

**ECE 457- Microwave Devices and Circuits**

Fall 2026

**Homework 4- Smith Chart and Lumped Matching Network Solutions****1. Extended Smith Chart**

The normalized admittance is related to the reflection coefficient by,

$$y = \frac{1 - \Gamma}{1 + \Gamma} = \frac{1 - u - jv}{1 + u + jv} = g + jb$$

Multiplying top and bottom by the conjugate of the denominator yields,

$$g + jb = \frac{(1 - u) - jv}{(1 + u) + jv} \cdot \frac{(1 + u) - jv}{(1 - u) - jv} = \frac{(1 - u^2 - v^2) - j2v}{(1 + u)^2 + v^2}$$

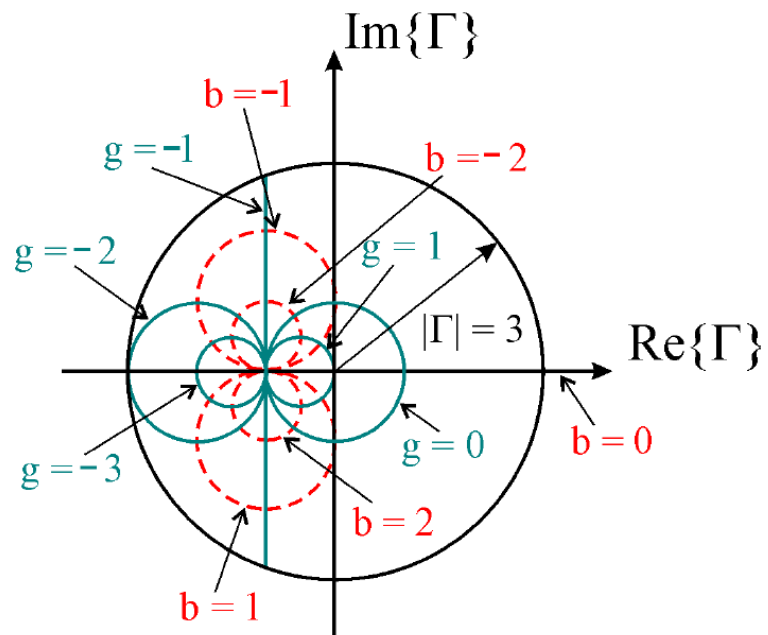
Equating the real and imaginary parts of each side, respectively, we obtain,

$$\begin{aligned} g(1 + u)^2 + gv^2 &= 1 - u^2 - v^2 & b(1 + u)^2 + bv^2 &= -2v \\ u^2(1 + g) + 2gu + v^2(1 + g) &= 1 - g & b(u + 1)^2 + bv^2 &= -2v \\ u^2 + 2\frac{g}{1 + g}u + v^2 &= \frac{1 - g}{1 + g} & (u + 1)^2 + v^2 + \frac{2}{b}v &= 0 \end{aligned}$$

Completing the squares (by adding  $(g/1 + g)^2$  to both sides on the left and  $1/b^2$  to both sides on the right) we have,

$$\left(u + \frac{g}{1 + g}\right)^2 + v^2 = \left(\frac{1}{1 + g}\right)^2 \quad (u + 1)^2 + \left(v + \frac{1}{b}\right)^2 = \frac{1}{b^2}$$

These two equations describe circles corresponding to contours of constant normalized conductance ( $g$ ) and susceptance ( $b$ ). Substituting in  $g = -3, -2, -1, 0, 1$ , and  $2$  and  $b = -2, -1, 0, 1$ , and  $2$ , we get the plot shown on the following page.

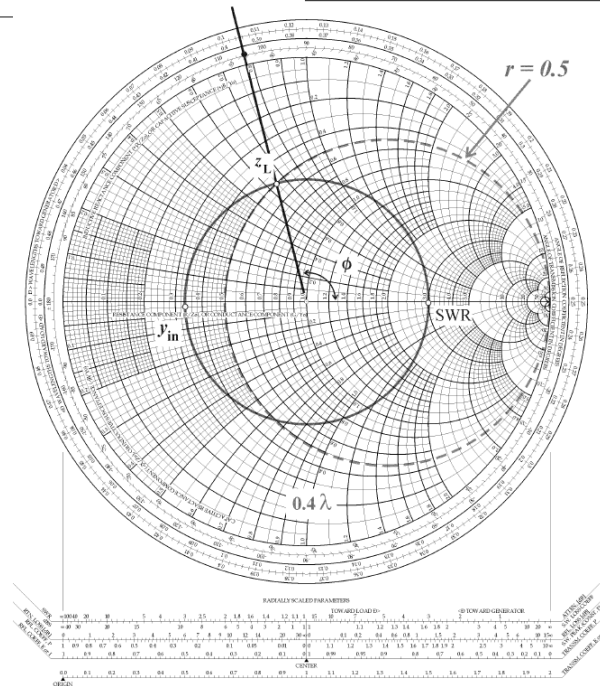


Problem 1. The Smith conductance chart for reflection coefficients with  $|\Gamma| \leq 3$ .

3. Refer to the Smith Chart shown below. Since we know the SWR is 3, we begin by plotting the SWR circle on the Smith Chart.

