ECE 463: Digital Communications Lab.

Lecture 5: Frame Synchronization & Modulation Part II

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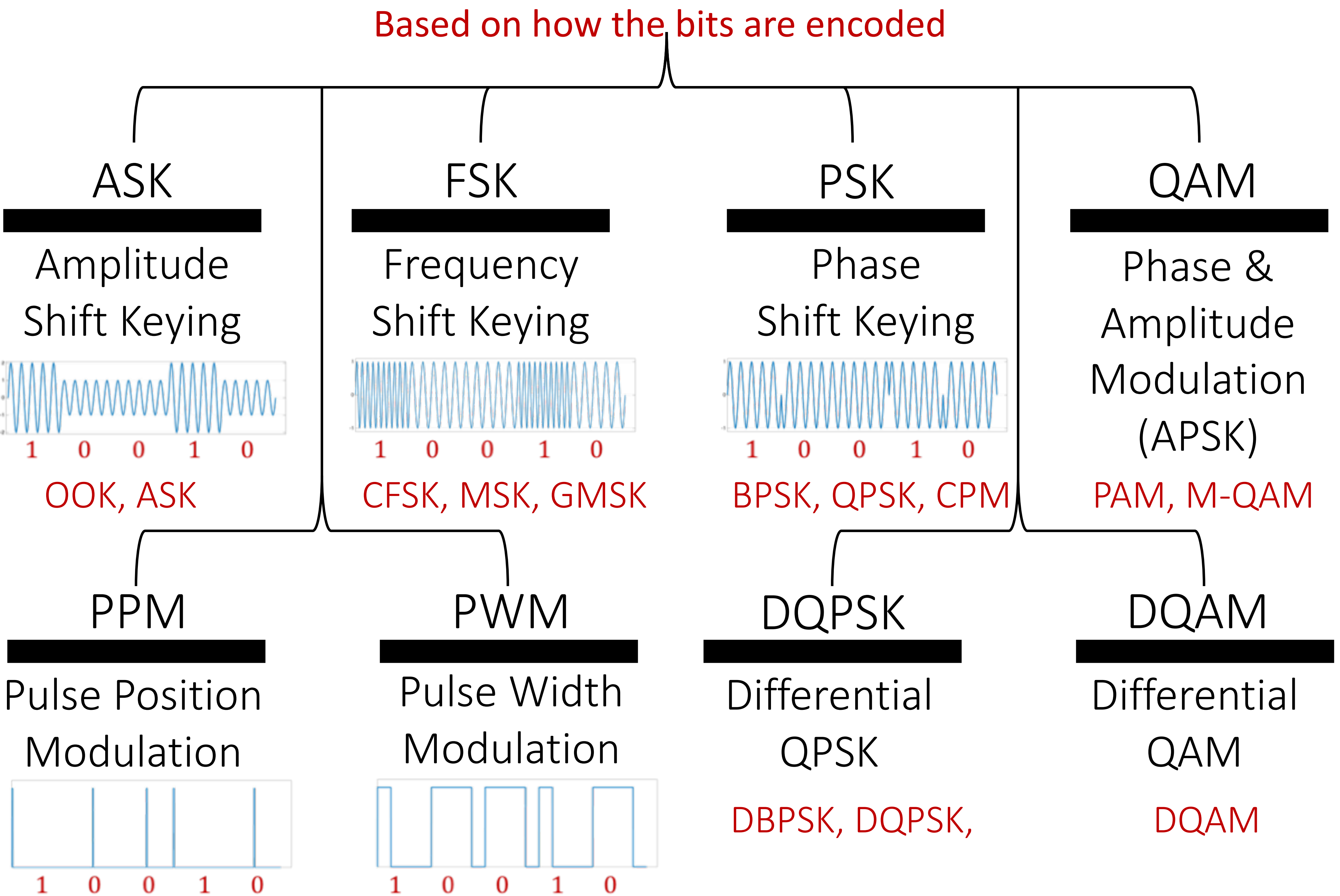
Previous Lecture:

- ✓ Channel Distortion
- ✓ Non-Coherent vs. Coherent Modulation
- ✓ DPBSK

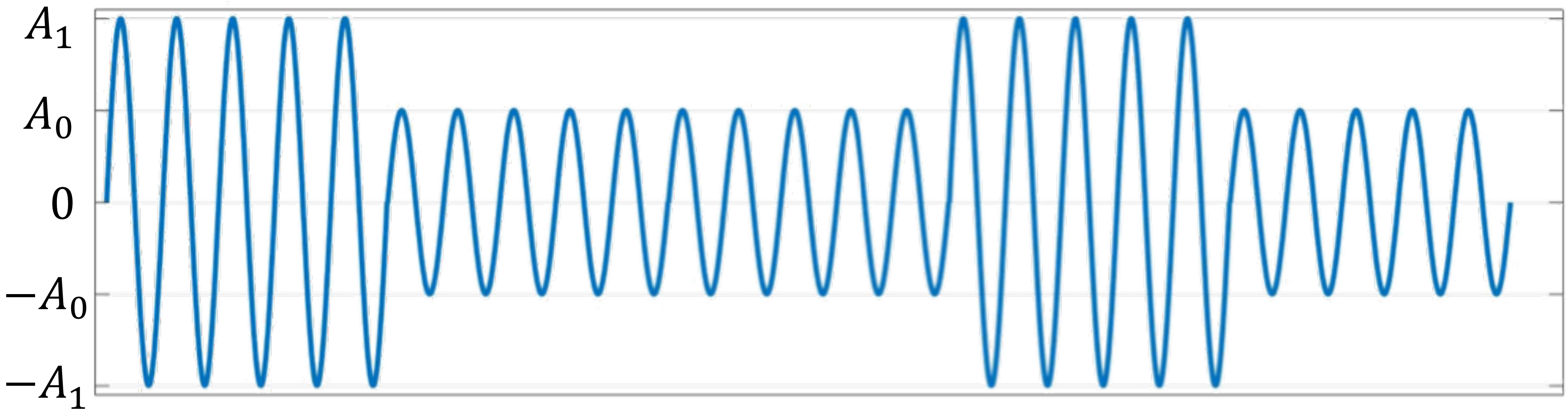
This Lecture:

- ❑ ASK Modulation
- ❑ FSK Modulation (Coherent & Non-Coherent)
- ❑ Frame Synchronization

Modulation


Based on how the bits are encoded
ASK
Amplitude Shift Keying
1 0 0 1 0
OOK, ASK
FSK
Frequency Shift Keying
1 0 0 1 0
CFSK, MSK, GMSK
PSK
Phase Shift Keying
1 0 0 1 0
BPSK, QPSK, CPM
QAM
Phase & Amplitude Modulation (APSK)
PAM, M-QAM
PPM
Pulse Position Modulation
1 0 0 1 0
PWM
Pulse Width Modulation
1 0 0 1 0
DQPSK
Differential QPSK
DBPSK, DQPSK,
DQAM
Differential QAM
DQAM

ASK: Amplitude Shift Keying



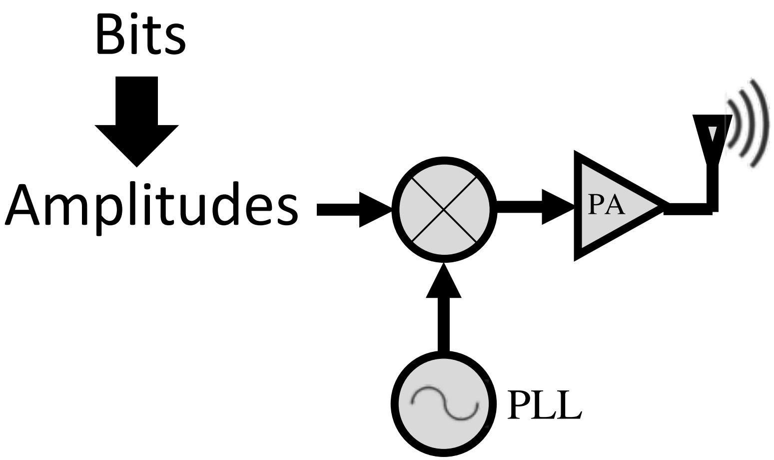
Binary ASK:

$$1 \rightarrow A_1 \cos 2\pi f_c t$$
$$0 \rightarrow A_0 \cos 2\pi f_c t$$

Special case $A_0 = 0$ → ON-OFF Keying

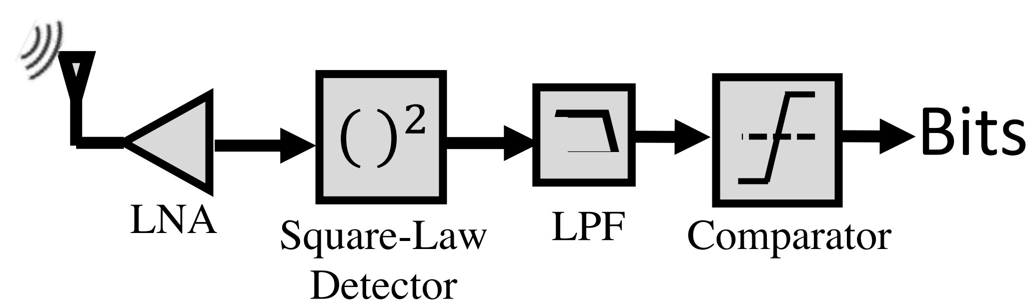
Binary ASK

ASK TX



$$x(t) = A_i \cos 2\pi f_c t$$

ASK RX



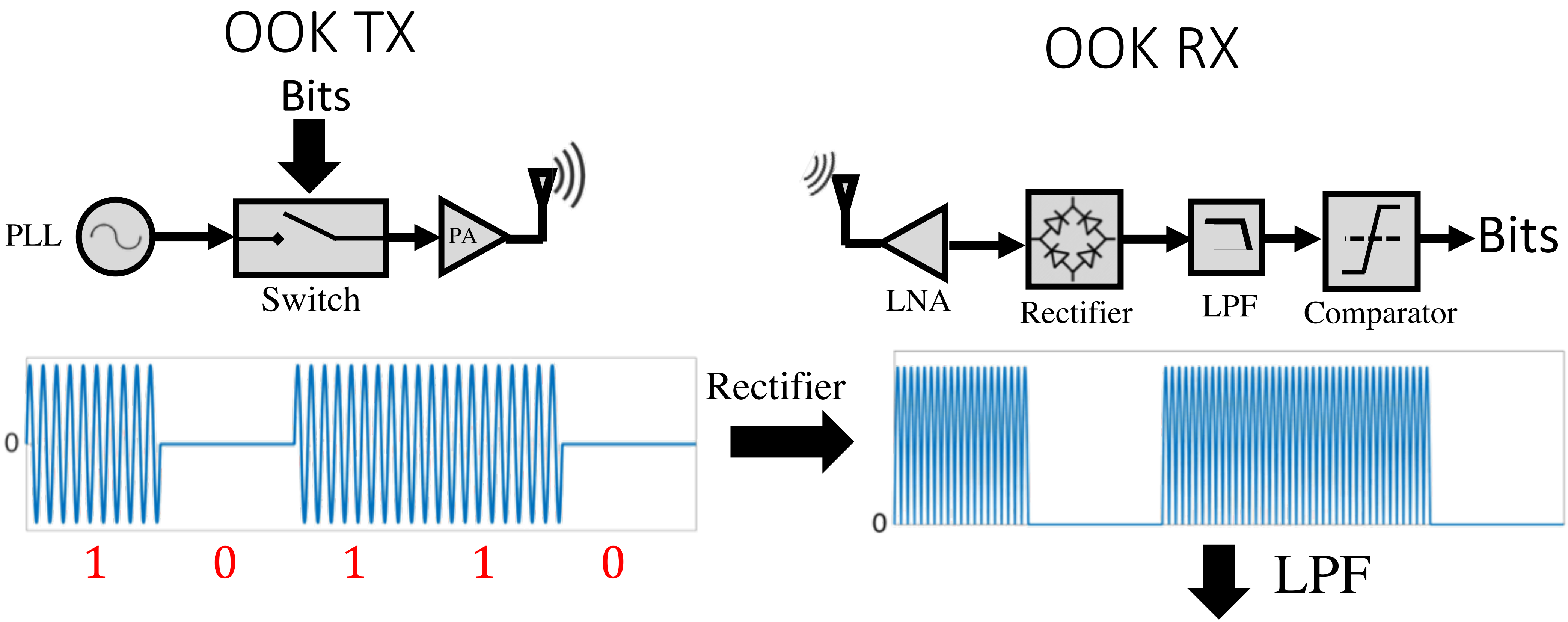
$$(A_i)^2 \cos^2 2\pi f_c t$$

$$= \frac{(A_i)^2}{2} + \frac{(A_i)^2}{2} \cos 2\pi(2f_c)t$$

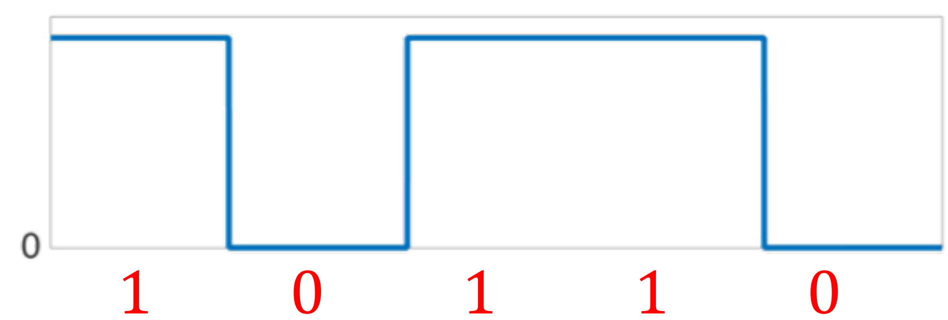
Low Pass Filter

$$= \frac{(A_i)^2}{2}$$

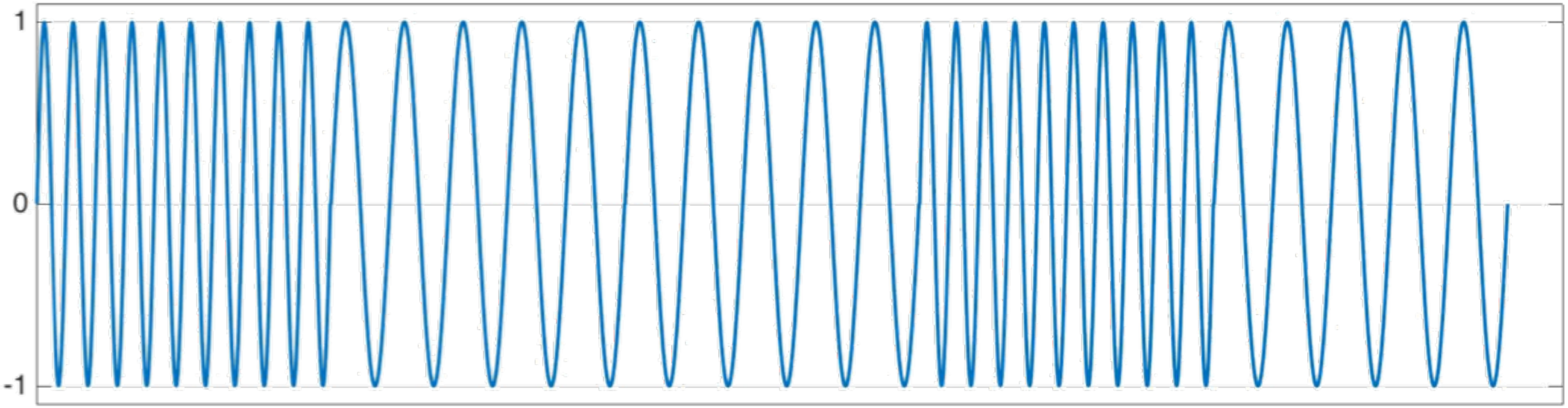
On-Off Keying



- Simple TX/RX Architecture
- Used for Low Power IoT
- Used for Optical Fiber



FSK: Frequency Shift Keying

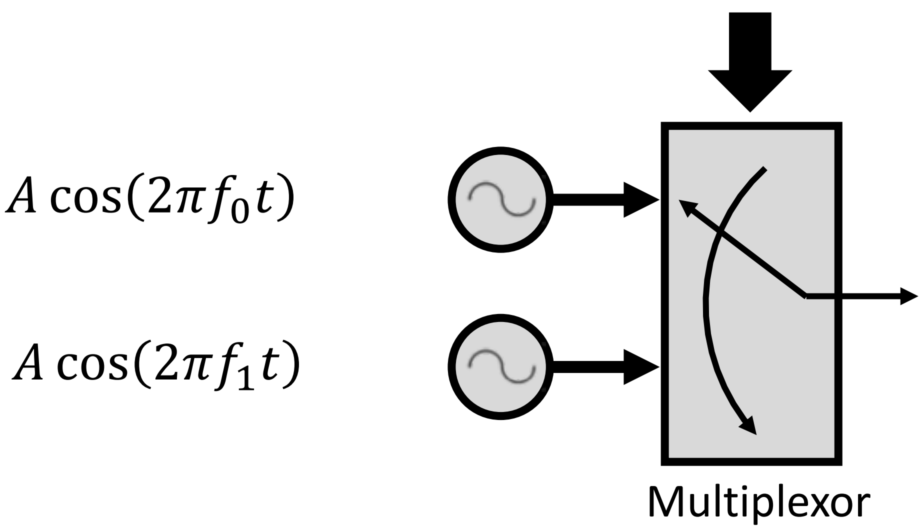


Binary FSK:

