Laser Dynamics and Pulsed Lasers

ECE 455 Optical Electronics

Gary Eden
Tom Galvin

If changes need to be made to these notes,
please contact Kavita Desai: kvdesai2@illinois.edu

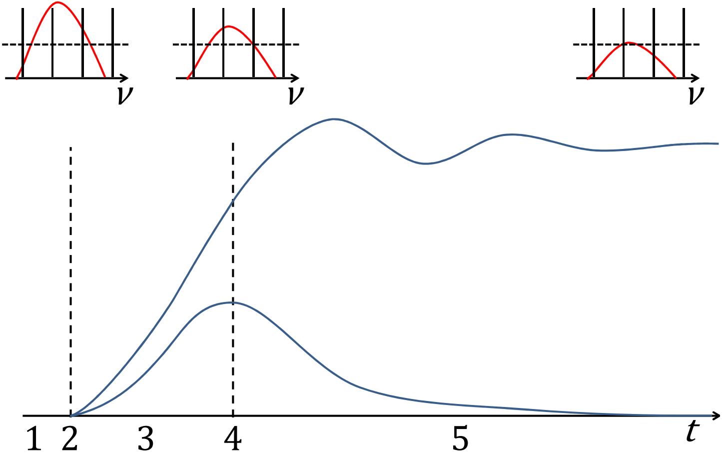
ECE Illinois

Introduction

In this section, the following subjects will be covered:

- Non-steady state behavior of lasers
- Motivations for pulsing lasers
- Methods for pulsing lasers

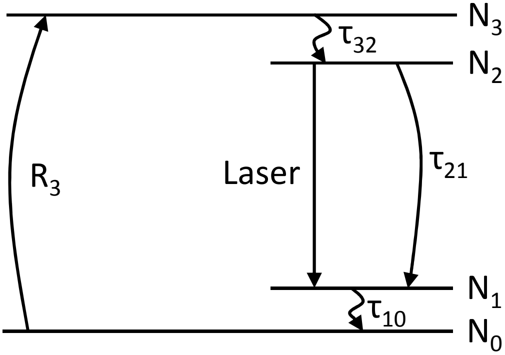
Starting a Laser I - Diagram



Starting a Laser II

1. Before the cavity has begun lasing, the cavity has defined discrete optical modes which may oscillate.
2. When the laser is started these modes are seeded by photons emitted via spontaneous emission.
3. All modes which are above threshold when the gain is unsaturated grow exponentially, but the mode with the highest net gain will grow more rapidly than all others.
4. The mode with the highest gain will reach a level where it saturates the medium before the other modes. The gain for all modes falls until only one is above threshold.
5. The other modes decay away and a single mode is left lasing.

Starting a Laser Level Diagram



Throughout the analysis that follows, we'll analyze a four-level system as shown above

Starting a Laser III Seeding Rate

The first question to answer is how long will it take for a cavity mode above threshold to become seeded? The density of states for photons per unit volume and frequency is:

$$\rho(\nu)d\nu = \frac{8\pi n^3\nu^2}{c^3}d\nu \tag{1}$$

Spontaneous emission can emit into any of these modes. But only one of these modes is the mode with the highest gain

$$\begin{aligned}\frac{d\phi}{dt} &= \frac{N_2 V_c}{\tau_{21}} g(\nu_{21})\Delta\nu_{21}\frac{1}{V_c\rho(\nu)\Delta\nu_{21}} &(2)\\ &= \frac{N_2}{\tau_{21}} g(\nu_{21})\frac{1}{\rho(\nu)} &(3)\\ &= \eta_{seed}\frac{N_2}{\tau_{21}} &(4)\end{aligned}$$

Starting a Laser IV Buildup Time

How long will it take for a single photon to stabilize into laser output? The small signal gain form of the gain equation may be used because saturation effects can be ignored while the intracavity intensity is low.

$$I_{sat} = \frac{h\nu}{A\tau_{21}} e^{(\gamma_0 - \gamma_{th})ct/n} \tag{5}$$

Therefore another way to estimate the buildup time is:

$$\tau_{buildup} = \frac{n}{(\gamma_0 - \gamma_{th})c} ln\left(\frac{I_{sat} A\tau_{21}}{h\nu}\right) \tag{6}$$

Starting a LaserV Buildup Time II

There are many ways to estimate the buildup time. The net gain per round trip is

$$G_{RT} = R_1 R_2 \exp\left[2\gamma_0 L_g\right] \tag{7}$$

The number or round trips necessary to reach a total gain of G is

$$N = \frac{\log G}{\log G_{RT}} \tag{8}$$

The round trip time

$$t_{RT} = \frac{2nL_g + 2(L - L_g)}{c} \tag{9}$$

Therefore the buildup time is

$$\tau_{buildup} = \frac{2nL_g + 2(L - L_g)}{c} \frac{\log G}{\log G_{RT}} \tag{10}$$

Example: Laser Buildup Time

Problem: Estimate the buildup time of a

Solution:

