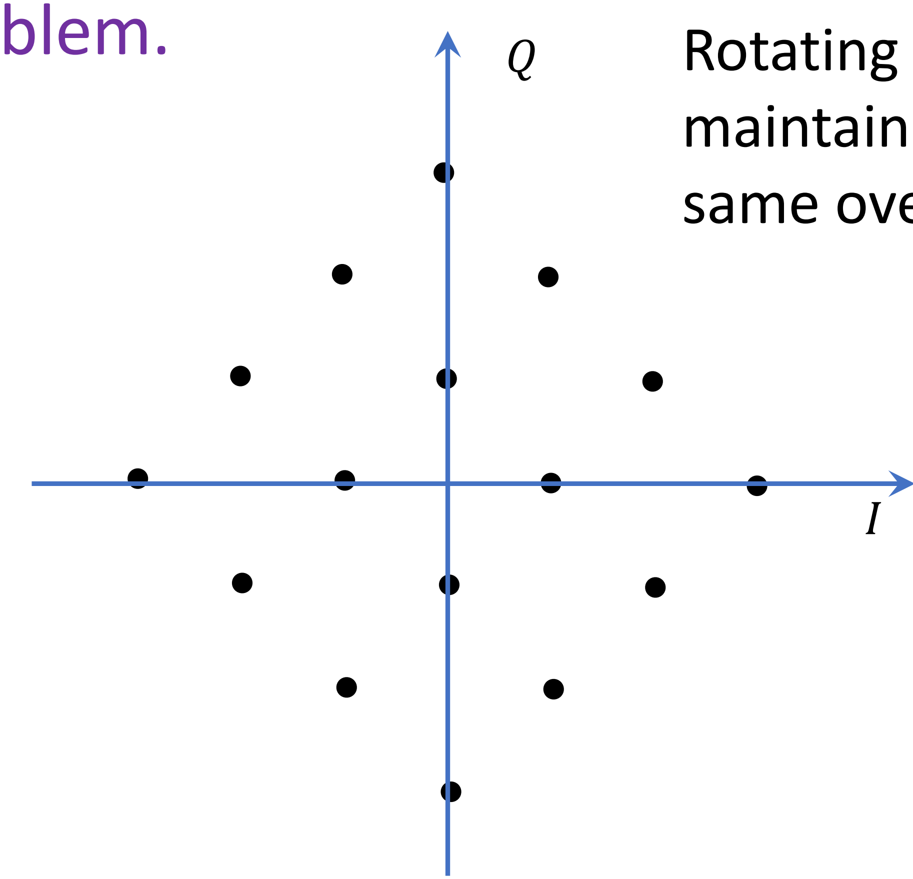




Rotating the constellation maintains same d_{min} for same overall power

QAM Is Popular!

- WiFi

  802.11n: BPSK, QPSK, 16-QAM, 64-QAM

  802.11ac: BPSK, QPSK, 16-QAM, 64-QAM, 256-QAM

  802.11ax (2019): BPSK, QPSK, 16-QAM, 64-QAM, 256-QAM, 1024-QAM

- LTE: QPSK, 16-QAM, 64-QAM

- Digital TV:

  DVB-C: 16-QAM, 64-QAM, 256-QAM

  DVB-C2: 16-QAM, 64-QAM, 256-QAM, 1024-QAM, 4096-QAM

  Next Gen. to include 16364-QAM & 65536-QAM

- Ethernet, Phone lines, Power lines…: 1024-QAM, 4096-QAM

- ADSL: 32768-QAM

65536-QAM

- Number of Constellation Points: $M = 65536$
- Bits per symbol: $\log_2 M = 16$
- QAM Grid: 256×256
- Need 8 bits to represent I and 8 bits to represent Q

What is the problem?

ADCs need to have a large
dynamic range!

Quantization noise kicks in!