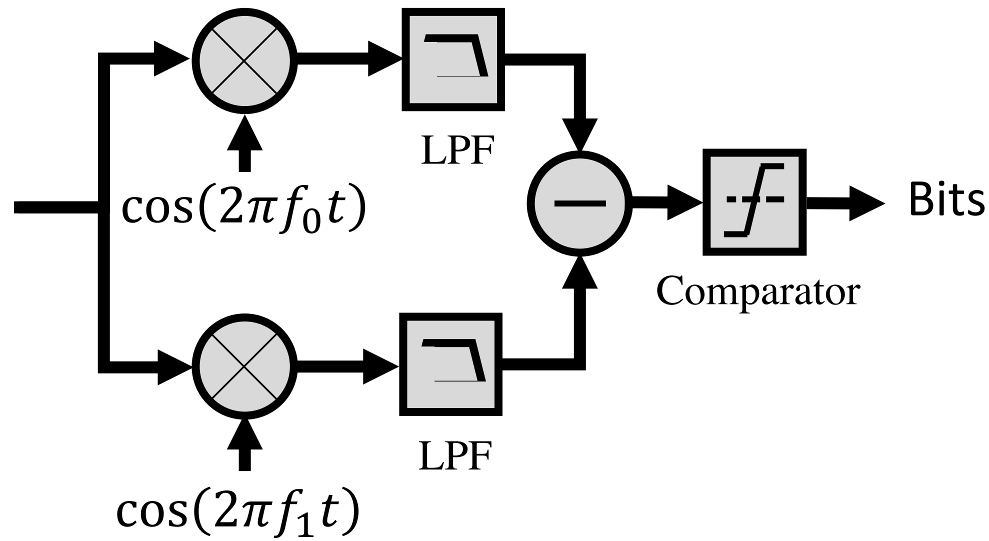
$$1 \rightarrow A\cos 2\pi f_1 t$$
$$0 \rightarrow A\cos 2\pi f_0 t$$

FSK: Frequency Shift Keying

BFSK TX



BFSK RX

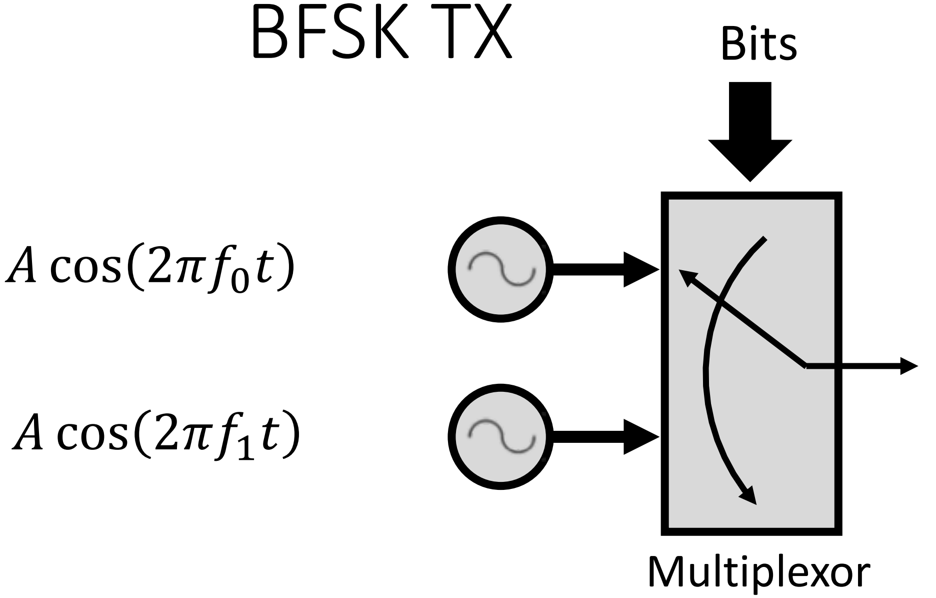


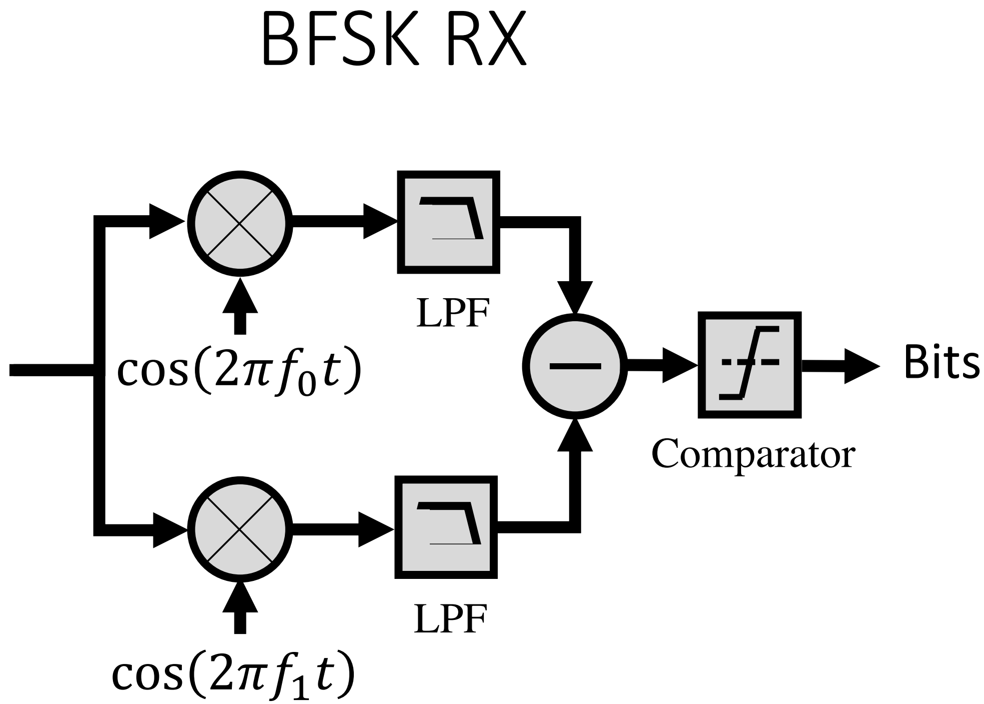
Assume bit is 0, we send $A\cos(2\pi f_0 t)$:

$$A\cos(2\pi f_0 t)\cos(2\pi f_0 t) = \frac{A}{2} + \frac{A}{2}\cos(2\pi 2 f_0 t)$$

$$A\cos(2\pi f_0 t)\cos(2\pi f_1 t) = \frac{A}{2}\cos(2\pi(f_0 - f_1)t) + \frac{A}{2}\cos(2\pi(f_0 + f_1)t)$$

FSK: Frequency Shift Keying





Assume bit is 0, we send $A\cos(2\pi f_0 t)$:

$$A\cos(2\pi f_0 t)\cos(2\pi f_0 t) = \frac{A}{2} + \frac{A}{2}\cos(2\pi 2 f_0 t) = \frac{A}{2}$$

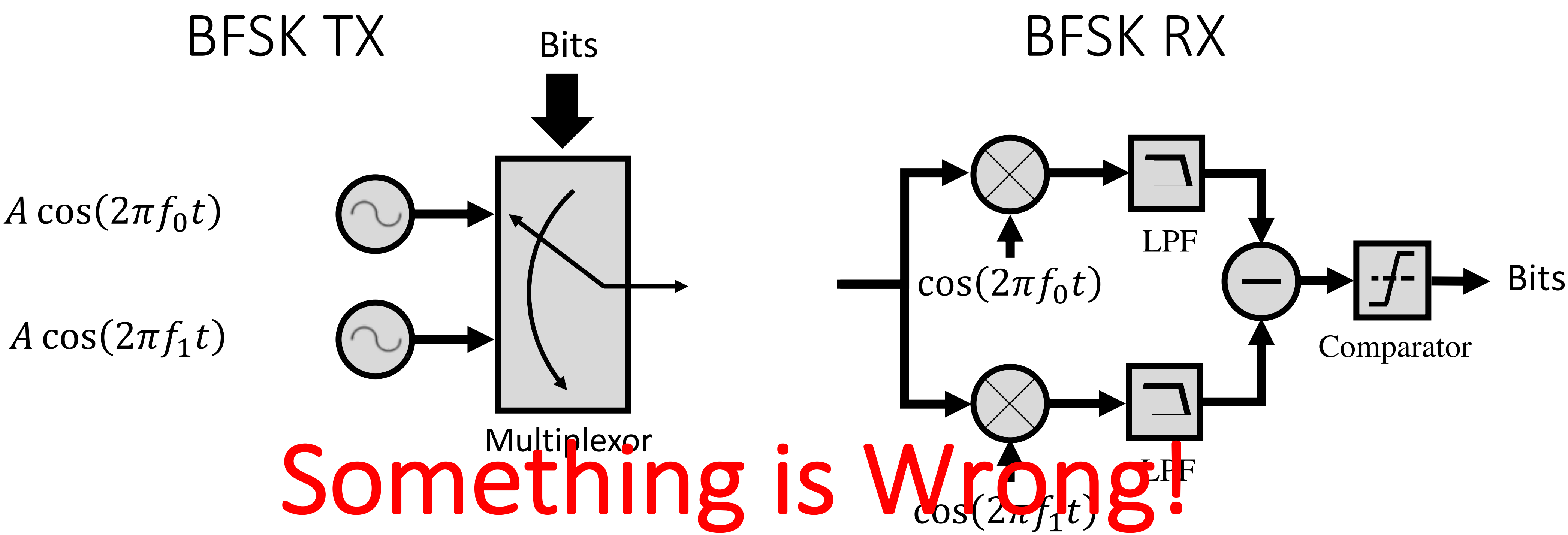
Low Pass Filter

$$A\cos(2\pi f_0 t)\cos(2\pi f_1 t) = \frac{A}{2}\cos(2\pi(f_0 - f_1)t) + \frac{A}{2}\cos(2\pi(f_0 + f_1)t) = 0$$

Low Pass Filter

Low Pass Filter

FSK: Frequency Shift Keying



Something is Wrong!

Assume bit is 0, we send $A\cos(2\pi f_0 t)$:

$$A\cos(2\pi f_0 t)\cos(2\pi f_0 t) = \frac{A}{2} + \frac{A}{2}\cos(2\pi 2 f_0 t) = \frac{A}{2}$$

Low Pass Filter

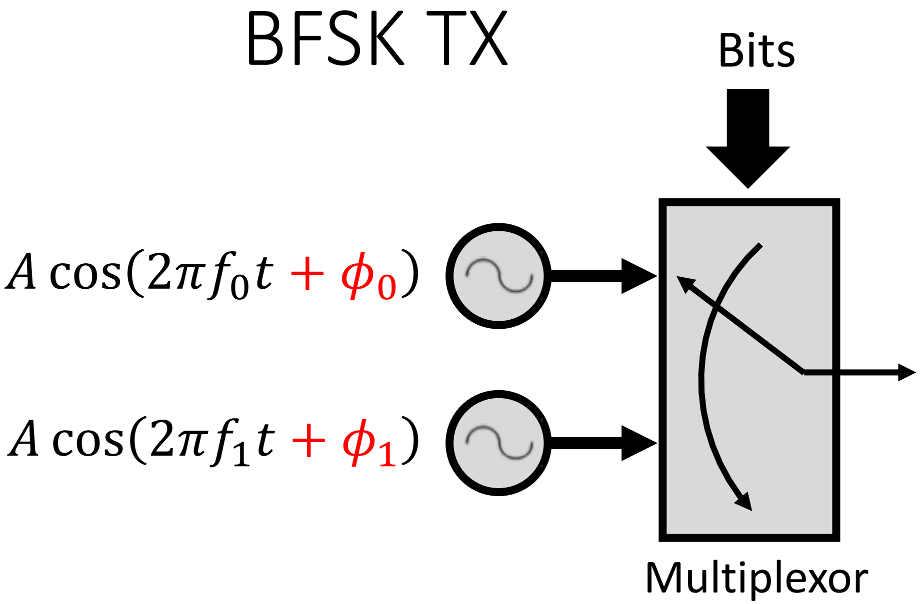
$$A\cos(2\pi f_0 t)\cos(2\pi f_1 t) = \frac{A}{2}\cos(2\pi(f_0 - f_1)t) + \frac{A}{2}\cos(2\pi(f_0 + f_1)t) = 0$$

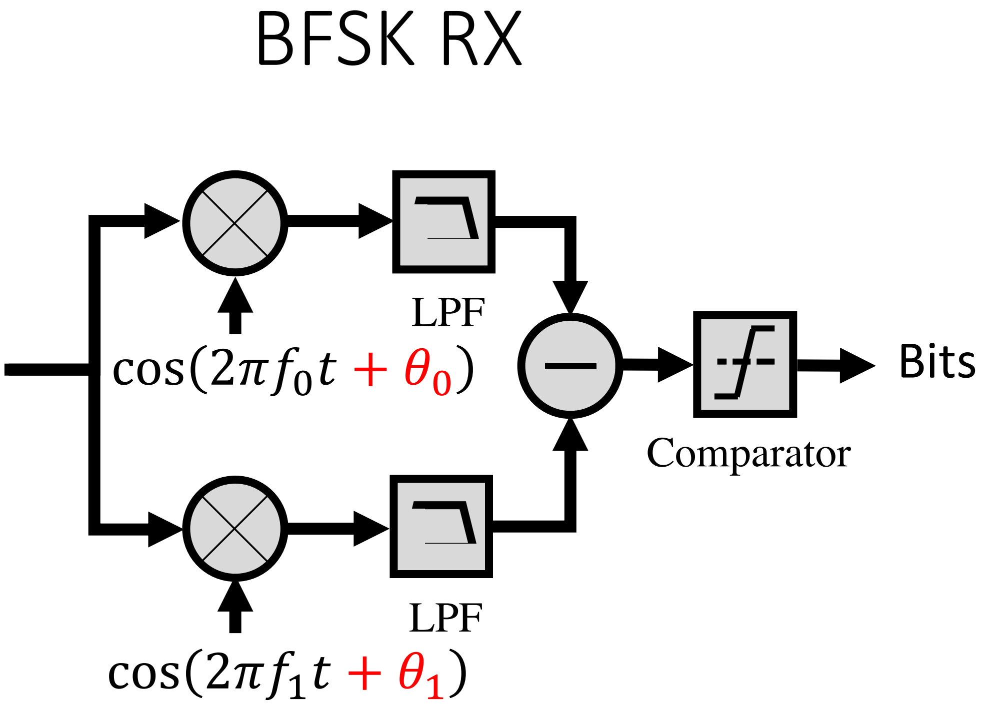
Low Pass Filter

Low Pass Filter

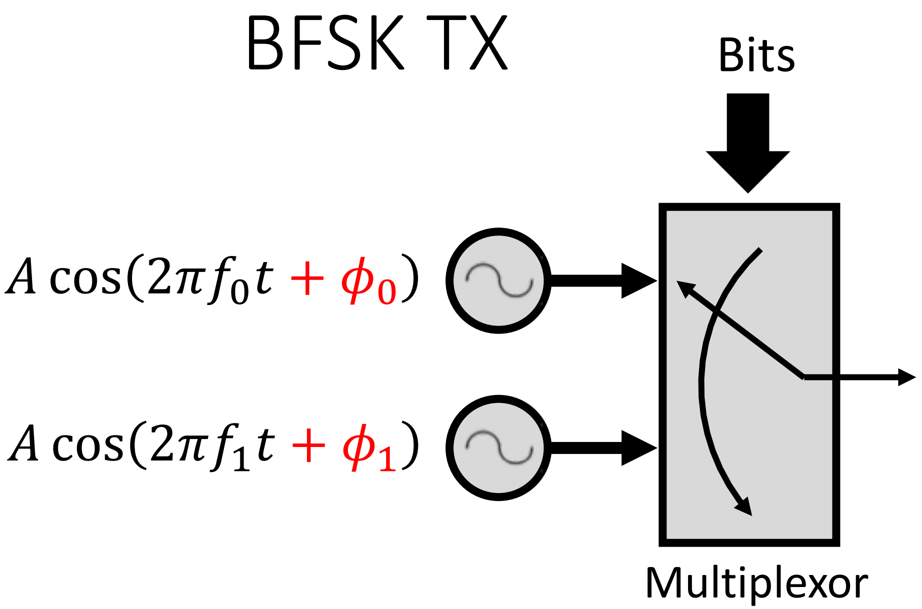
Assumed TX and RX are Coherent!

