ECE 463 Lab 7: Carrier Frequency Offset Estimation and Correction

1. Introduction

In most of communication systems, the carrier frequency at the transmitter does not match the carrier frequency at the receiver. This is known as carrier frequency offset (CFO). In this lab, we will detect the training sequence in the presence of CFO which is in general impossible using the correlation with the known training sequence. After detecting the training sequence, we will estimate the CFO and correct it to receive the correct symbols.

1.1. Contents

1.2. Report

Submit the answers, figures and the discussions on all the questions. The report is due as a hard copy at the beginning of the next lab.

2. CFO Estimation & Correction

In this section, you will implement the following subVIs.

- FrameSync_selfref.gvi
- CFO_estimation.gvi
- CFO_correction.gvi
- sim_CFO.gvi

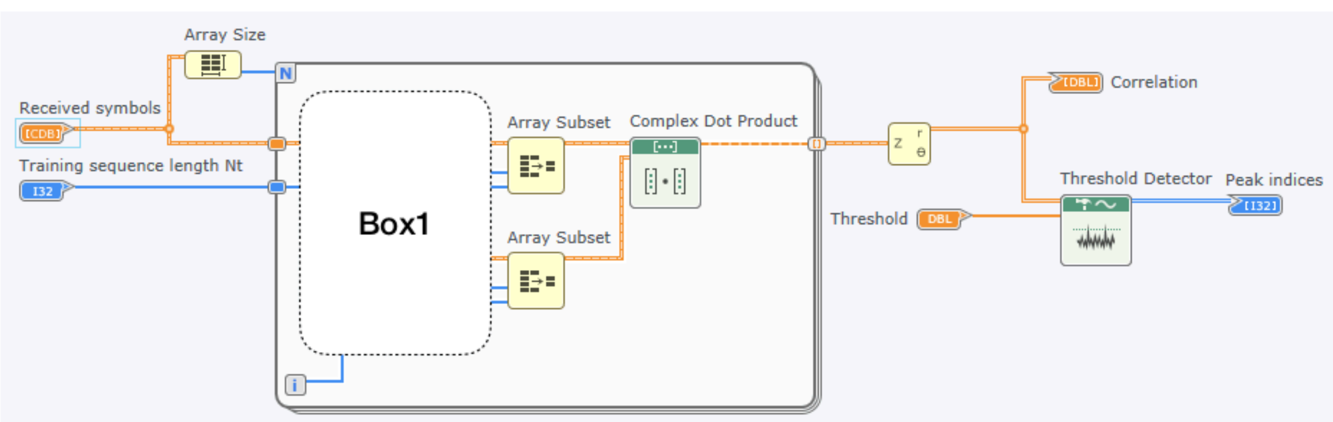
2.1. Self-reference Frame Sync

To detect the training sequence in the presence of CFO, we will implement the self-reference frame synchronization discussed in the class. Make sure **the transmitter sends two sets of training sequence** (e.g. two 13-Barker sequence). Correlate the first $\boldsymbol{N_t}$ symbols with the next $\boldsymbol{N_t}$ symbols as

$$R[n] = \left| \sum_{k=0}^{N_t-1} y^*[n+k]y[n+k+N_t] \right|$$

where $\boldsymbol{N_t}$ is the number of one set of the training sequence. Complete the figure below to implement "FrameSync_selfref.gvi".

FrameSync_selfref.gvi	Terminal name	Type	Description
Input	*Received symbols*	Complex Double array (1D)	
	Training sequence length	Double	
	Threshold	Double	
Output	*Correlation*	Double	
	Peak indices	Signed integer array (1D)	



2.2. CFO Estimation

Consider a repetition of two training sequences (e.g. two sets of 13-bit Barker code). Suppose there exists a carrier frequency offset (f_e). For $n = 0, \dots, N_t - 1$, the first half of the received training sequence is

$$y[n] = e^{j2\pi f_e n T_s}\, t[n]$$

where T_s is the symbol period. Then, the second half of the received training sequence can be written as

$$\begin{aligned} y[n+N_t] &= e^{j2\pi f_e (n+N_t) T_s}\, t[n+N_t] \\ &= e^{j2\pi f_e N_t T_s}\, y[n] \\ &= \alpha\, y[n] \end{aligned}$$