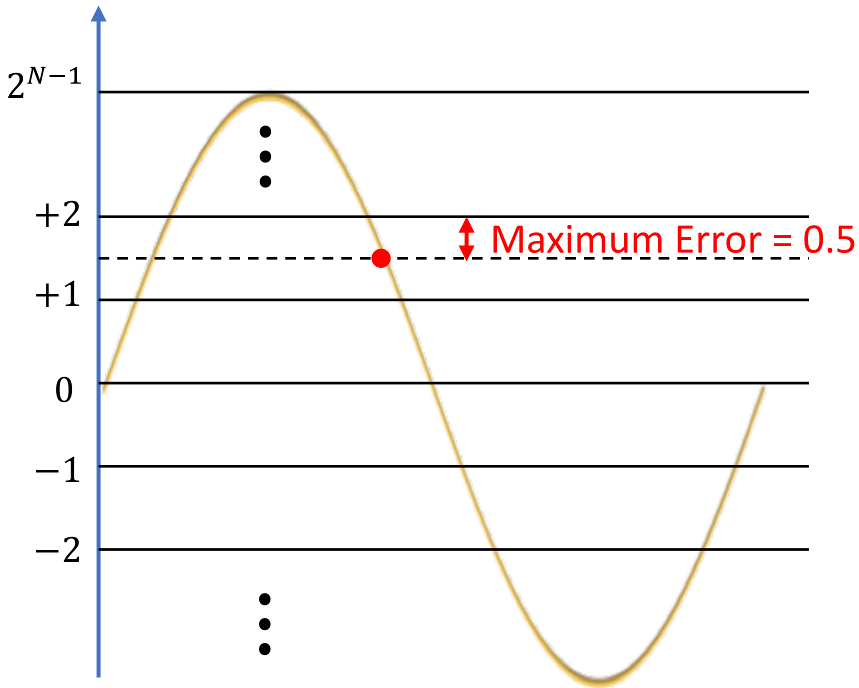
65536-QAM

- Number of Constellation Points: $M = 65536$
- Bits per symbol: $\log_2 M = 16$
- QAM Grid: 256×256
- Need 8 bits to represent I and 8 bits to represent Q
- ADC with K bits typically supports $N < K$ bit samples

ENOB: Effective Number of Bits $N < K$

Quantization Noise

- Consider N bit quantization



$$SNR_{quant.} = \frac{(Signal\ Amplitude)^2}{(Quantization\ Noise)^2}$$

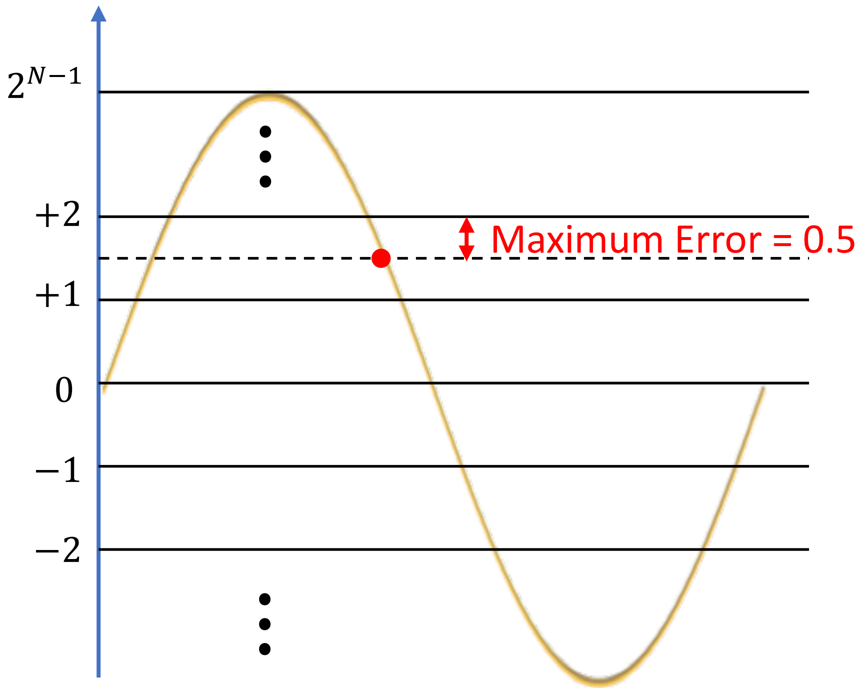
$$= \frac{(2^{N-1})^2}{(0.5)^2} = \frac{2^{2N-2}}{2^{-2}} = 2^{2N}$$