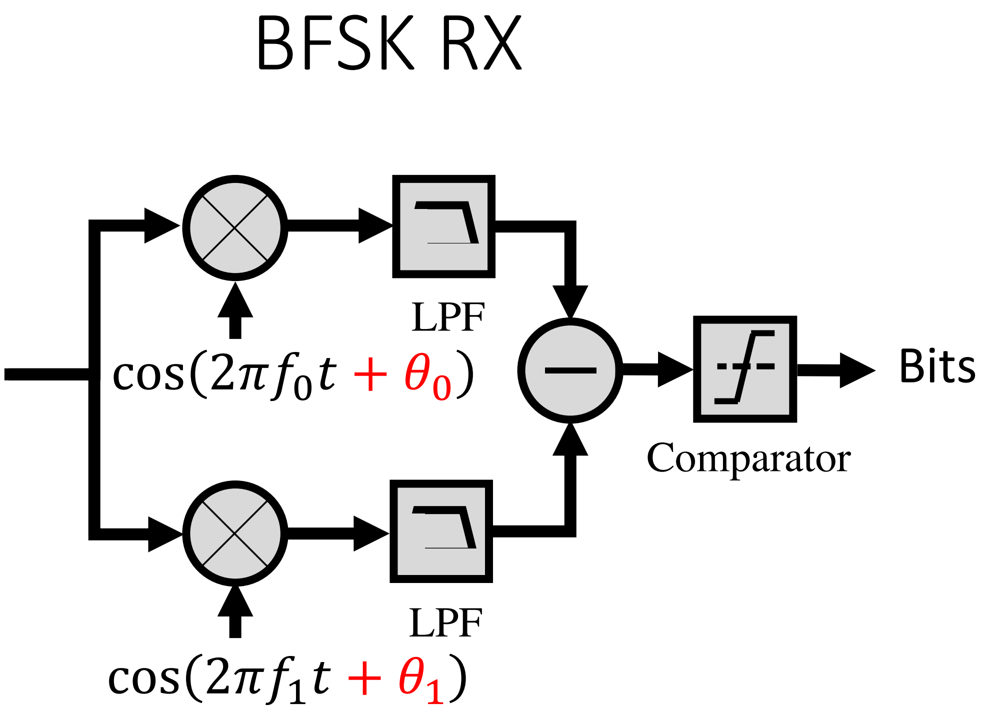
FSK: Frequency Shift Keying





FSK: Frequency Shift Keying





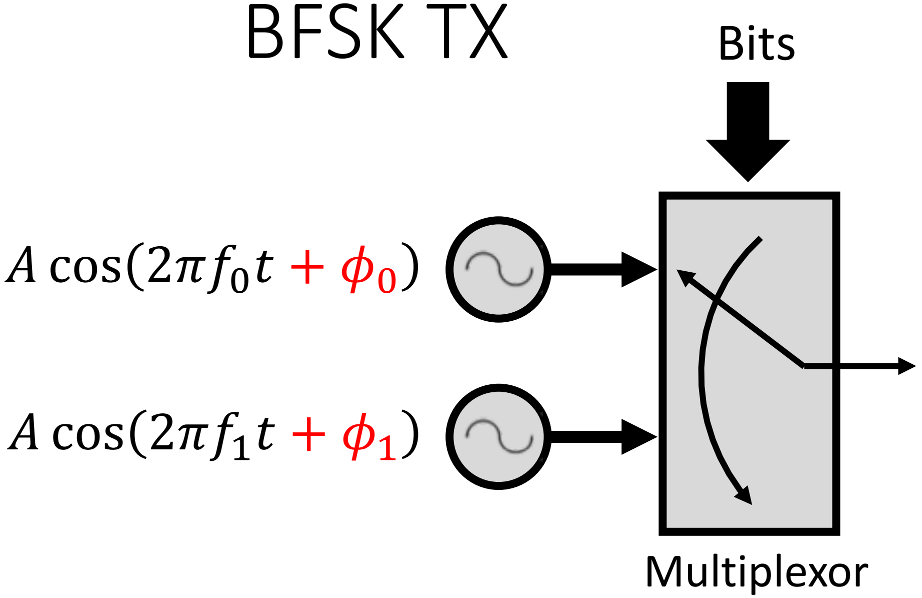
Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0)$$

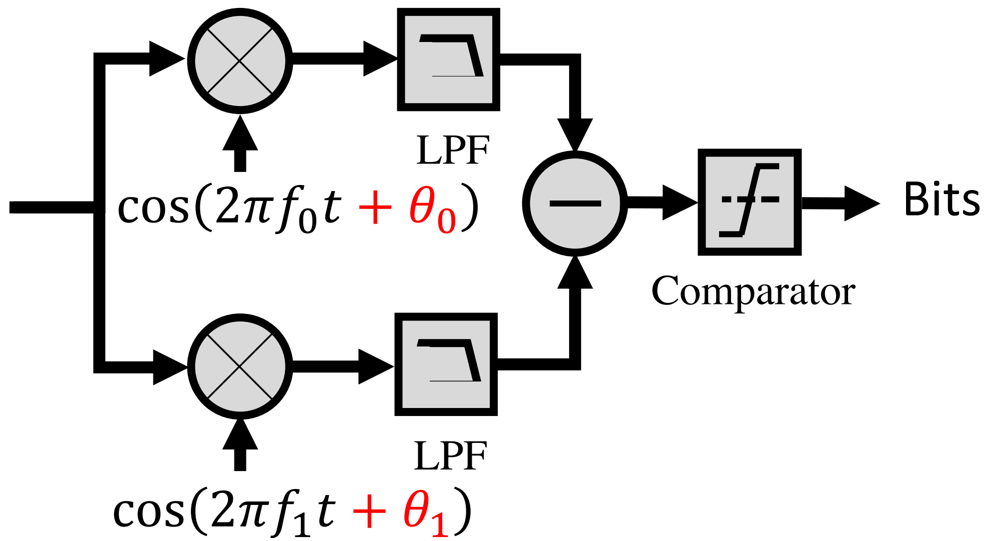
$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1)$$

FSK: Frequency Shift Keying

BFSK TX



BFSK RX



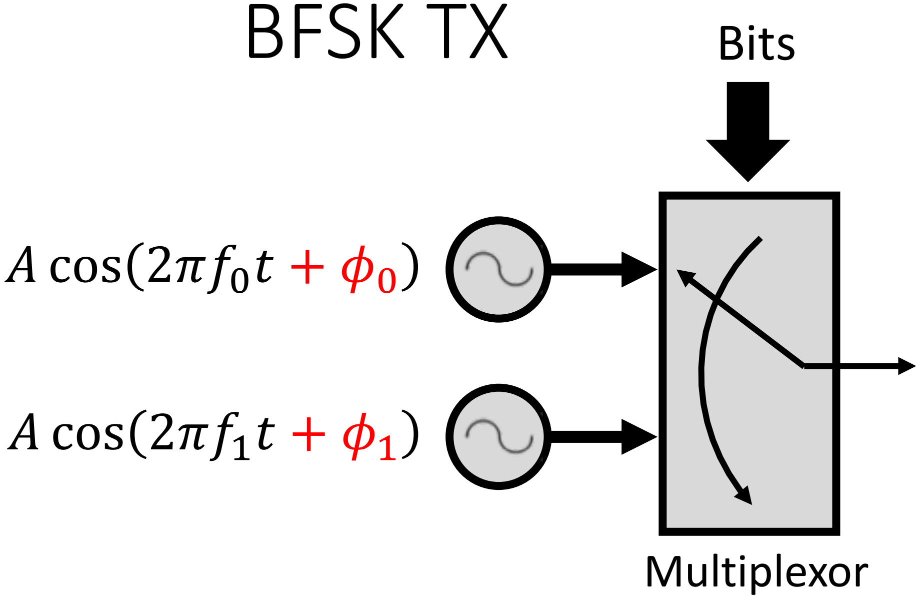
Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) = \frac{A}{2}\cos(\phi_0 - \theta_0) + \frac{A}{2}\cos(2\pi 2 f_0 t + \phi_0 + \theta_0)$$

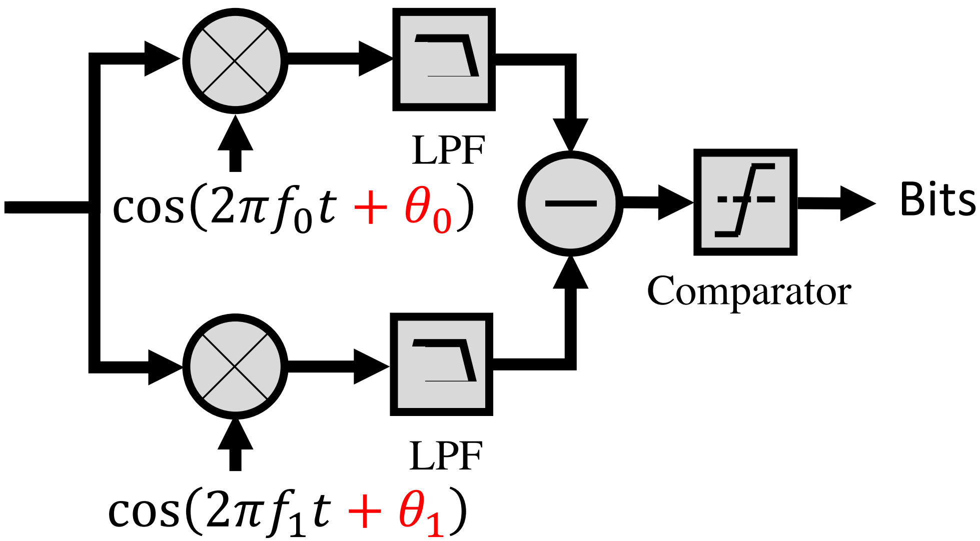
$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1) = \frac{A}{2}\cos(2\pi(f_0 - f_1)t + \phi_0 - \theta_0) + \frac{A}{2}\cos(2\pi(f_0 + f_1)t + \phi_0 + \theta_0)$$

FSK: Frequency Shift Keying

BFSK TX



BFSK RX



Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) = \frac{A}{2}\cos(\phi_0 - \theta_0) + \frac{A}{2}\cos(2\pi 2 f_0 t + \phi_0 + \theta_0)$$

Low Pass Filter

$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1) = \frac{A}{2}\cos(2\pi(f_0 - f_1)t + \phi_0 - \theta_0)$$

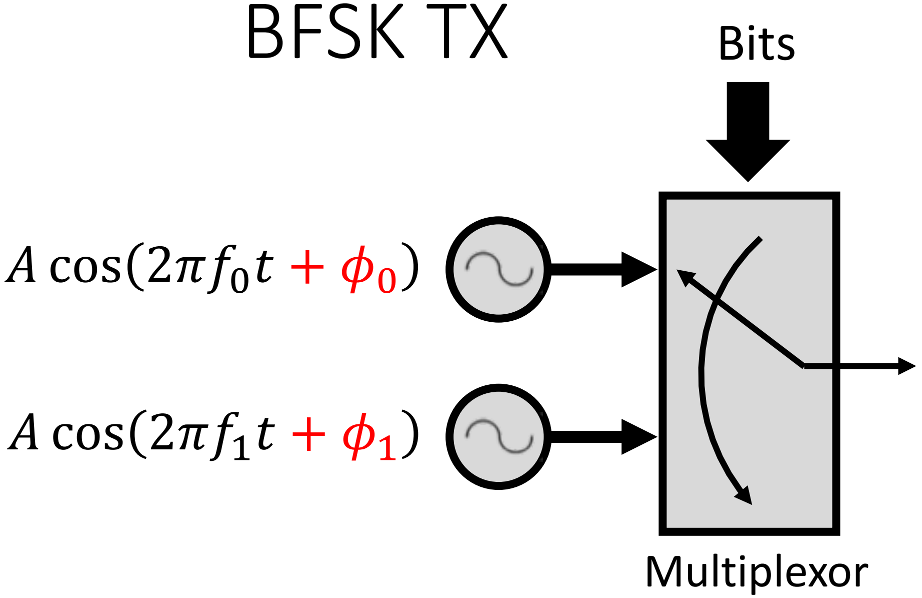
Low Pass Filter

$$+ \frac{A}{2}\cos(2\pi(f_0 + f_1)t + \phi_0 + \theta_0)$$

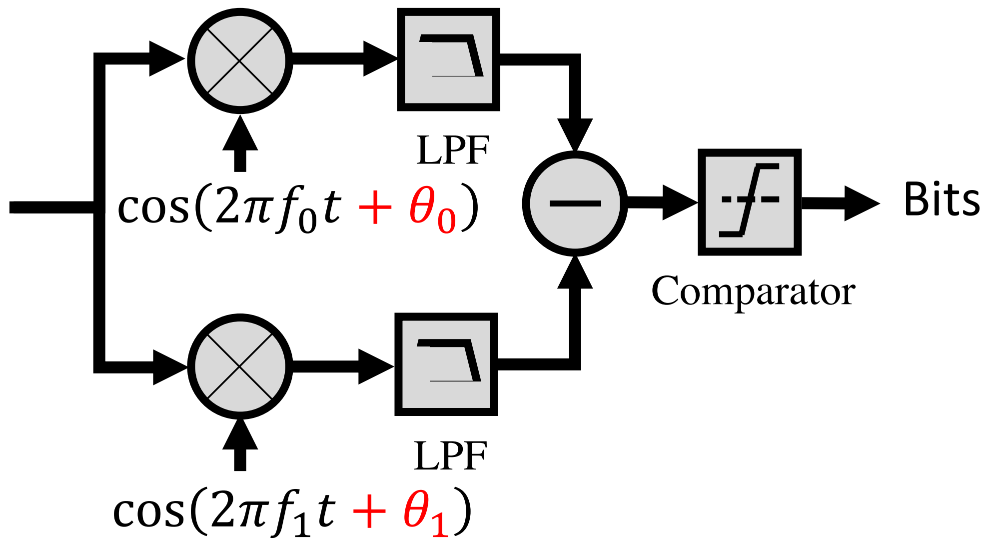
Low Pass Filter

FSK: Frequency Shift Keying

BFSK TX



BFSK RX



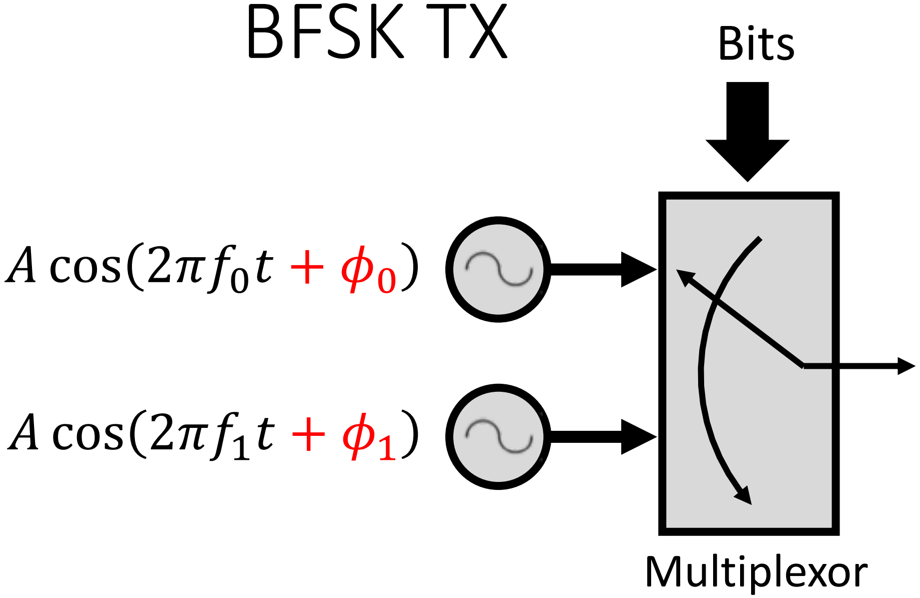
Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) = \frac{A}{2}\cos(\phi_0 - \theta_0)$$

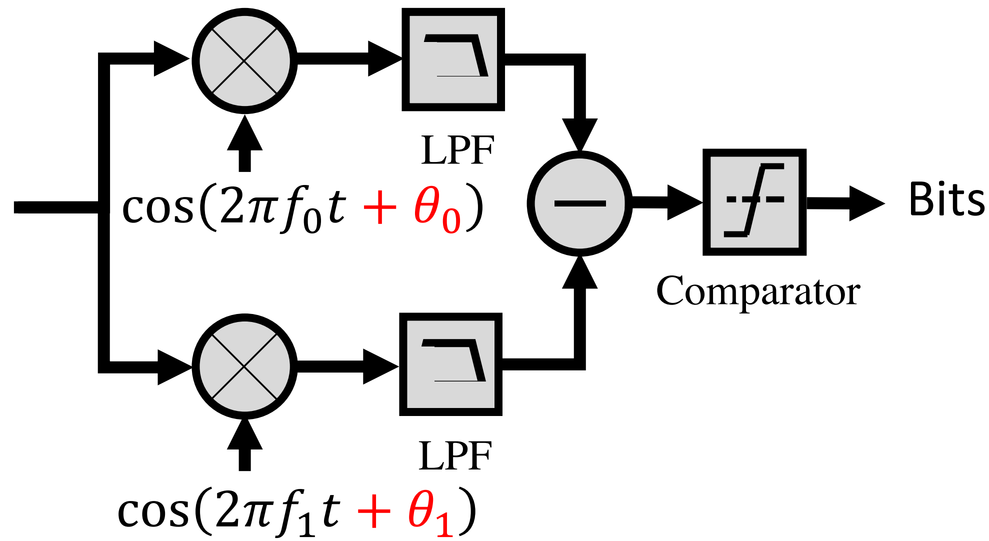
$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1) = 0$$