The least square solution for α is

$$\alpha = \frac{\sum_{n=0}^{N_t-1} y^*[n] y[n+N_t]}{\sum_{n=0}^{N_t-1} y^*[n] y[n]}$$

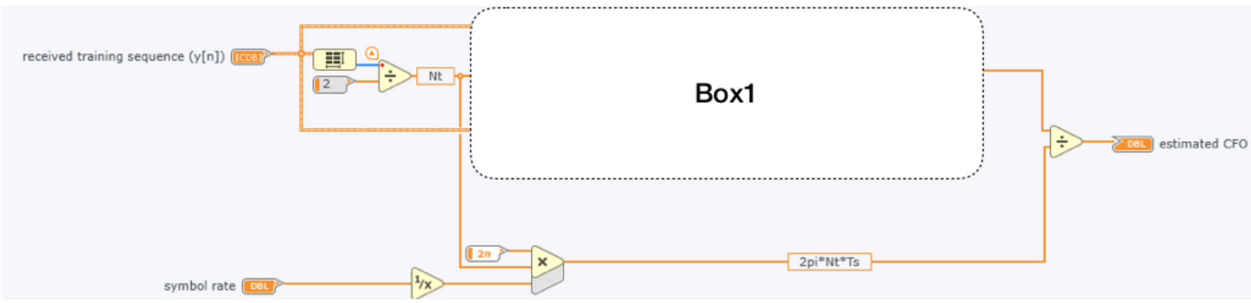
Since we are only interested in its phase, the estimated CFO is

$$\hat{f}_e = \frac{Phase\left(\sum_{n=0}^{N_t-1} y^*[n] y[n+N_t]\right)}{2\pi N_t T_s}$$

Design "CFO_estimation.gvi" that estimates the carrier frequency offset, f_e.

CFO_estimation.gvi	Terminal name	Type	Description
Input	*Received training symbols*	Complex Double array (1D)	
	Symbol rate	Double	
Output	*Estimated CFO*	Double	

- Complete **Box1** in the following figure of "CFO_estimation.gvi".



2.3. CFO Correction

Once the CFO is estimated, the CFO corrected symbols can be written as

$$\hat{y}[n] = e^{-j2\pi \hat{f}_e n T_s}\, y[n], \quad n = n_d, n_d + 1, \ldots, n_d + N - 1$$

where $\hat{f}_e$ is the estimated CFO, n_d is the initial offset, and N is the number of symbols. Design "CFO_correction.gvi" that takes the estimated CFO as input and corrects the symbols.

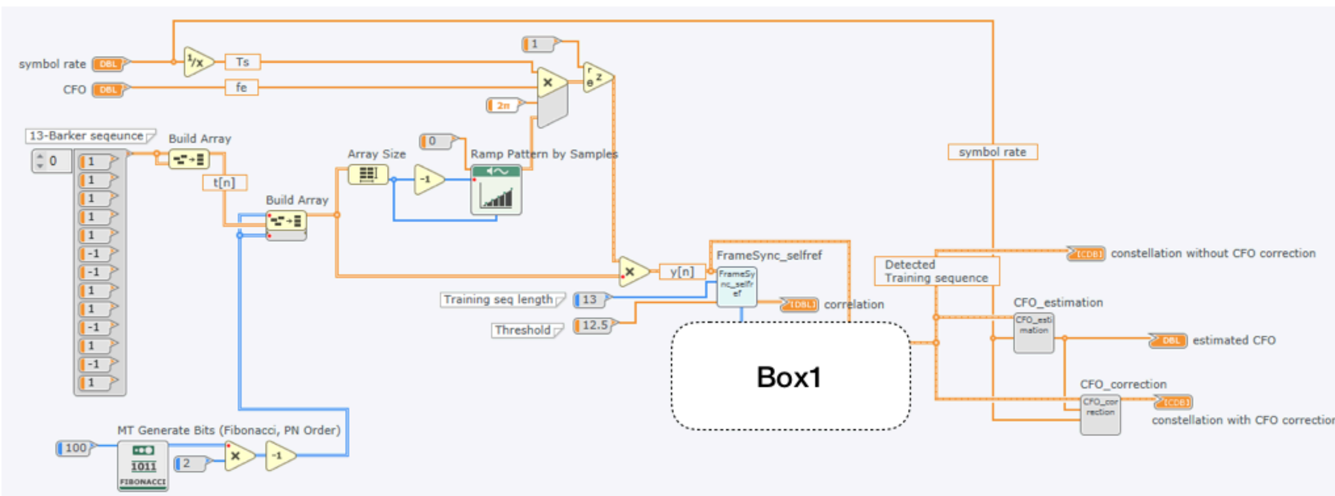
CFO_correction.gvi	Terminal name	Type	Description
Input	*Estimated CFO*	Double	
	Symbol rate	Double	
	Symbols	Complex Double array (1D)	
	Initial offset	Singed integer	
Output	*Corrected symbols*	Complex Double array (1D)	