$$SNR_{quant.}(\mathrm{dB}) = 10\log_{10} 2^{2N} = 20N\log_{10} 2 = 6.02N$$

$$\approx 6 \text{ dB per bit}$$

Quantization Noise

- Consider N bit quantization



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$$SNR_{quant.}(\text{dB}) = 10\log_{10} 2^{2N} = 20N\log_{10} 2 = 6.02N$$

$\approx$ 6 dB per bit

Need: $SNR_{quant.} > SNR_{thermal}$

65536-QAM: $BER = \frac{1}{4}Q\left(\sqrt{\frac{3\ SNR}{65535}}\right)$ ➡ Need: $SNR > 52$ dB to achieve $BER < 10^{-3}$

➡ Need: $N > 9$ bits

65536-QAM

- Number of Constellation Points: $M = 65536$
- Bits per symbol: $\log_2 M = 16$
- QAM Grid: 256×256
- Need 8 bits to represent I and 8 bits to represent Q
- ADC with K bits typically supports $N < K$ bit samples

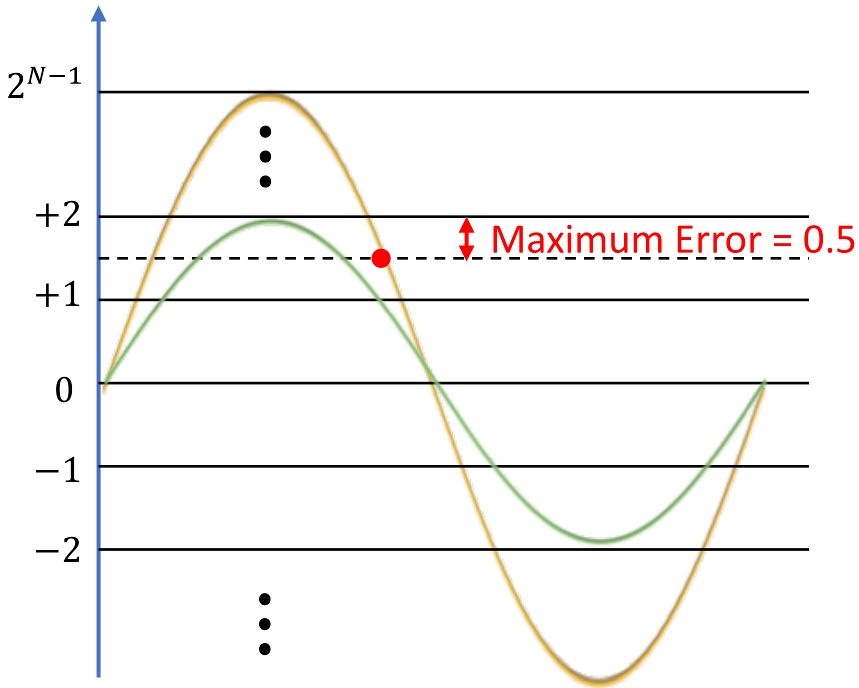
ENOB: Effective Number of Bits $N < K$

- Need very high SNR to achieve reasonable BER.
- Need to minimize quantization noise: $N \geq 12$