FSK: Frequency Shift Keying

BFSK TX



BFSK RX



Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) = \frac{A}{2}\cos(\phi_0 - \theta_0) + \text{noise}$$

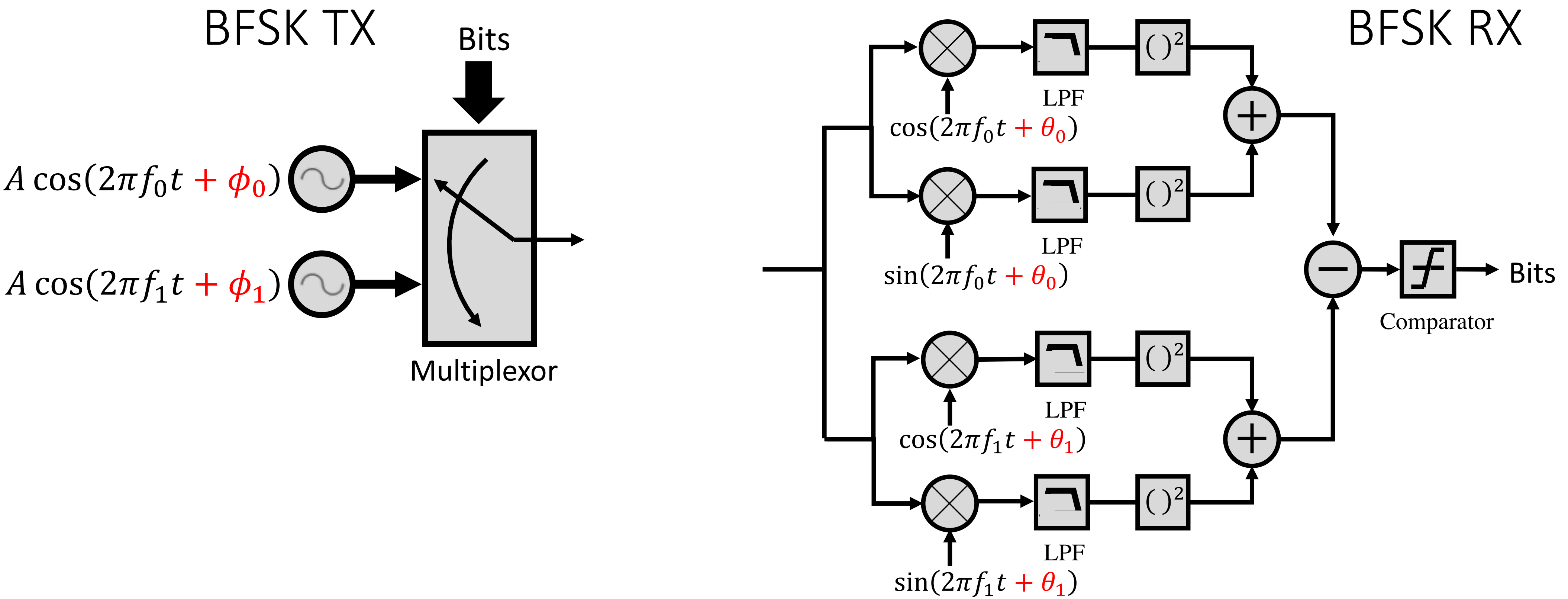
$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1) = 0 + \text{noise}$$

Not necessarily decoded as 0.

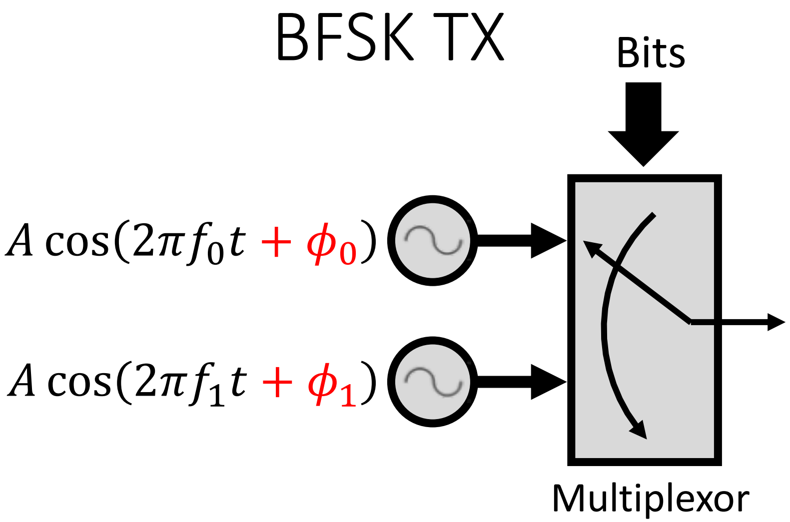
Require $\theta_0 = \phi_0$ and $\theta_1 = \phi_1$, to decode correctly

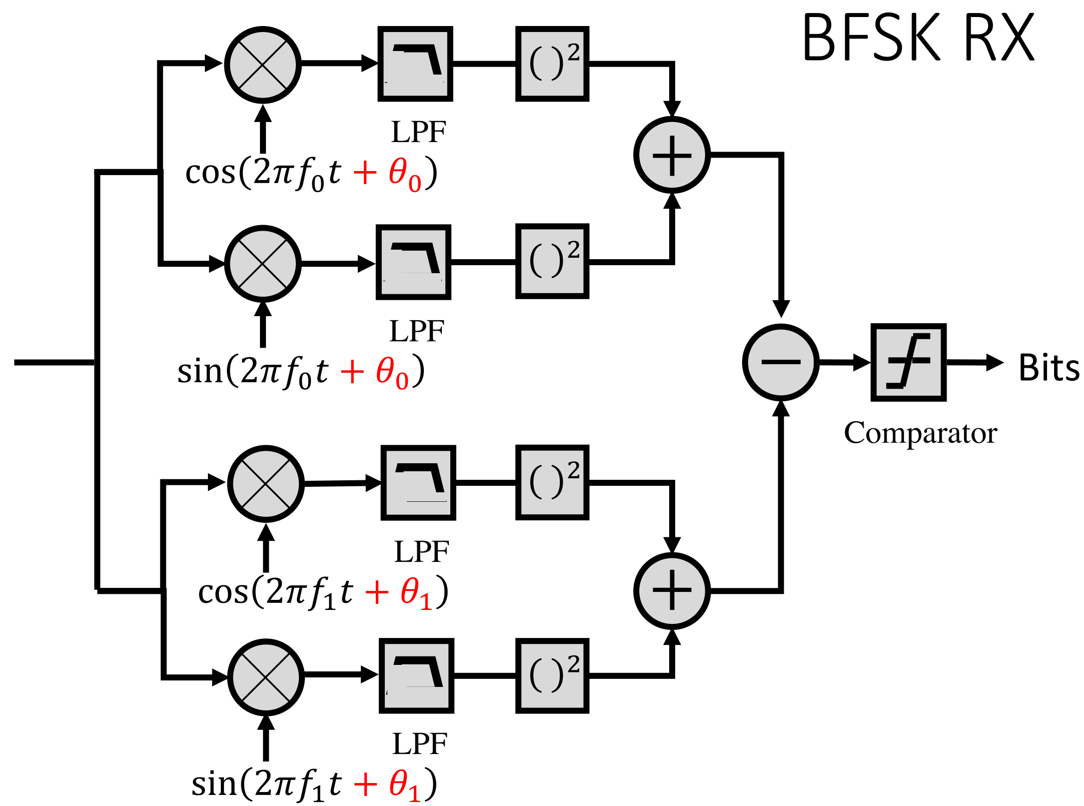
→ Need coherent decoder

Non-Coherent FSK



Non-Coherent FSK





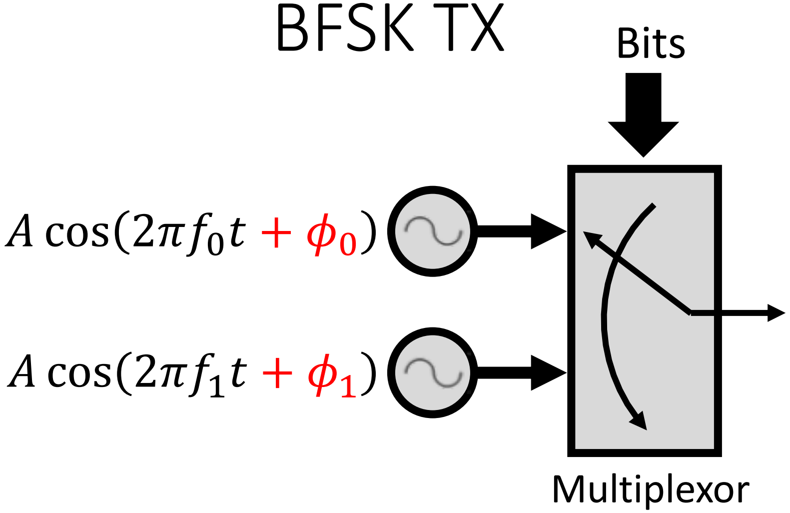
Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0)$

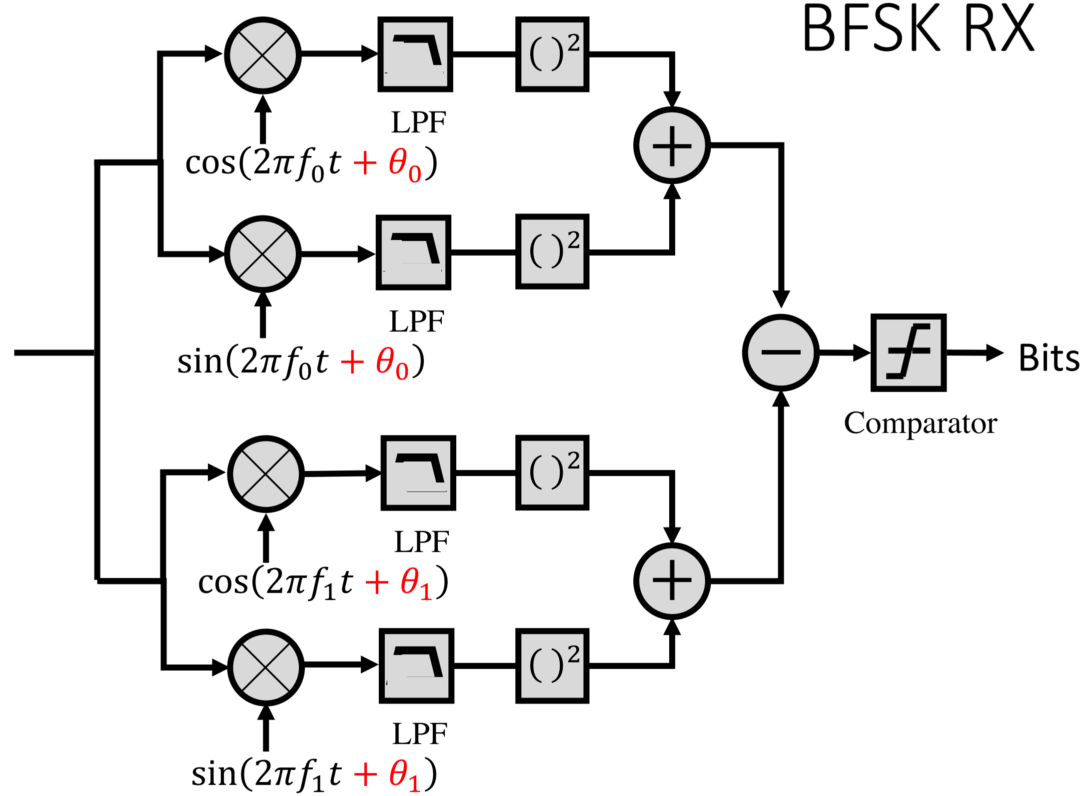
Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0)$

Non-Coherent FSK

BFSK TX



BFSK RX

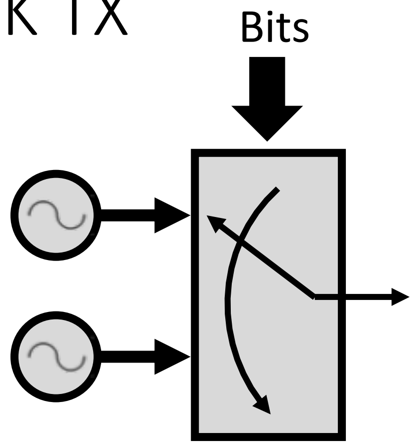


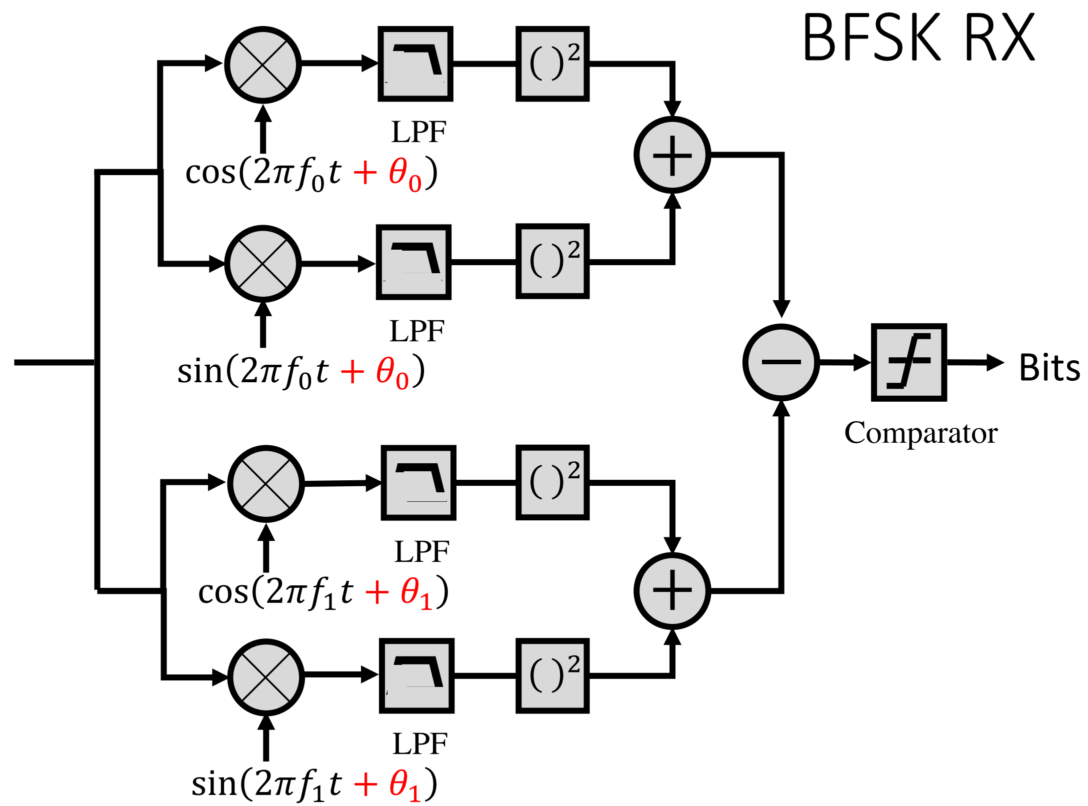
Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) = \frac{A}{2}\cos(\phi_0 - \theta_0) + \frac{A}{2}\cos(2\pi 2 f_0 t + \phi_0 + \theta_0)$

Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) = \frac{A}{2}\sin(\theta_0 - \phi_0) + \frac{A}{2}\sin(2\pi 2 f_0 t + \phi_0 + \theta_0)$

Non-Coherent FSK





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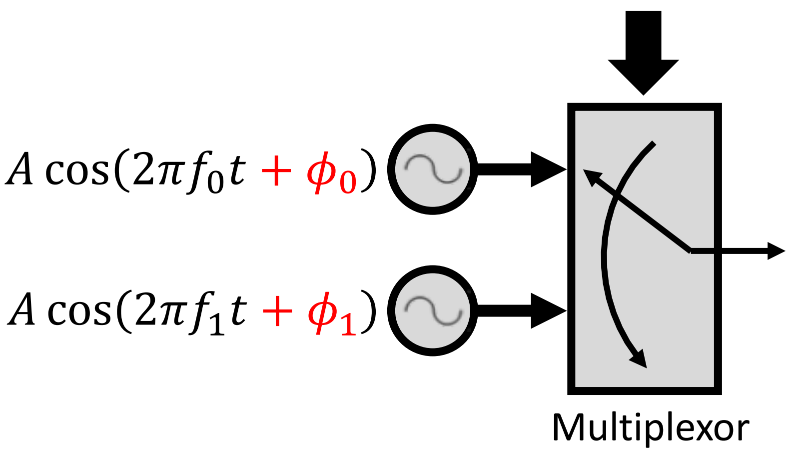
Low Pass Filter

Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) = \frac{A}{2}\sin(\theta_0 - \phi_0) + \frac{A}{2}\sin(2\pi 2 f_0 t + \phi_0 + \theta_0)$

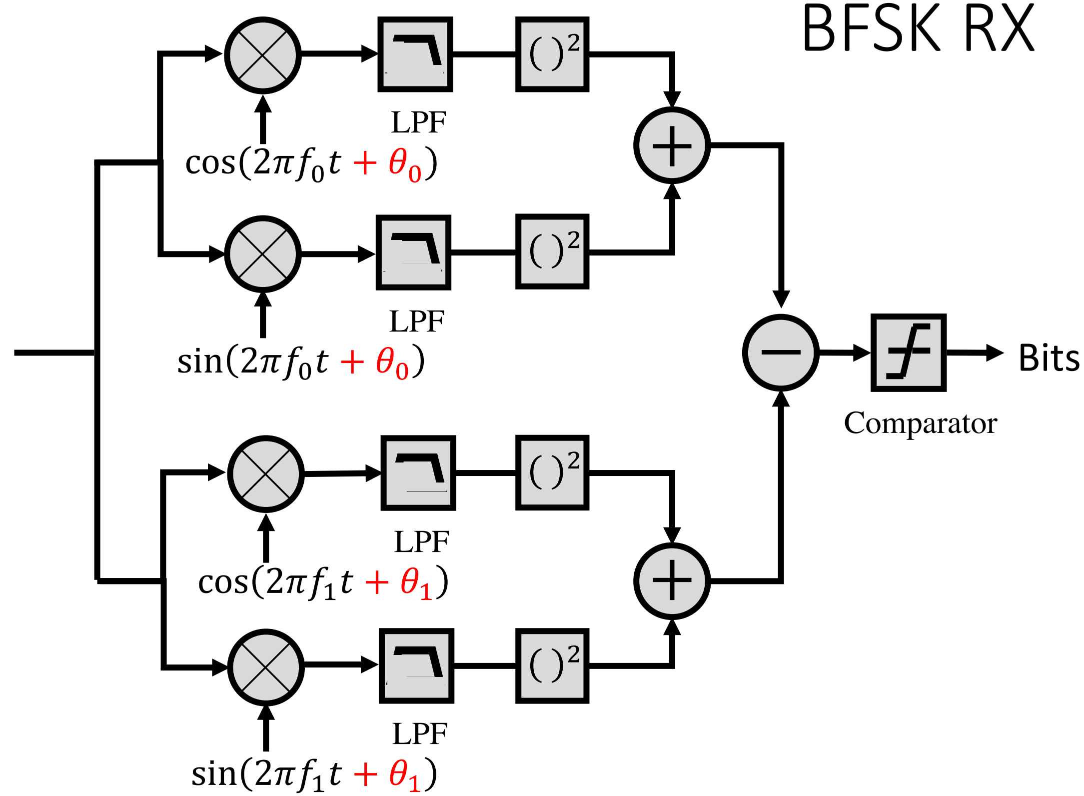
Low Pass Filter

Non-Coherent FSK

BFSK TX



BFSK RX



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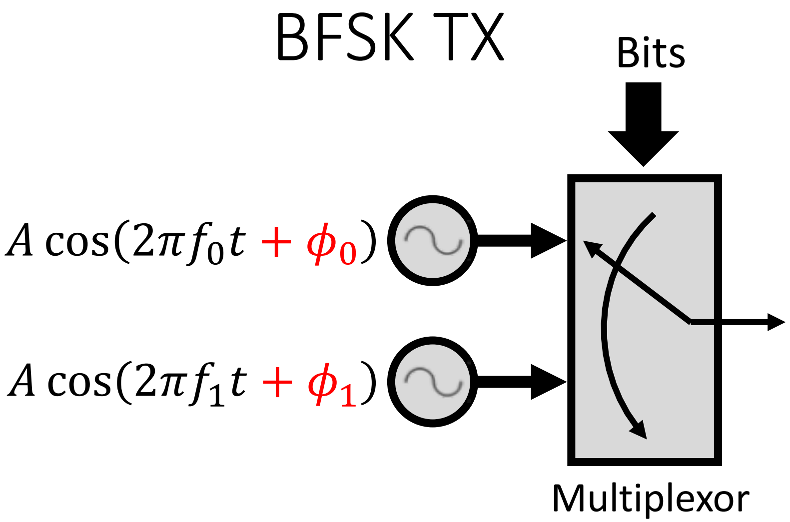
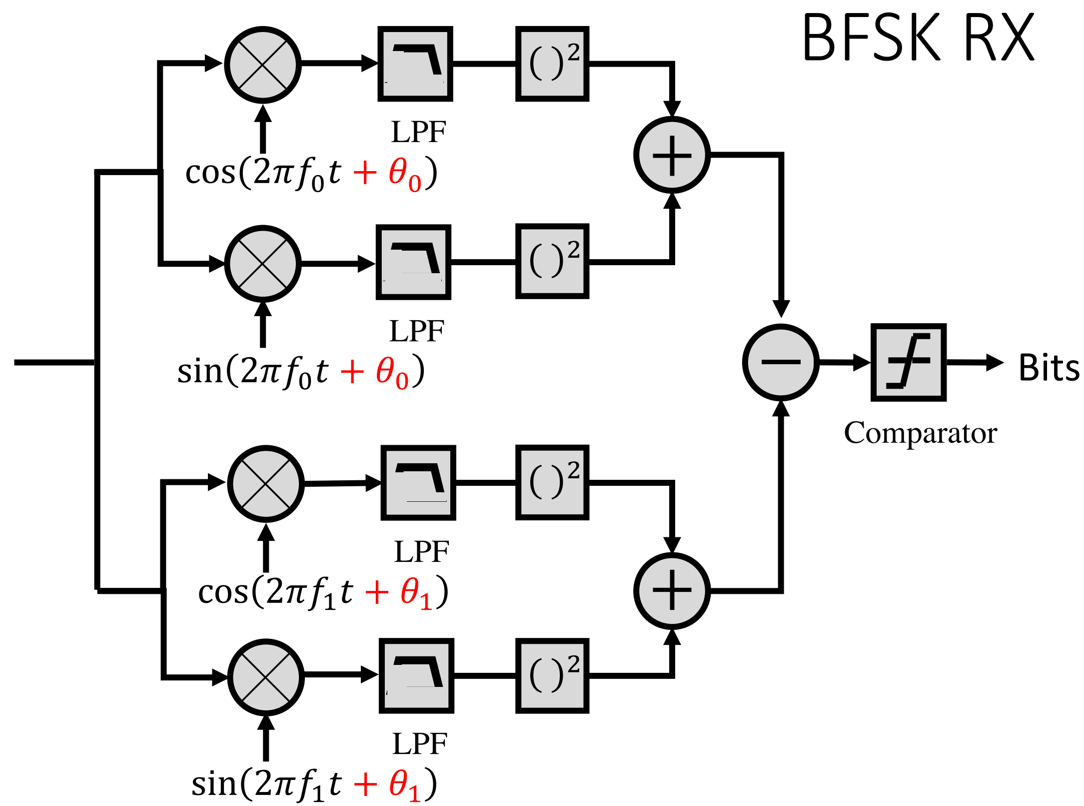
Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) \rightarrow \frac{A}{2}\sin(\theta_0 - \phi_0)$

Non-Coherent FSK



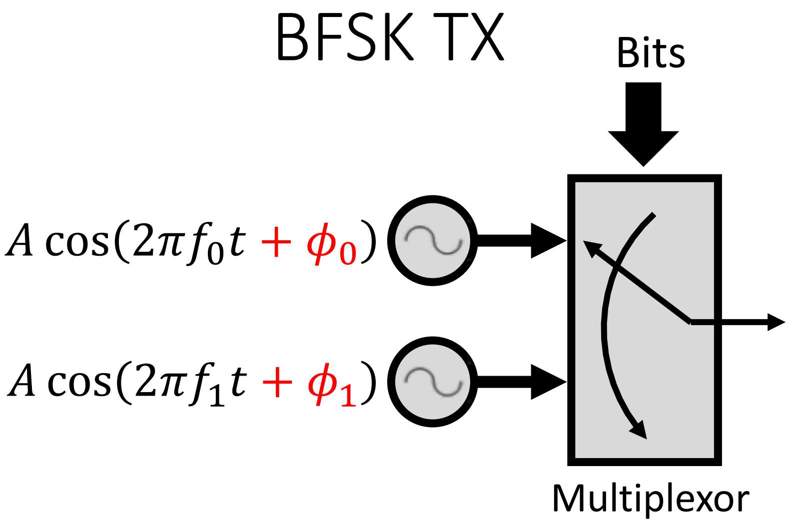


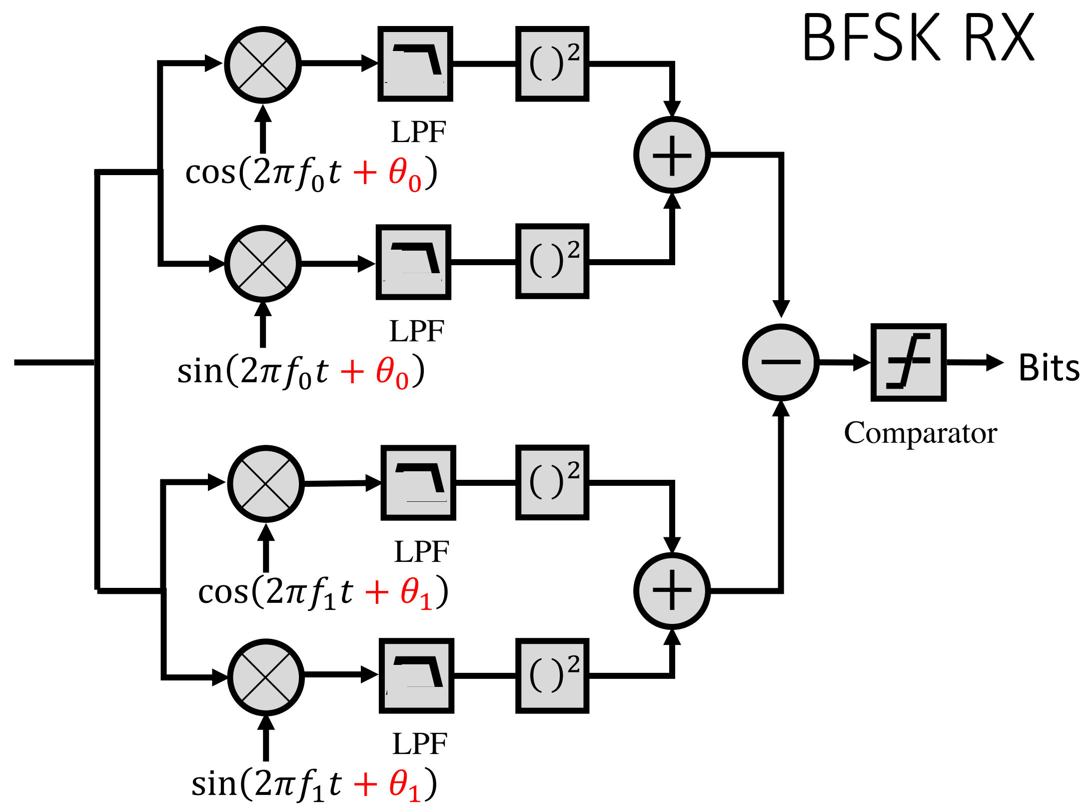
Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) \rightarrow \dfrac{A^2}{4}\cos^2(\phi_0 - \theta_0)$

Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) \rightarrow \dfrac{A^2}{4}\sin^2(\theta_0 - \phi_0)$

Non-Coherent FSK





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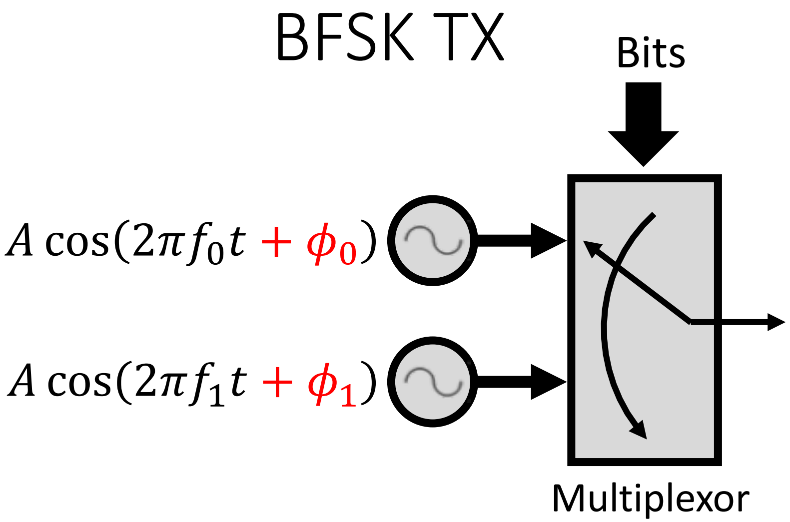
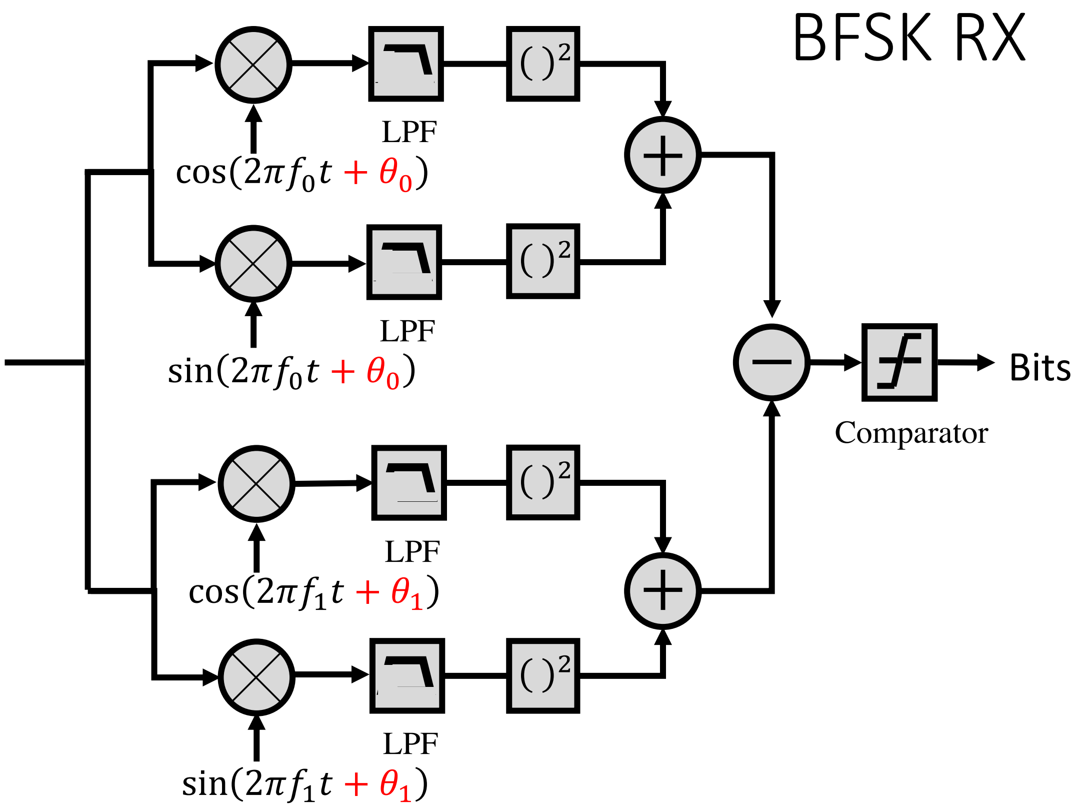
In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\cos^2(\phi_0 - \theta_0)$

Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\sin^2(\theta_0 - \phi_0)$

$+ = \frac{A^2}{4}$

Non-Coherent FSK





Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

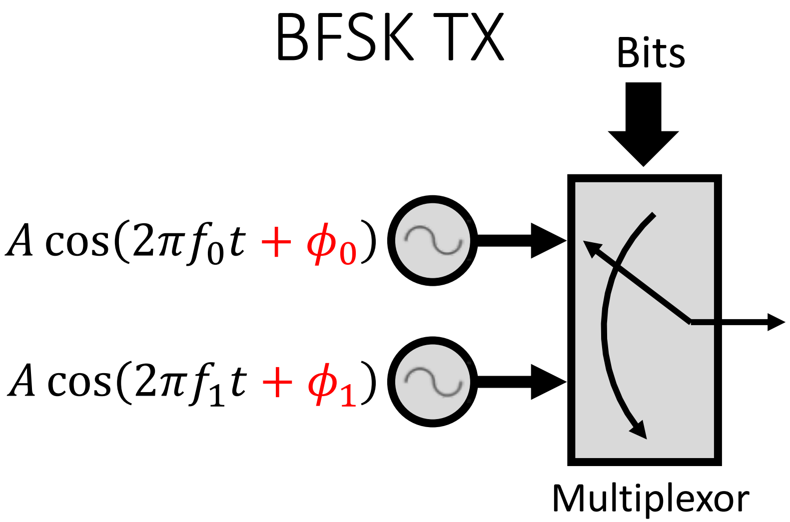
In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\cos^2(\phi_0 - \theta_0)$

