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ECE 457- Microwave Devices and Circuits

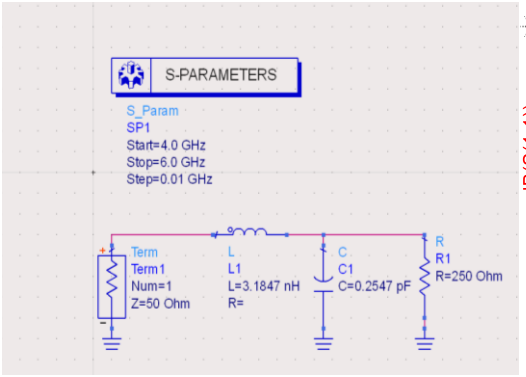
Fall 2026

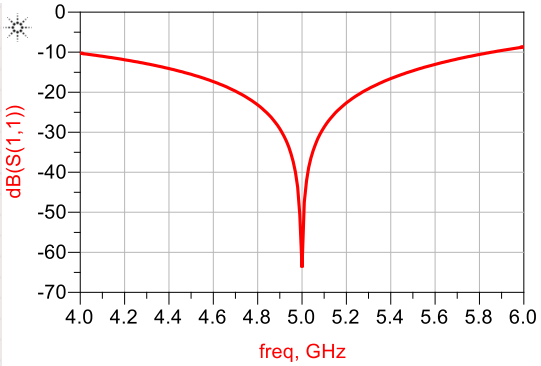
Homework #5- Impedance Matching with TLines

Solutions

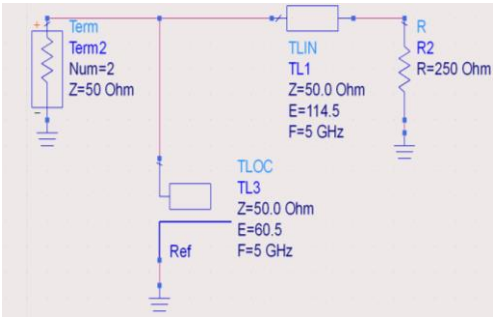
4 Matching networks.

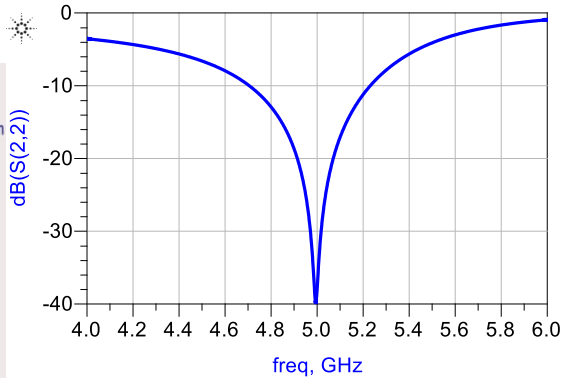
Lumped Elements:



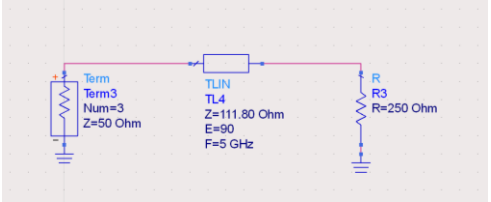


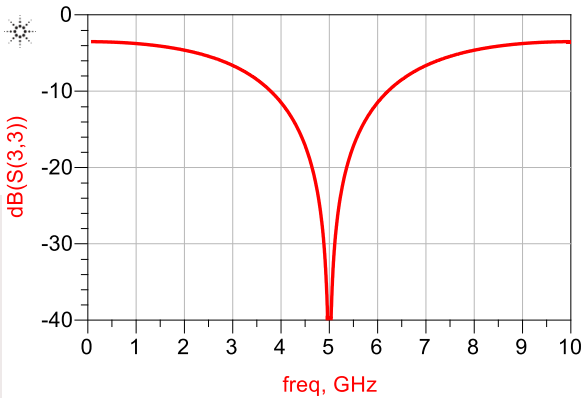
Single Stub Matching:



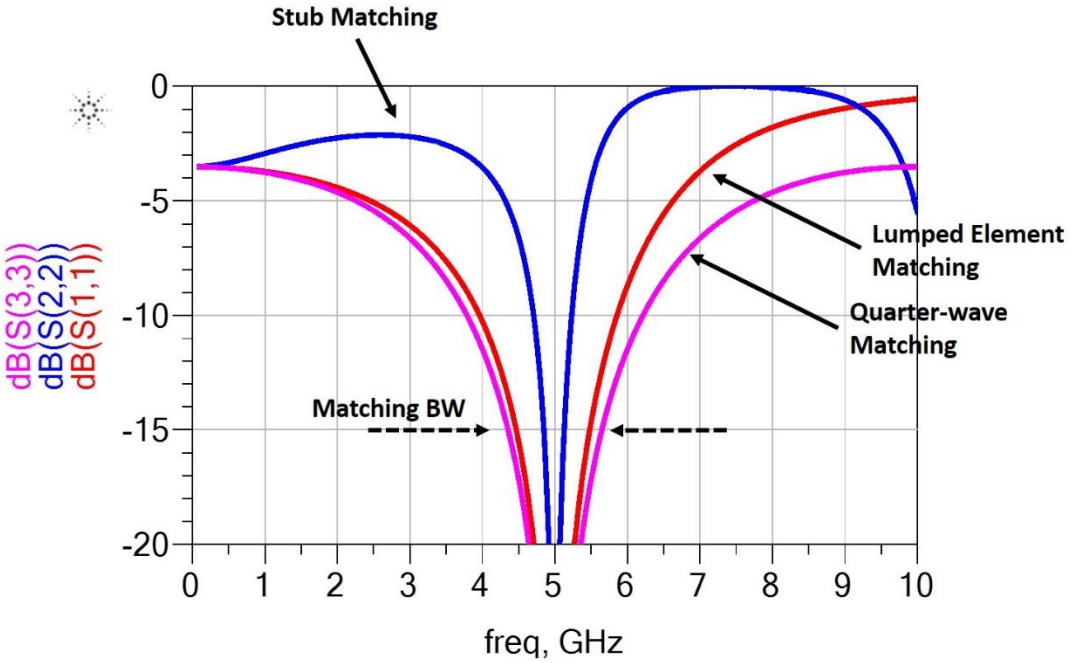


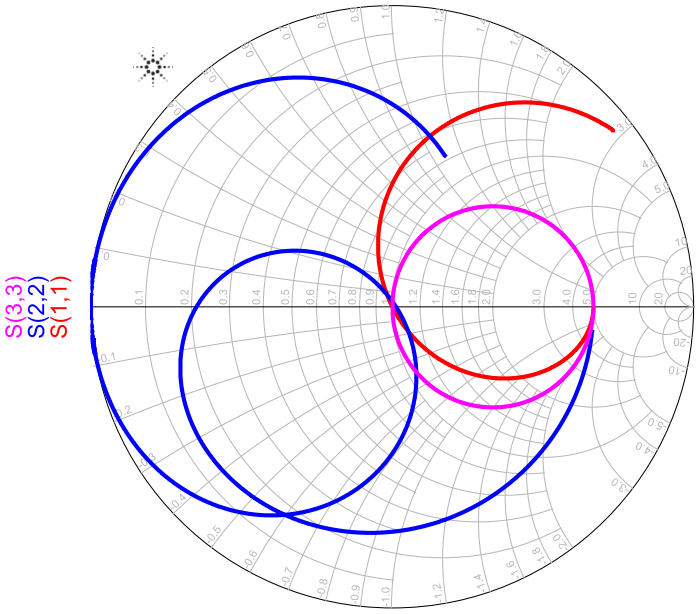
Quarter-wave Transformer:





All together 0.1-10GHz





Use Markers, We find the -15 dB BW of the matching networks as follows:

1. Quarter-wave transformer: BW=1.167 GHz
2. Lumped-element matching: BW=1.0 GHz
3. Single-stub matching: BW=333 MHz

5. Slug Matching

Let's consider the output first. The impedance looking into the slug tuner circuit from the left (see the figure below) must be Z_0 for a match. From the properties of the $\lambda/4$ transformer, we have

$$Z_{\text{in}} = \frac{Z_M^2}{Z_0} = 12.5\,\Omega$$

because $Z_M = 25\,\Omega$ and $Z_0 = 50\,\Omega$. Thus, the magnitude of the reflection coefficient *on transmission line* θ_2 must be,

$$|\Gamma_{\text{in}}| = \left|\frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}\right| = \left|\frac{Z_m^2 - Z_0^2}{Z_M^2 + Z_0^2}\right| = \frac{3}{5},$$

in order to match at the output. Note that the magnitude of Γ_{in} does not change on the line $\Rightarrow$ only the phase. Thus, the strategy is to use the first 50 Ω line (θ_1) and first $\lambda/4$ transformer to produce an input impedance Z' with reflection coefficient magnitude of $3/5$. The second transmission line (θ_2) is then used to move the impedance to the real axis (to normalized value of 0.25). The final $\lambda/4$ transformer then transforms this load to $50\,\Omega$.

Consider the first two transmission lines (again, refer to the figure below):

$$Z = \frac{Z_M^2}{Z'} = Z_0\frac{1+\Gamma}{1-\Gamma} \quad \text{and} \quad Z' = \frac{Z_M^2}{Z_0}\frac{1-\Gamma}{1+\Gamma}$$

but

$$\Gamma' = \frac{Z' - Z_0}{Z' + Z_0} = \frac{Z_M^2(1-\Gamma) - Z_0^2(1+\Gamma)}{Z_M^2(1-\Gamma) + Z_0^2(1+\Gamma)} = \frac{\gamma - \Gamma}{1 - \gamma\Gamma}$$

where,

$$\gamma \equiv \frac{Z_M^2 - Z_0^2}{Z_M^2 + Z_0^2}.$$

Now, we want

$$|\Gamma'|^2 = \left(\frac{3}{5}\right)^2 = \left(\frac{\gamma - \Gamma}{1 - \gamma\Gamma}\right)\left(\frac{\gamma - \Gamma}{1 - \gamma\Gamma}\right)^* \equiv \rho^2 = \frac{\gamma^2 - \gamma\Gamma - \gamma\Gamma^* + |\Gamma|^2}{1 - \gamma\Gamma - \gamma\Gamma^* + \gamma^2|\Gamma|^2}$$

$$|\Gamma|^2 - \Gamma\frac{\gamma(1-\rho^2)}{1-\gamma^2\rho^2} - \Gamma^*\frac{\gamma(1-\rho^2)}{1-\gamma^2\rho^2} = \frac{\rho^2-\gamma^2}{1-\rho^2\gamma^2}$$

completing the square gives us the equation of a circle:

$$\left|\Gamma - \frac{\gamma(1-\rho^2)}{1-\gamma^2\rho^2}\right|^2 = \frac{\rho^2(1-\gamma^2)^2}{(1-\gamma^2\rho^2)^2} \qquad (A)$$

with center, Γ_c of,

$$\Gamma_c = \frac{\gamma(1-\rho^2)}{1-\gamma^2\rho^2}$$

and radius, r of

$$r = \frac{\rho(1-\gamma^2)}{1-\gamma^2\rho^2}$$

Expression (A) gives the contour on which Γ (The reflection coefficient seen on line θ_1 at the input to the first slug) must lie in order for the slug tuners to produce a match. With $\rho = 3/5$ and $\gamma = -3/5$, we have $\Gamma_c = -0.4412$ and $r = 0.4412$. This circle is plotted in the figure below. The magnitude of Γ_L must lie somewhere on this circle after the phase θ_1 is appropriately adjusted. Thus, the allowed region of the Smith Chart is found by sweeping this circle about the Smith Chart through all possible angles (again refer to the figure below).