Need at least a 14 bit or 16 bit ADC

Quantization Noise

- Consider N bit quantization



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$$= \frac{(2^{N-1})^2}{(0.5)^2} = \frac{2^{2N-2}}{2^{-2}} = 2^{2N}$$

$$SNR_{quant.}(\text{dB}) = 10\log_{10} 2^{2N} = 20N\log_{10} 2 = 6.02N$$

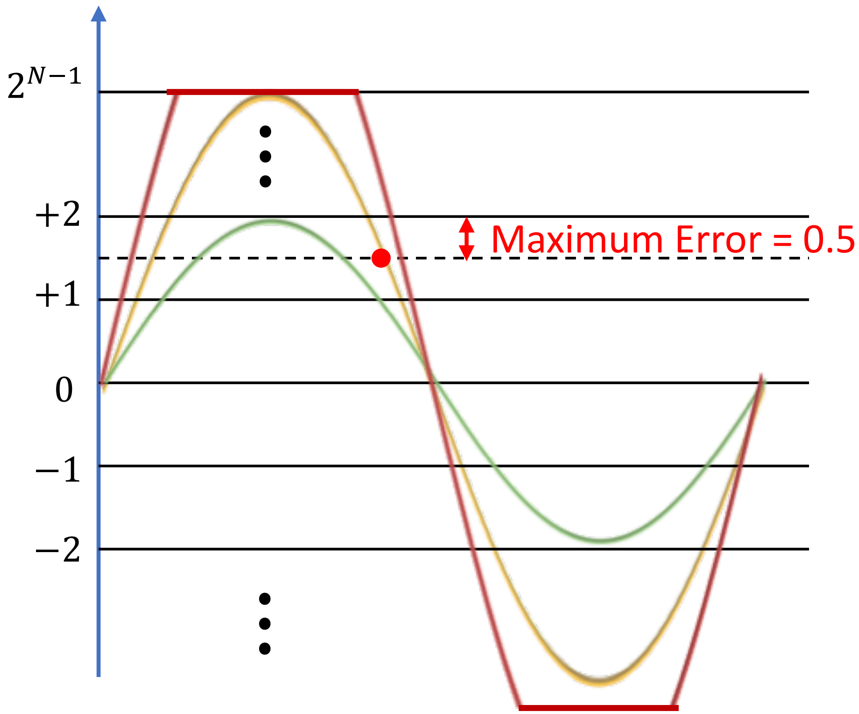
$\approx$ 6 dB per bit

Assumes signal amplitude fills maximum quantization level.

Signals arrives too attenuated → fill smaller quantization levels → Low $SNR_{quant.}$

Quantization Noise

- Consider N bit quantization



$$SNR_{quant.} = \frac{(Signal\ Amplitude)^2}{(Quantization\ Noise)^2}$$

$$= \frac{(2^{N-1})^2}{(0.5)^2} = \frac{2^{2N-2}}{2^{-2}} = 2^{2N}$$

$$SNR_{quant.}(\text{dB}) = 10\log_{10} 2^{2N} = 20N\log_{10} 2 = 6.02N$$

$\approx$ 6 dB per bit

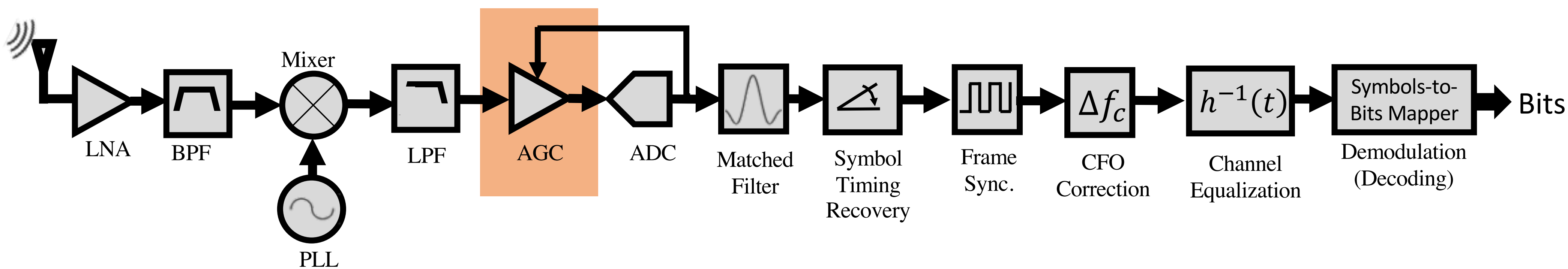
Assumes signal amplitude fills maximum quantization level.