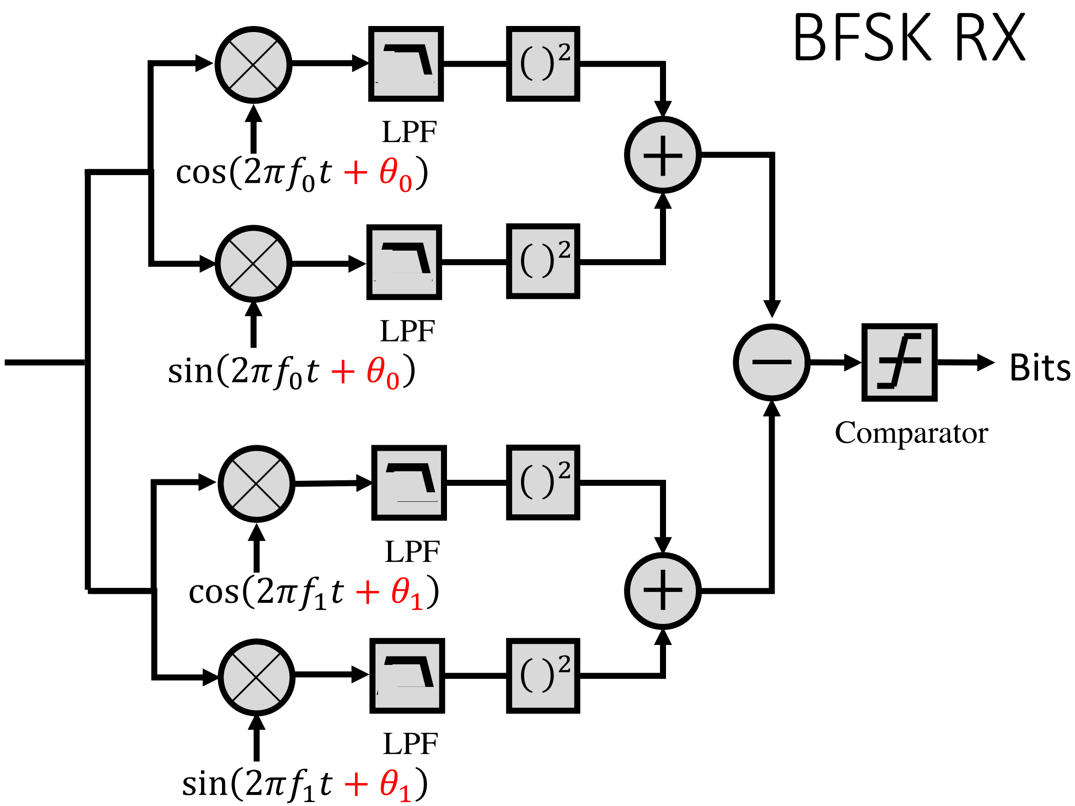
Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\sin^2(\theta_0 - \phi_0)$

$+ = \frac{A^2}{4}$

In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1)$

Non-Coherent FSK





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Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\sin^2(\theta_0 - \phi_0)$

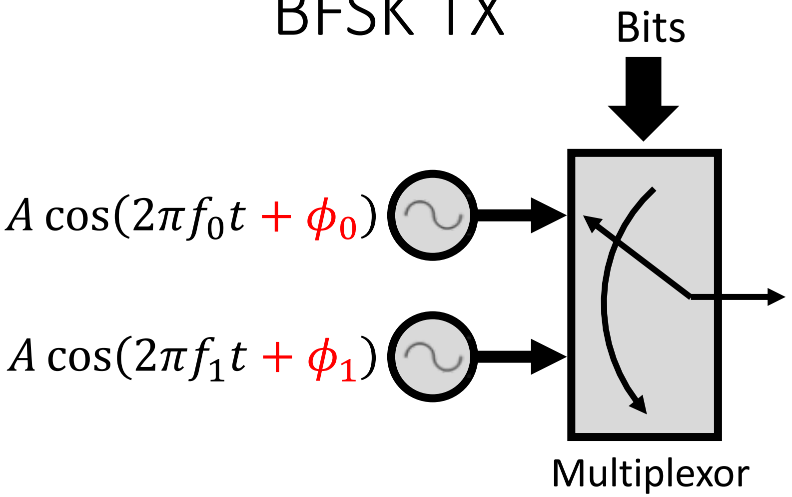
$+ = \frac{A^2}{4}$

In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1)$

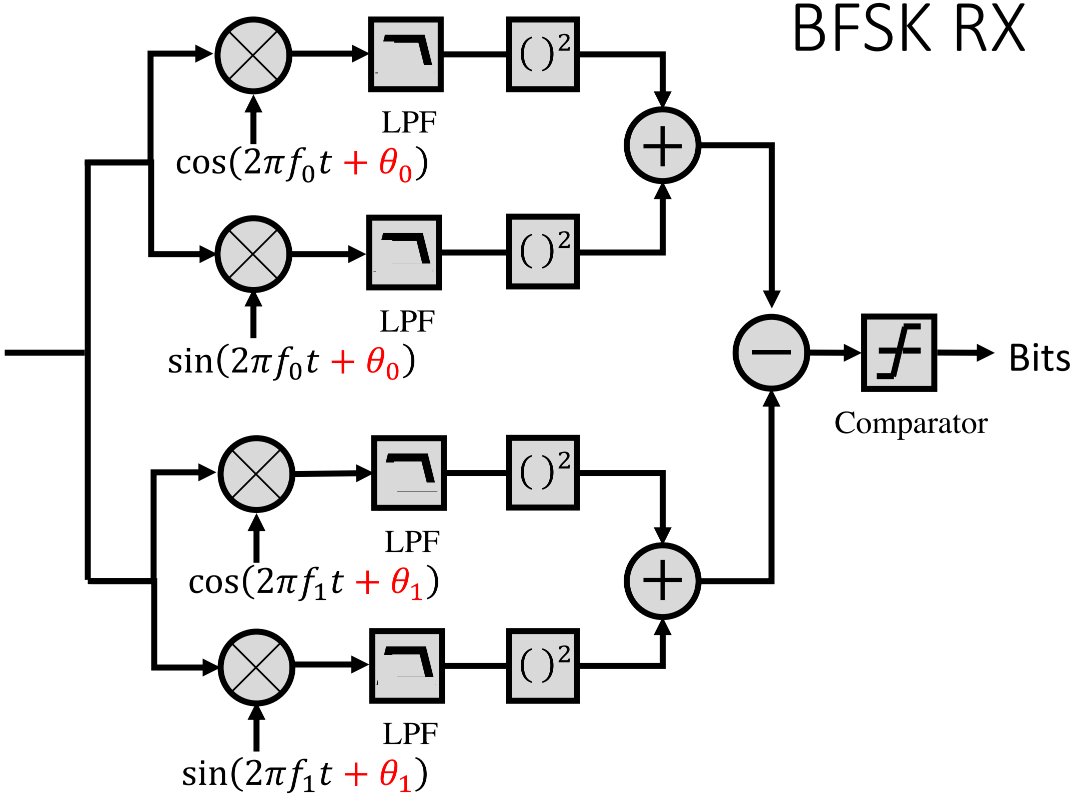
$$= \frac{A}{2}\cos(2\pi(f_0 - f_1)t + \phi_0 - \theta_1) + \frac{A}{2}\cos(2\pi(f_0 + f_1)t + \phi_0 + \theta_1)$$

Non-Coherent FSK

BFSK TX



BFSK RX



Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\cos^2(\phi_0 - \theta_0)$

Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\sin^2(\theta_0 - \phi_0)$

$+ = \frac{A^2}{4}$

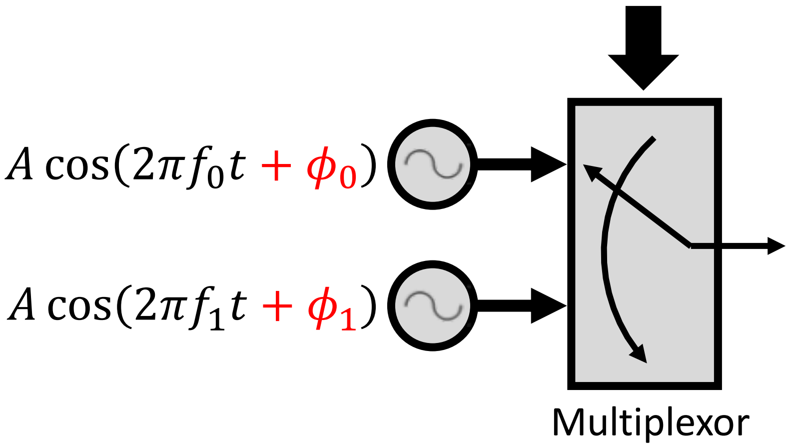
In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1)$

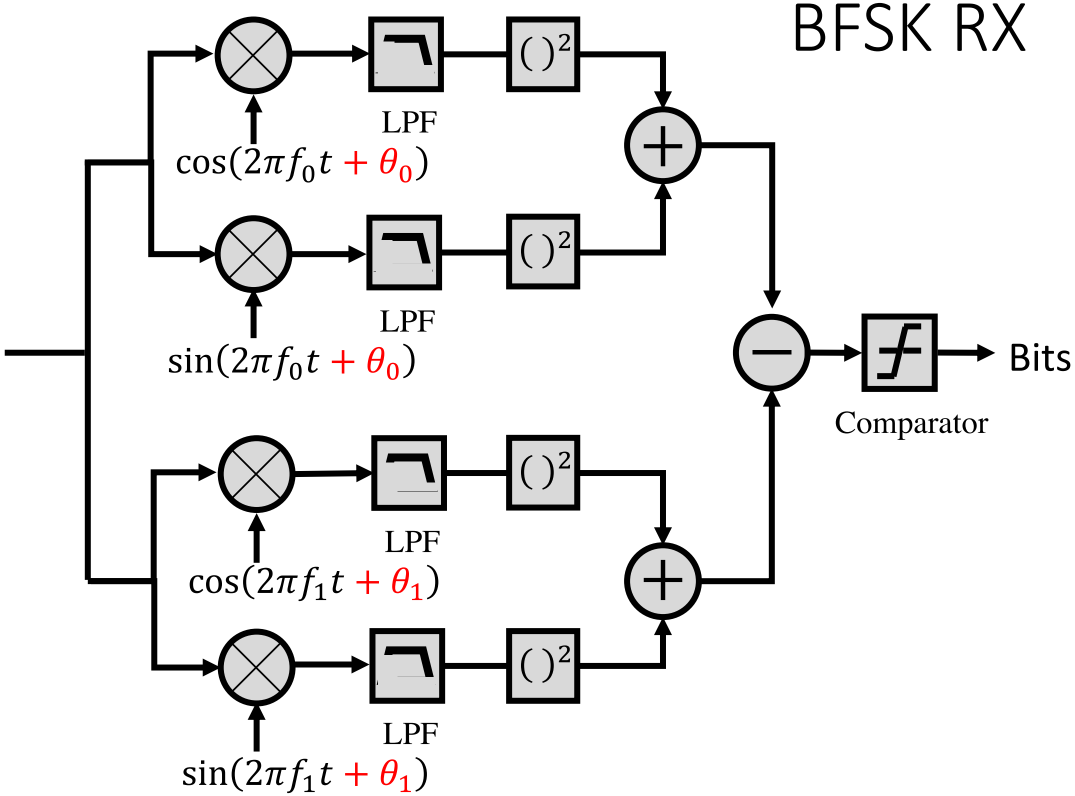
$$= \frac{A}{2}\cos(2\pi(f_0 - f_1)t + \phi_0 - \theta_1) + \frac{A}{2}\cos(2\pi(f_0 + f_1)t + \phi_0 + \theta_1)$$

Low Pass Filter

Low Pass Filter

Non-Coherent FSK





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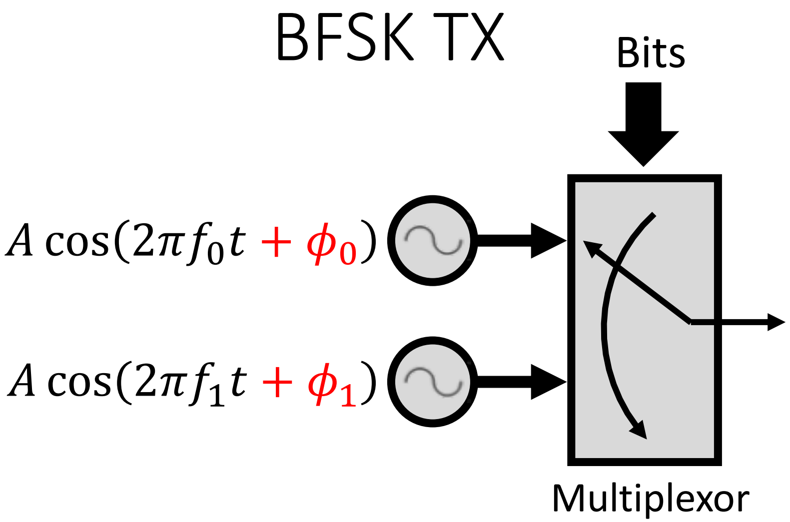
In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\cos^2(\phi_0 - \theta_0)$

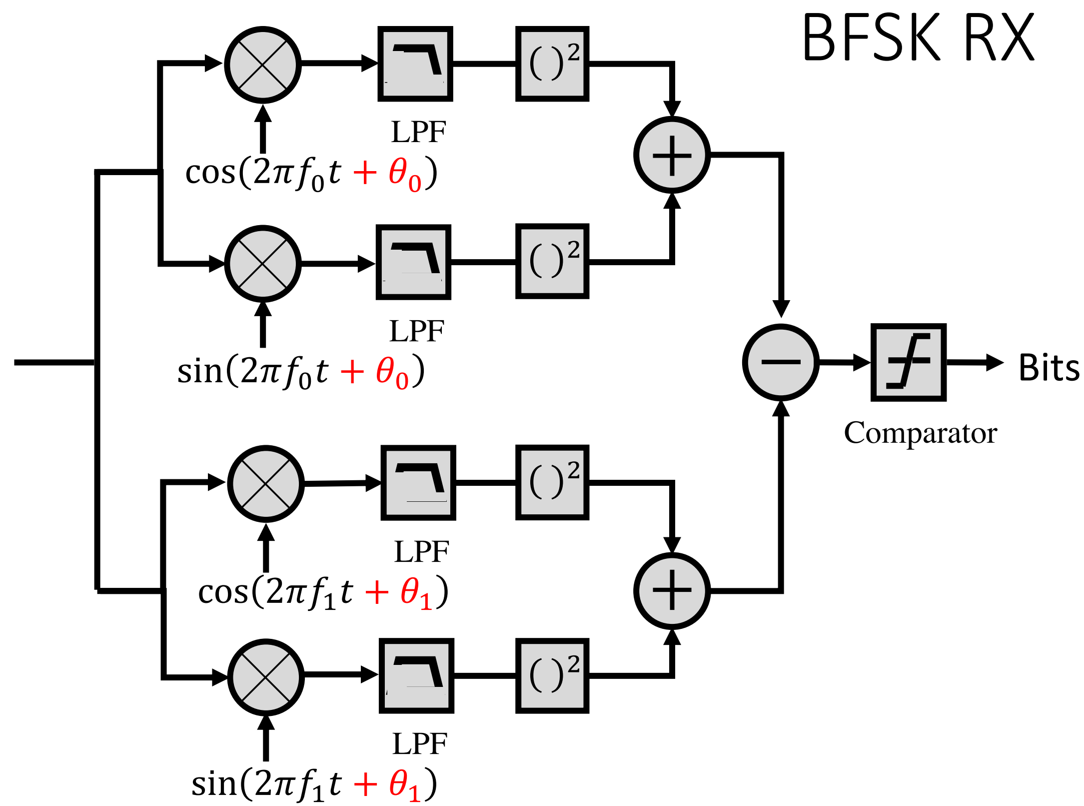
Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\sin^2(\theta_0 - \phi_0)$

$+ = \frac{A^2}{4}$

In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1) \rightarrow = 0$

Non-Coherent FSK





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In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\cos^2(\phi_0 - \theta_0)$