Reading off the chart, we find:

- (a)  $|\Gamma| = 0.5$
- (b) The SWR circle intersects the  $r = 0.5$  circle at  $x = \pm 0.65$ . Since the load is known to be *inductive*,  $x = 0.65$  and  $\phi = 104^\circ$ .
- (c) The reactive part of the load impedance is  $x = 0.65$  or  $X_L = \omega L = x \cdot Z_0 = 32.5 \Omega$
- (d) The first voltage maximum occurs at  $l_{\max} = 0.395\lambda - 0.25\lambda = 0.145\lambda$  from the load.
- (e) The input admittance to the line is  $y_{\text{in}} = 0.33 \Rightarrow Y_{\text{in}} = 6.6 \text{ mS}$ .



Problem 3. Smith chart plot for problem 3.

5. To begin, we draw the contours for  $Q_n = 5$  on the Smith Chart (refer to the Smith Chart shown below). The centers and radii of these circles are:

$$C_Q = (0, \pm 0.2) \quad R_Q = 1.02$$

We wish for  $Z_{\text{in}} = 10 - j15 \Omega$  or  $z_{\text{in}} = 0.2 - j0.3$ , if we use  $50 \Omega$  as a normalizing impedance. We will design a “T” matching section based on the circuit shown in figure 5. Working backwards, we use the following steps to produce  $Z_{\text{in}}$  with  $Q_n = 5$ :

- (i) Add  $X_1$  to move from  $50\Omega$  (center of the Smith Chart) to point  $A$  where  $z_A = 1.0 + j2.0$ . The required reactance is

$$X_1 = \frac{\omega L_1}{Z_0} = 2.0$$

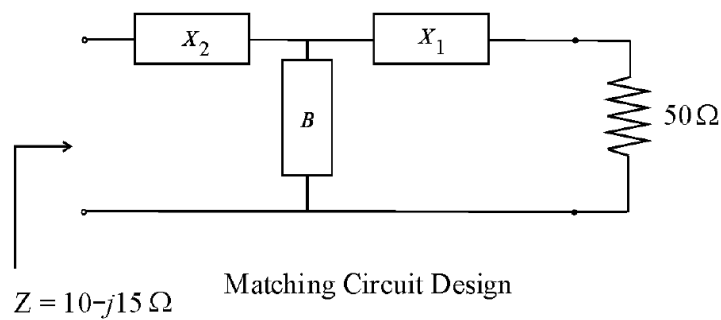
- (ii) Reflect through the origin of the Smith Chart to change to admittance. This point will lie on the rotated  $r = 1$  circle (point B) where  $y_B = 0.2 - j0.4$ .

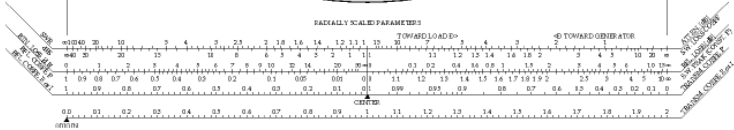
- (iii) Add the shunt susceptance  $B$  to move along the  $g = 0.2$  circle to point  $C$ , on the  $Q_n = 5$  circle (where  $y_C = 0.2 + j0.975$ ). The required value of the susceptance is,

$$B = \frac{\omega C}{Y_0} = 0.975 - (-0.4) = 1.375$$

- (iv) Reflect through the origin to change back to impedance (point  $D$ ) where  $z_D = 0.2 - j1.0$ .  
 (v) Add the series reactance  $X_2$  to move along the  $r = 0.2$  circle to point  $E$  where  $z_E = 0.2 - j0.3$ , as desired. The value of this final reactance is,

$$X_2 = \frac{\omega L_2}{Z_0} = -0.3 - (-1.0) = 0.7$$





**Pozar 2.17**

$$Z_L = jX$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX - Z_0}{jX + Z_0}$$

$$|\Gamma|^2 = \Gamma\Gamma^* = \frac{(jX - Z_0)(-jX - Z_0)}{(jX + Z_0)(-jX + Z_0)} = \frac{X^2 - jZ_0X + jZ_0X + Z_0^2}{X^2 + Z_0^2} = 1$$

**Pozar 5.1**

a)  $Z_L = 150 - j200\Omega$ ,  $z_L = 1.5 - j2$ , inside  $1 + jx$  circle

1. Shunt  $C$ , Series  $L$

$$b_1 = 0.107 \Rightarrow C = \frac{b}{2\pi f Z_0} = 0.0568 pF$$

$$x_1 = 1.78 \Rightarrow L = \frac{xZ_0}{2\pi f} = 9.44 nH$$

2. Shunt  $L$ , Series  $C$

$$b_2 = -0.747 \Rightarrow L = \frac{-Z_0}{2\pi f b} = 7.10 nH$$

$$x_2 = -1.78 \Rightarrow C = \frac{-1}{2\pi f x Z_0} = 0.298 pF$$

b)  $Z_L = 20 - j90\Omega$ ,  $z_L = 0.2 - j0.9$ , outside  $1 + jx$  circle

1. Series  $L$ , Shunt  $C$

$$x_1 = 1.30 \Rightarrow L = \frac{xZ_0}{2\pi f} = 6.90 nH$$

$$b_1 = 2.00 \Rightarrow C = \frac{b}{2\pi f Z_0} = 1.06 pF$$

2. Series  $L$ , Shunt  $L$

$$x_2 = 0.5 \Rightarrow L = \frac{-Z_0}{2\pi f} = 2.65 nH$$

$$x_2 = -2.00 \Rightarrow C = \frac{-Z_0}{2\pi f b} = 2.65 nH$$