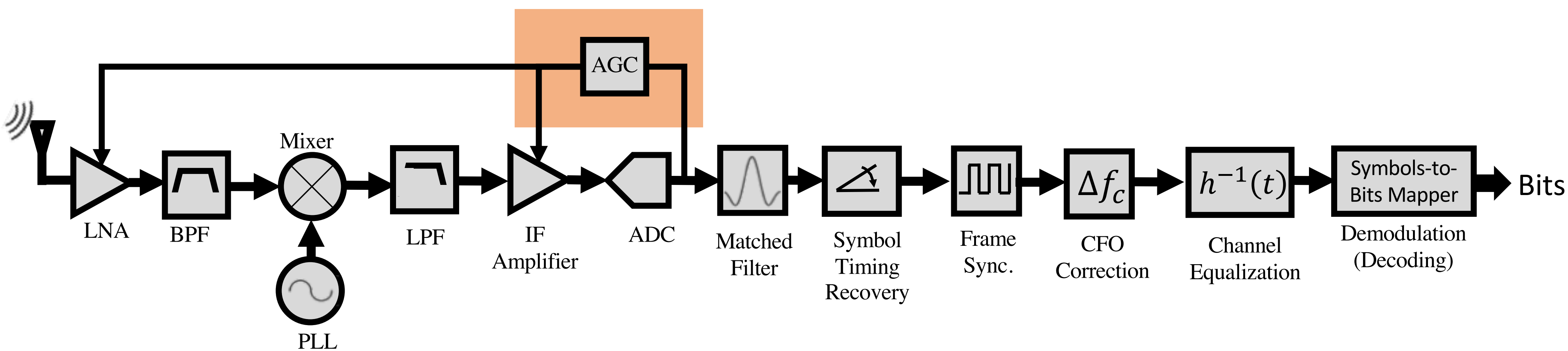
Signals arrives too attenuated → fill smaller quantization levels → Low $SNR_{quant.}$

Signals arrives too amplified → Clipping

AGC: Automatic Gain Control

- Consists of Variable Gain Amplifier and Control Circuit
- Adjust the gain to minimize quantization noise & avoid clipping.
- Receiver Circuit:





Definitions & Variables

- $x(t)$: Transmitted Signal
- $v(t)$: Additive Gaussian Noise
- $y(t)$: Received Signal
- h: Single Tap Channel Coefficient.
- N_o': Gaussian Noise Energy
- N_0: Gaussian Noise Scaled by Channel
- $\mathrm{E}[\quad]$: Expectation
- SNR: Signal to Noise Ratio
- E_s: Energy per Symbol
- E_b: Energy per Bit
- $P(\quad)$: Probability Distribution
- $P(\ |\ )$: Conditional Probability Distribution
- σ: Std. Dev. of Gaussian Noise.
- b: Transmitted Bit
- $\hat{b}$: Decoded Bit
- BER: Bit Error Rate
- SER: Symbol Error Rate
- P_b: Probability of Bit Error
- P_S: Probability of Symbol Error
- $Q(\quad)$: Q Function
- M: Number of Constellation Points
- d_{min}: Minimum Distance between constellation points.
- $\#nn$: Number of nearest neighbors in the constellation.
- K: Number of ADC bits
- N: Effective Number of ADC bits
- $\alpha_M, \beta_M, \hat{\alpha}_M, \hat{\beta}_M$: Parameters of BER Q function