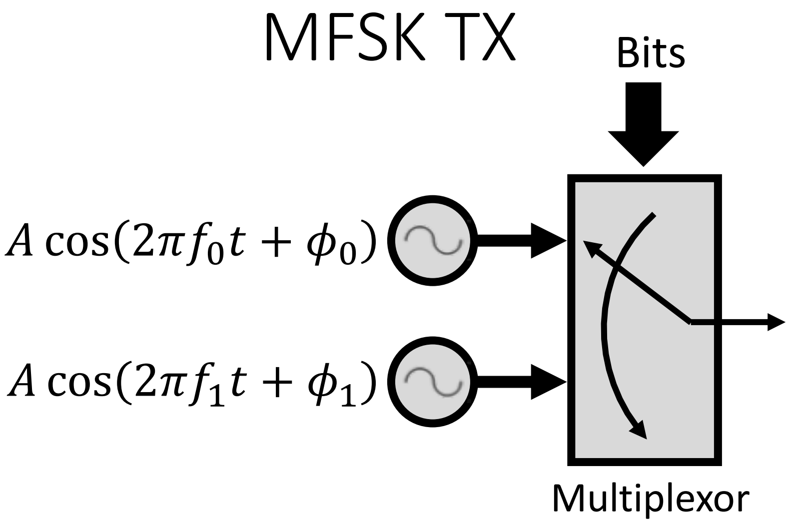
$$+ = \frac{A^2}{4}$$

Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \theta_0) \rightarrow \frac{A^2}{4}\sin^2(\theta_0 - \phi_0)$

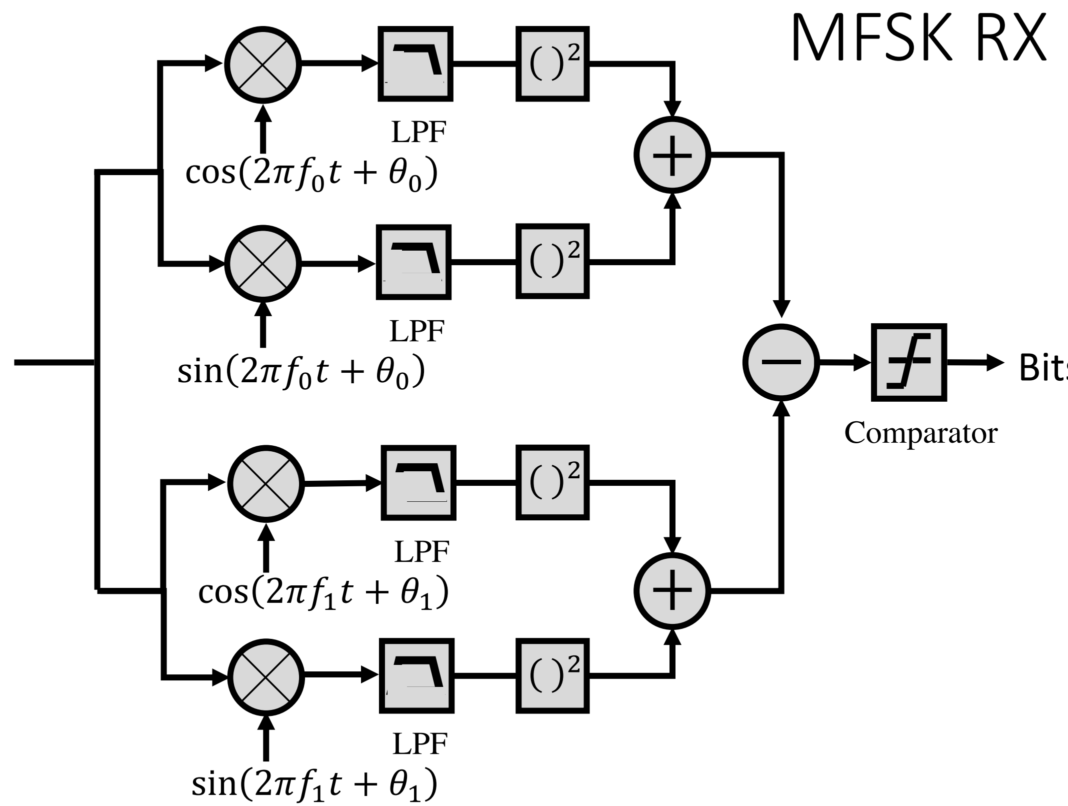
In-phase branch: $A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1) \rightarrow = 0$

Quadrature branch: $A\cos(2\pi f_0 t + \phi_0)\sin(2\pi f_1 t + \theta_1) \rightarrow = 0$

MFSK

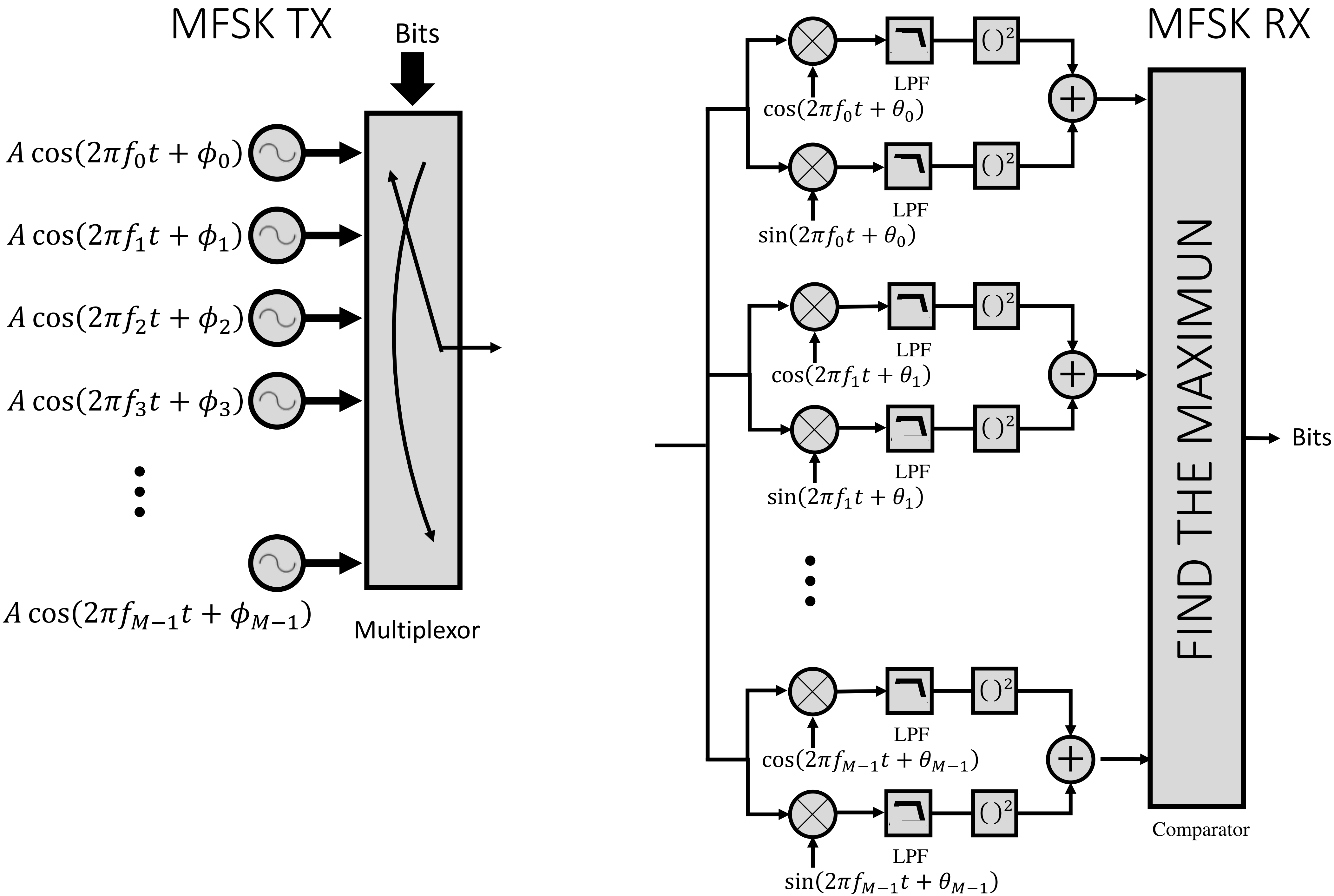


MFSK TX
Bits
$A\cos(2\pi f_0 t + \phi_0)$
$A\cos(2\pi f_1 t + \phi_1)$
Multiplexor

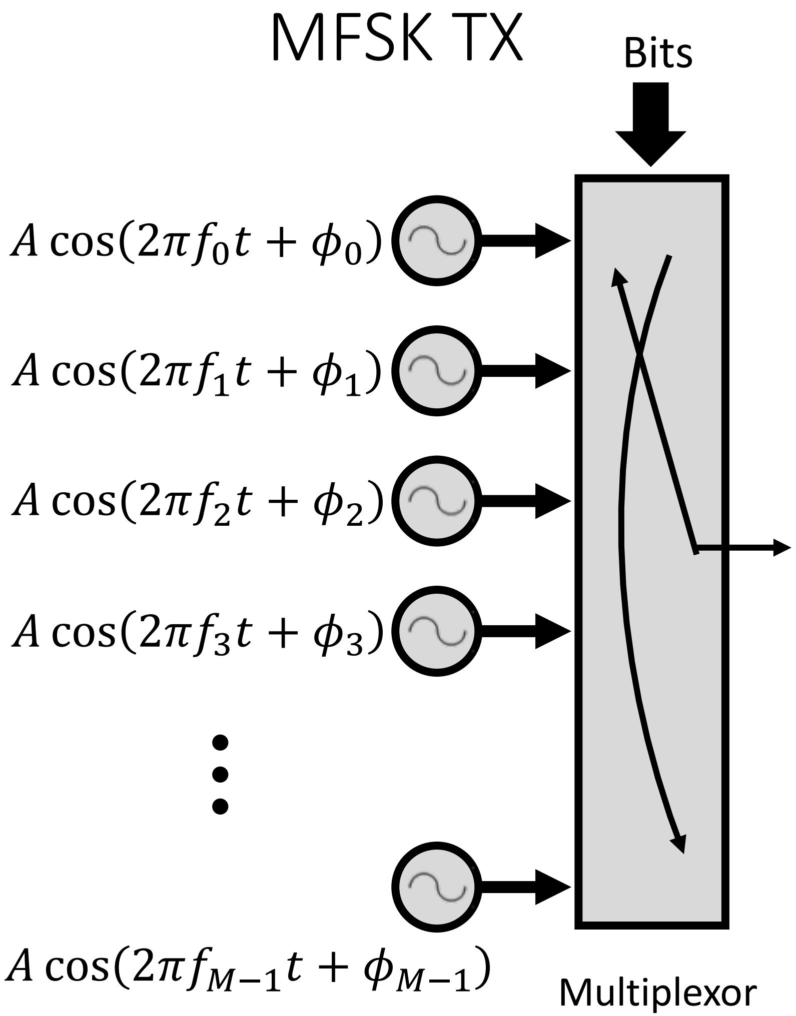


MFSK RX
LPF
$()^2$
$\cos(2\pi f_0 t + \theta_0)$
$\sin(2\pi f_0 t + \theta_0)$
$\cos(2\pi f_1 t + \theta_1)$
$\sin(2\pi f_1 t + \theta_1)$
Comparator
Bits

MFSK


MFSK TX
Bits
$A\cos(2\pi f_0 t + \phi_0)$
$A\cos(2\pi f_1 t + \phi_1)$
$A\cos(2\pi f_2 t + \phi_2)$
$A\cos(2\pi f_3 t + \phi_3)$
$A\cos(2\pi f_{M-1} t + \phi_{M-1})$
Multiplexor
MFSK RX
LPF
$()^2$
$\cos(2\pi f_0 t + \theta_0)$
$\sin(2\pi f_0 t + \theta_0)$
$\cos(2\pi f_1 t + \theta_1)$
$\sin(2\pi f_1 t + \theta_1)$
$\cos(2\pi f_{M-1} t + \theta_{M-1})$
$\sin(2\pi f_{M-1} t + \theta_{M-1})$
FIND THE MAXIMUN
Comparator
Bits

MFSK



$$x(t) = A\cos(2\pi f_i t + \phi_i)$$

$$= A\cos(2\pi f_c t + 2\pi(2i + 1 - M)\Delta f t + \phi_i)$$

Minimum Frequency Separation: $2\Delta f$

What is the problem here?

Phase discontinuity:

$\phi_0 \neq \phi_1 \neq \phi_2 \neq \ldots \neq \phi_{M-1}$

→ Wide spectral leakage

CPFSK: Continuous Phase FSK

$$x(t) = A\cos\left(2\pi f_c t + 2\pi\Delta f \int_{-\infty}^{t} u(\tau)d\tau\right)$$

$u(t) = \sum_{n} s[n]p(t - nT_b)$: Pulse Amplitude Modulation from Lecture 3

Leakage Side-lobe Power decays $\frac{1}{f^4}$ in CPFSK vs $\frac{1}{f^2}$ in FSK

FSK Bandwidth

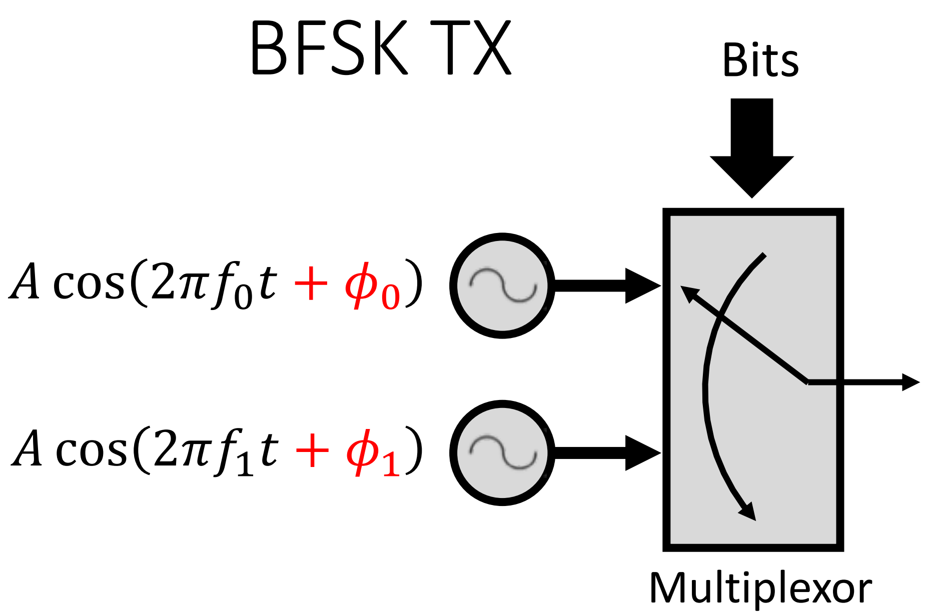
- Frequency separation: $2\Delta f$
- Symbol Time: T_b
- Pulse Bandwidth: $B = \frac{1}{T_b}$

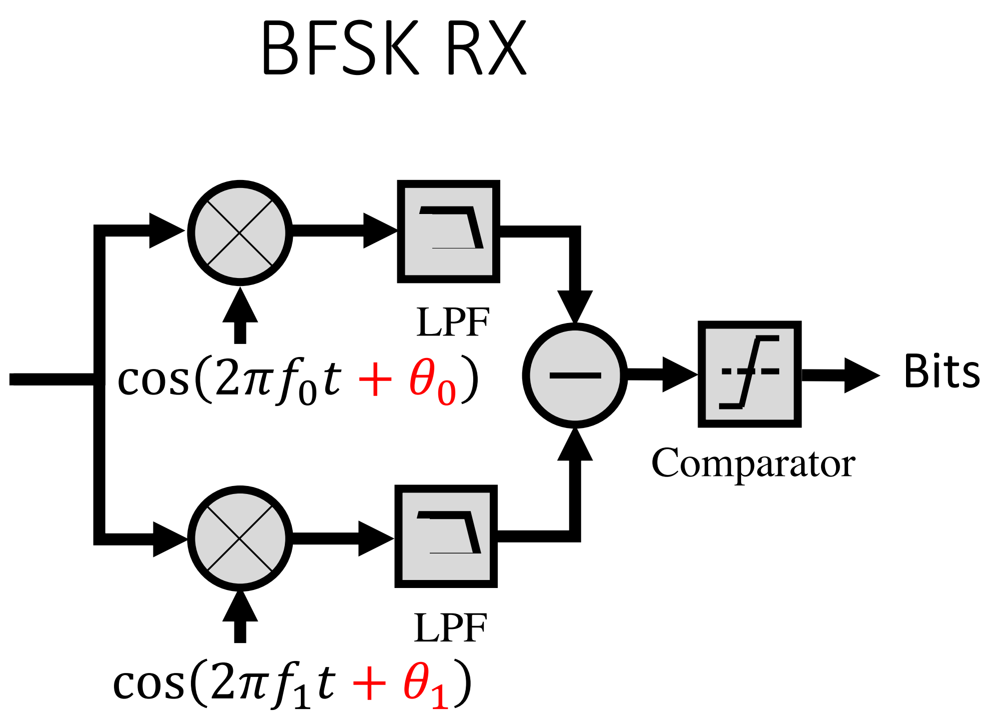
Bandwidth: $2\Delta f + B$ for BFSK

$2M\Delta f + B$ for MFSK

What is the minimum Δf?