2.4. CFO Simulation

Implement the following diagram to simulate and verify the subVI's that you created. The first half of the diagram is simulating the symbols with CFO as

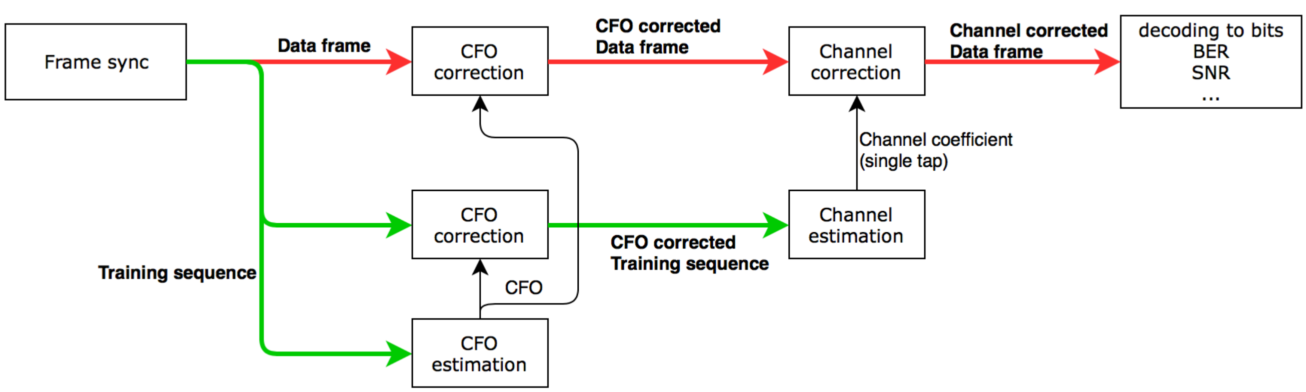
$$y[n] = e^{j2\pi f_e n T_s}\ t[n]$$

Complete Box1 to extract the training sequence from the received symbols y[n]. Use "CFO_estimation" and "CFO_correction" to correct the CFO. Plot and compare the constellation before and after CFO correction (See Question 2.6.1. and 2.6.2).



2.5. CFO and Channel correction in BPSK

Load BPSK receiver with the channel estimation implemented in Lab6. Replace the old frame sync block with the self-reference frame sync. Add CFO estimation/correction before the channel estimation. See the following diagram.



- Separate the training sequence and data frame using the index returned by the new frame sync block.

- The input symbols for the new frame sync block are no longer "real" valued symbols. Make sure the input symbols are normalized complex-double-type symbols.

- Extract the training sequence and estimate CFO.
- Perform the channel estimation using the CFO corrected training sequence.
- Correct the estimated CFO and channel coefficient on the data frame.

2.6. Questions

2.6.1. (Simulation) Run the simulation with the symbol rate 100k. Set the CFO as 1kHz.
Report 1) estimated CFO, and plot 2) constellation without CFO correction, 3) constellation with CFO correction and 4) correlation from the frame sync block.

2.6.2. (Simulation) Given the symbol rate 100k, increase the CFO until CFO estimation/correction fails. What is the maximum CFO value you can estimate? Compare with the theoretical value in the lecture.

2.6.3. (USRP) Use the following setup for the transmitter and the receiver.
-Transmitter

- IQ rate: 1M
- Symbol rate: 100k
- Carrier frequency: 1GHz

-Receiver

- IQ rate: 1M
- Symbol rate: 100k
- Carrier frequency: 1GHz **+ 1kHz (CFO)**

Plot the constellations of 1) the raw data frame, 2) the CFO corrected data frame, and 3) the channel corrected data frame.

2.6.4. (USRP) Try higher CFO than the maximum CFO value that can be estimated in the given configuration. Report the estimated CFO and plot the constellation after CFO correction. Explain the results.