ECE 457- Microwave Devices and Circuits

Fall 2026

Homework #2 – Plane Waves

Due: Monday Sept 14th

1. (a) The phasor $\vec{H}$-field of a uniform plane wave traveling in a source-free, unbounded medium is given by,

$$\vec{H} = \frac{1}{120\pi}(\hat{x} - 2\hat{y})e^{-jk_0 z}.$$

Find the corresponding phasor $\vec{E}$-field and the time-averaged power density ($Re\{\vec{E} \times \vec{H}^*\}$).

(b) Suppose the phasor electric field of a plane wave is given by,

$$\vec{E} = (\hat{x} + j\hat{z})e^{-jk_0 y} + (2\hat{x} - j\hat{z})e^{+jk_0 y}.$$

Assuming an unbounded, source-free medium, find the corresponding magnetic field and the time-averaged power density.

2. Consider the superposition of two plane waves traveling in opposite directions:

$$\vec{E} = \hat{x}Ae^{-jk_0 z} + \hat{x}Be^{jk_0 z},$$

where A and B are the rms amplitudes of the forward and reverse waves, respectively (assume A and B are real). Such a superposition of traveling waves is known as a "standing wave."

(a) Show that the magnitude of the electric field given above is a function of position z and find its maximum and minimum values. How far apart (in terms of the wavelength λ_0) are adjacent maxima and minima?

(b) Find the magnetic field $\vec{H}$ that is associated with the electric field given above.

(c) The ratio of the electric field phasor amplitude to the magnetic field phasor amplitude is called the "wave impedance" (note this, in general, is different from the "intrinsic wave impedance" which relates $\vec{E}$ and $\vec{H}$ for a traveling wave). Find the wave impedance for the standing wave given above. You should notice that the impedance is a function of position z.

(d) Suppose that $B = -A$ (as in the case of a plane wave reflected by a perfect conductor). Find the maximum and minimum electric field magnitudes, the corresponding magnetic field, and the wave impedance as a function of position, z.

3. The "standing wave ratio" or SWR is defined to be the ratio of the magnitude of *maximum* electric field amplitude to the magnitude of the *minimum* electric field amplitude,

$$SWR = \frac{|E|_{\max}}{|E|_{\min}}$$

(a) What is the SWR of a uniform plane wave traveling in an unbounded, lossless medium?

(b) Suppose the uniform plane wave in part (a) is incident from air onto the planar surface of a dielectric with refractive index of n. Find the SWR in the air.

(c) A plane wave is normally incident from air upon the (planar) surface of some unknown dielectric material. The SWR in air is measured to be 3 and the distance from the interface to the *first* minimum in the electric field magnitude is $5\lambda_0/16$, where λ_0 is the wavelength in free space. Find the refractive index of the dielectric.

4. A plane wave in free space is normally incident on a half-space of a very good conductor (with electrical parameters μ_0, ϵ, σ). (a) Show that the ratio of the reflected to the incident average power is given approximately by,

$$|\Gamma_0|^2 \approx 1 - 2k_0\delta$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ and δ is the skin depth. (b) Using this result calculate and compare the fraction of incident microwave power at 10 GHz that would be dissipated in a thick block of copper, brass, and stainless steel (refer to page 704 of Pozar for the relevant conductivities). Express your answers in decibels $(10\log[P_{\text{diss}}/P_{\text{inc}}])$.

5. Pozar 1.7 A plane wave is normally incident upon a dielectric slab of permittivity ϵ_r and thickness d, where $d = \lambda_0/(4\sqrt{\epsilon_r})$ and λ_0 is the free-space wavelength of the incident wave. (a) If free-space exists on both sides of the slab, find the reflection coefficient of the wave reflected from the front of the slab. (b) Repeat for the case when $d = \lambda_0/(2\sqrt{\epsilon_r})$.