FSK: Frequency Shift Keying





Assume bit is 0, we send $A\cos(2\pi f_0 t + \phi_0)$:

$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_0 t + \theta_0) = \frac{A}{2}\cos(\phi_0 - \theta_0) + \frac{A}{2}\cos(2\pi 2 f_0 t + \phi_0 + \theta_0)$$

$$A\cos(2\pi f_0 t + \phi_0)\cos(2\pi f_1 t + \theta_1) = \frac{A}{2}\cos(2\pi(f_0 - f_1)t + \phi_0 - \theta_0) + \frac{A}{2}\cos(2\pi(f_0 + f_1)t + \phi_0 + \theta_0)$$

FSK Bandwidth

- Frequency separation: $2\Delta f$
- Symbol Time: T_b
- Pulse Bandwidth: $B = \frac{1}{T_b}$

Bandwidth: $2\Delta f + B$ for BFSK

$2M\Delta f + B$ for MFSK

What is the minimum Δf?

Need to Low Pass Filter $|f_0 - f_1| = 2\Delta f$

$$2\Delta f \geq \frac{B}{2} = \frac{1}{2T_b}$$

MSK: Minimum Shift Keying

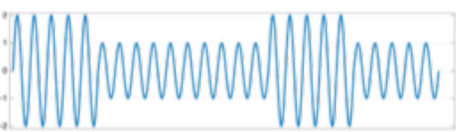
- FSK with Frequency separation: $2\Delta f = 1/2T_b$
- Symbol Time: T_b
- Avoid phase discontinuity by setting $\phi_0 = \phi_1$
- GMSK: Gaussian Minimum Shift Keying
- Data Stream is passed through Gaussian filter to reduce sidelobes.
- Used in GSM cellular systems

Modulation

Based on how the bits are encoded

ASK

Amplitude Shift Keying

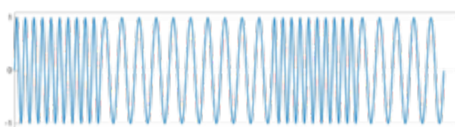


1 0 0 1 0

OOK, ASK

FSK

Frequency Shift Keying

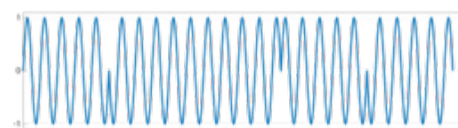


1 0 0 1 0

CFSK, MSK, GMSK

PSK

Phase Shift Keying



1 0 0 1 0

BPSK, QPSK, CPM

QAM

Phase & Amplitude Modulation (APSK)

PAM, M-QAM

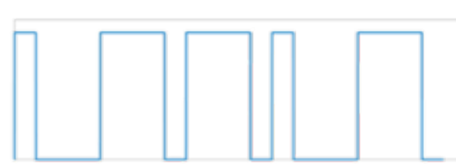
PPM

Pulse Position Modulation

1 0 0 1 0

PWM

Pulse Width Modulation



1 0 0 1 0

DQPSK

Differential QPSK

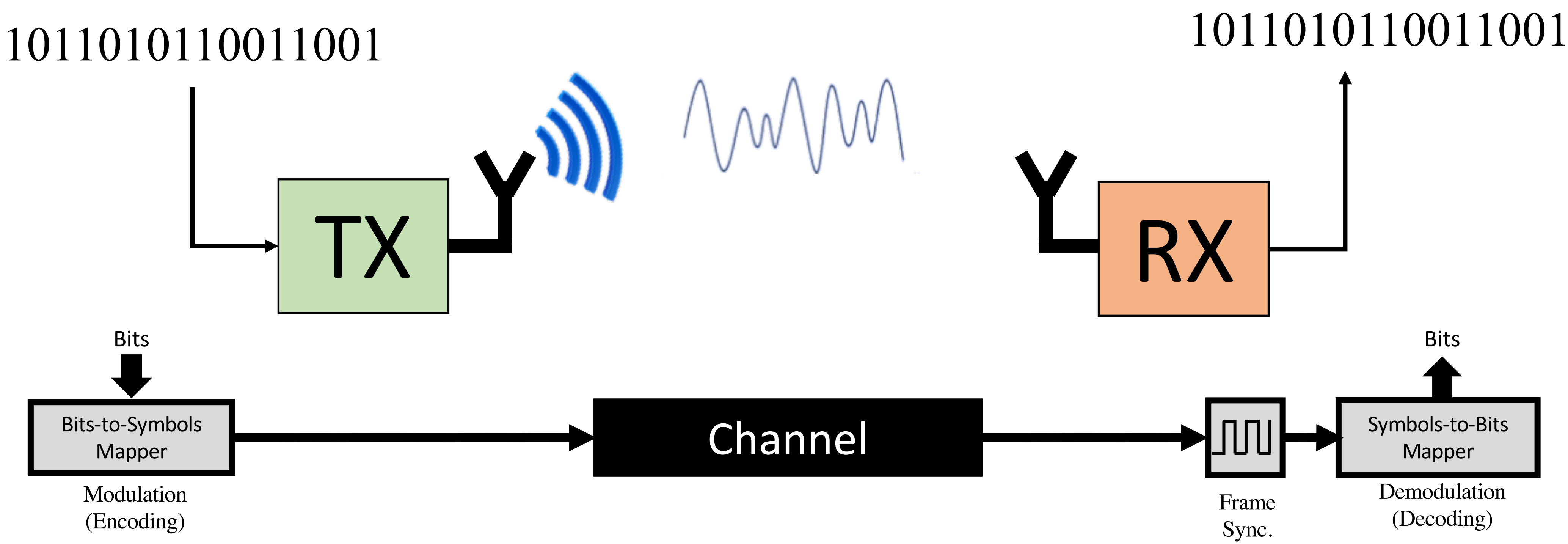
DBPSK, DQPSK,

DQAM

Differential QAM

DQAM

The Channel

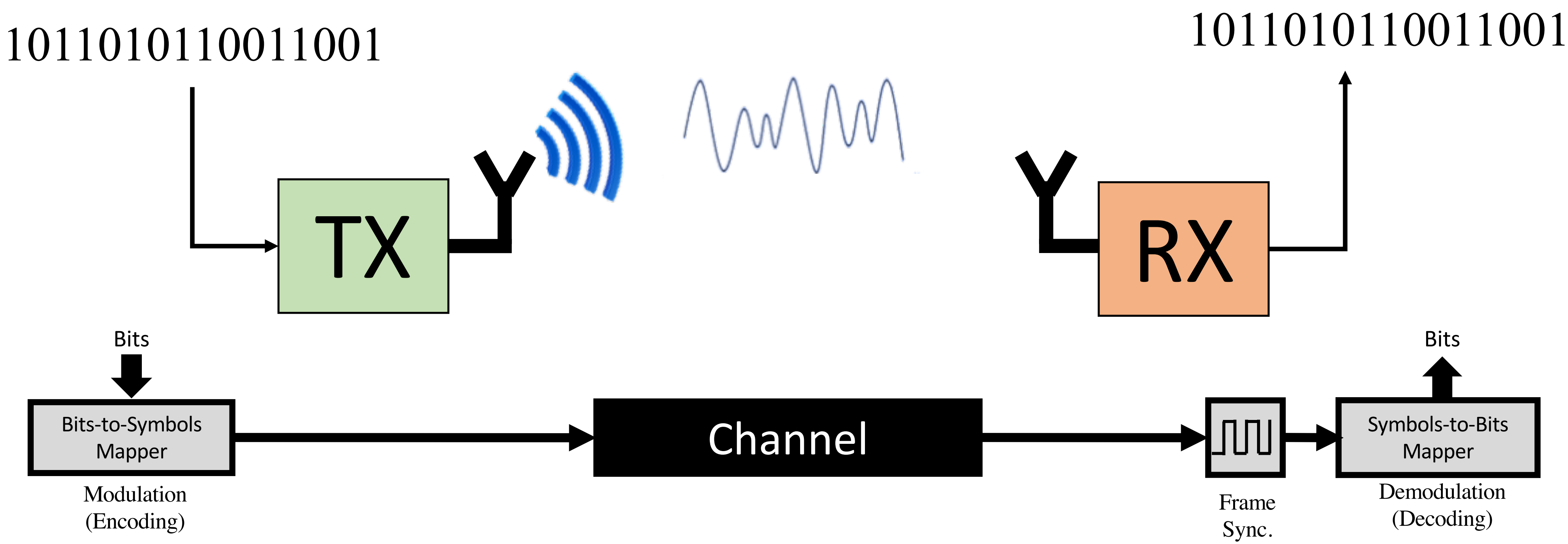


$x(t)$ ➡ ➡ $y(t) = \mathbf{h}\, x(t-\tau) + v(t)$

Channel:

- Adds Noise
- Attenuates the Signal
- Rotates the Phase of the Signal
- Delays the Signal

The Channel

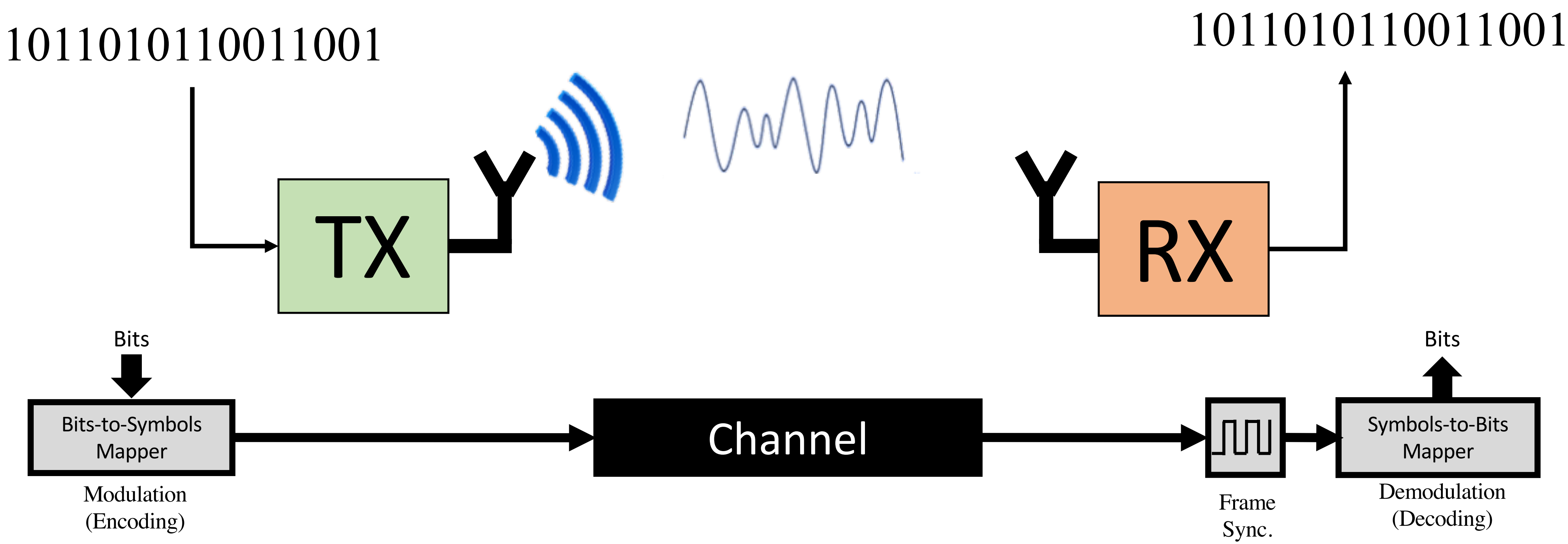


$x(t)$ ➡ ➡ $y(t) = \boldsymbol{h}\, x(t - \boldsymbol{\tau}) + v(t)$

Channel:

- Adds Noise
- Attenuates the Signal
- Rotates the Phase of the Signal
- Delays the Signal $x(t) \rightarrow x(t - \tau)$

The Channel



$x(t)$ ➡ ➡ $y(t) = \boldsymbol{h}\, x(t - \boldsymbol{\tau}) + v(t)$

Channel: Delays the Signal $x(t) \rightarrow x(t - \tau)$

⬇

We do not know where the packet starts and where to decode the bits (Don’t want to decode noise)

Frame Synchronization

- Data bits organized intro frames.
- Need to detect the beginning of the frame to extract the data message from the frame correctly.

$$\cdots -1 +1 -1 -1 +1 +1 +1 -1 \cdots$$

How to detect the beginning of the frame?

Send training sequence at the beginning

$-1 +1 -1 -1 +1$	$\cdots -1 +1 -1 -1 +1 +1 +1 -1 \cdots$

Frame Synchronization

$-1+1-1-1+1$ | $\cdots -1+1-1-1+1+1+1-1\cdots$

- Training sequence is a known sequence of bits sent at the beginning of each frame.
- Training sequence agreed upon between TX and RX before transmission.
- Correlate with Training sequence to find the start of the frame.
- Training sequence should have good auto-correlation properties.
- Many types: Baker sequences, Gold sequences, …

Frame Synchronization

$-1+1-1-1+1$	$\cdots -1+1-1-1+1+1+1-1\cdots$

- $t[n]$ is training sequence of length N
- Discrete samples: $y[n] = h \cdot t[n-d] + v[n]$

$$R[n] = \frac{1}{N}\left|\sum_{k=0}^{N-1} t^*[k]y[n+k]\right|^2$$

$$= \frac{1}{N}\left|\sum_{k=0}^{N-1} t^*[k]h \cdot t[n+k-d] + \sum_{k=0}^{N-1} t^*[k]v[n+k]\right|^2$$

Frame Synchronization

$-1+1-1-1+1$ $\cdots-1+1-1-1+1+1+1-1\cdots$

- $t[n]$ is training sequence of length N
- Discrete samples: $y[n] = h \cdot t[n-d] + v[n]$

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$$= \frac{1}{N}\left|\sum_{k=0}^{N-1} t^*[k]h \cdot t[n+k-d] + \sum_{k=0}^{N-1} t^*[k]v[n+k]\right|^2$$

0

Frame Synchronization

$-1+1-1-1+1$	$\cdots -1+1-1-1+1+1+1-1\cdots$

- $t[n]$ is training sequence of length N
- Discrete samples: $y[n] = h \cdot t[n-d] + v[n]$

$$R[n] = \frac{1}{N}\left|\sum_{k=0}^{N-1} t^*[k]y[n+k]\right|^2$$

$$= \frac{1}{N}\left|\sum_{k=0}^{N-1} t^*[k]h \cdot t[n+k-d]\right|^2 = |h|^2 \begin{cases} 1 & if \quad n = d \\ 1/N & if \quad n \neq d \end{cases}$$

Frame Synchronization

$-1+1-1-1+1$	$\cdots -1+1-1-1+1+1+1-1\cdots$

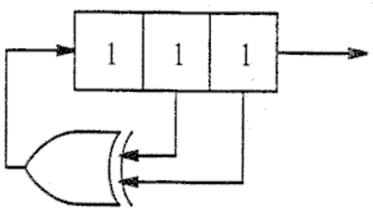
- $t[n]$ is Baker Sequence

Code Length	Code
2	$[-1+1]$ or $[-1-1]$
3	$[-1-1+1]$
4	$[-1+1-1-1]$ or $[-1+1+1+1]$
5	$[-1-1-1+1-1]$
7	$[-1-1-1+1+1-1+1]$
11	$[-1-1-1+1+1+1-1+1+1-1+1]$
13	$[-1-1-1-1-1+1+1-1-1+1-1+1-1]$

MLS: Maximum Length Sequence

$-1+1-1-1+1$ $\cdots -1+1-1-1+1+1+1-1\cdots$

- Type of Pseudo Random Noise Sequence (PN)
- Also known as m-sequence
- Balances 1s $(+1)$ and 0s (-1)
- Implemented using LFSR (Linear Feedback Shift Register)



- Auto-Correlation Property:

$$R[n]=\frac{1}{N}\sum_{k=0}^{N-1} t^*[k]t[n+k \bmod N]=\begin{cases} 1 & if \quad n=0 \\ -1/N & if \quad n\neq 0 \end{cases}$$

Definitions & Variables

- A_i, A_0, A_1: Amplitude levels of ASK signals
- f_0, f_1, f_i: Frequencies of FSK signals
- A: Amplitude of FSK signals
- ϕ_0, ϕ_1, ϕ_i: Phase of frequency tones at transmitter.
- $\theta_0, \theta_1,\ \theta_i$: Phase of frequency tones at receiver.
- M: Number of tones in FSK modulated signals.
- Δf: Frequency separation of FSK tones.
- $u(t)$: PAM modulated & pulse shaped signal.
- $p(t)$: Pulse of pulse shaping filter.
- $s[n]$: PAM modulated symbols.
- T_b: Symbol time
- B: Bandwidth
- $x(t)$: Transmitted Signal
- $v(t)$: Additive Gaussian Noise
- $y(t)$: Received Signal
- τ: Time delay of the signal
- h: Channel Coefficient.
- n: Symbol index
- $t[n]$: Training Sequence
- $y[n]$: Sampled Received Signal
- $R[n]$: Cross Correlation
- N: Length of training Sequence
- d: Signal Delay in number of samples
- f_c: Carrier Frequency