Starting a Laser: Alternative View

Laser dynamics can be approximated with the following model

$$\frac{d\phi}{dt} = -\frac{\phi}{\tau_c} + \sigma_{se}\left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{cL_g}{L + L_g(n-1)} \tag{11}$$

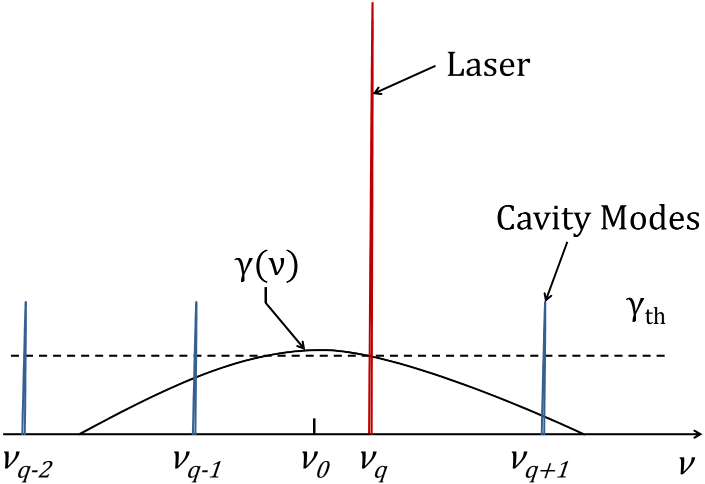
$$\frac{dN_1}{dt} = -\frac{N_1}{\tau_1} + \sigma_{se}\left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c}{A(L + L_g(n-1))} + \frac{N_2}{\tau_2} \tag{12}$$

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau_2} - \sigma_{se}\left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c}{A(L + L_g(n-1))} + P(t) \tag{13}$$

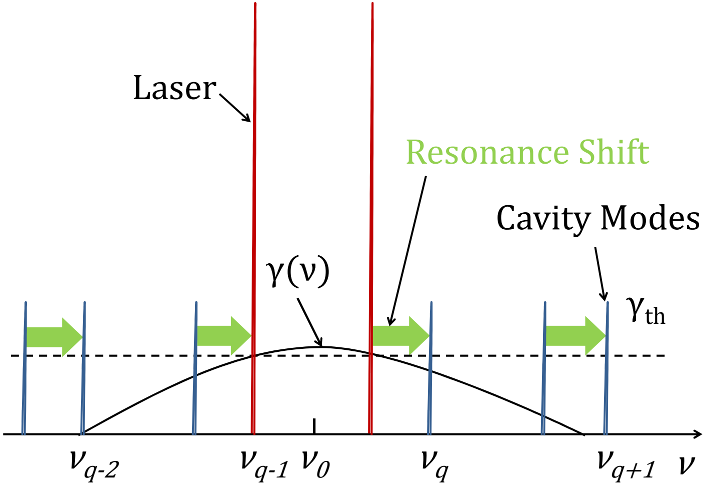
There is no simple analytic solution to this nonlinear system of equations; they must be solved numerically.

Laser Spiking

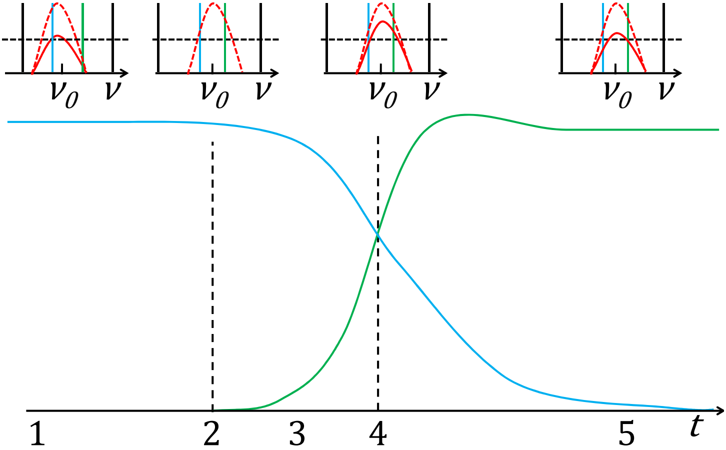
Mode Hopping



Mode Hopping



Mode Hopping



Mode Hopping

1. Laser minding own business
2. Change in laser cavity
3. New highest-gain mode grows exponentially
4. New highest-gain mode suppresses original mode
5. Old mode decays away

Mode Hopping

$$\frac{d\phi_{mpq}}{dt} = (\gamma - \gamma_{th})\phi_{mpq} - \frac{\phi_{mpq}}{\tau_c} \tag{14}$$

Mode Hopping

Mode hopping may be caused by:

- Changes in cavity resonance
  - Mirror separation changes (only need $\lambda/4$)
  - Index of refraction changes (thermal effects)
- Elements in the cavity which provide feedback at selected wavelengths
  - Intracavity etalon
  - Diffraction grating as end mirror (Littrow, Litman-Metcalf)

The mode with the most gain will always win

Relaxation Oscillations

Relaxation oscillations occur in lasers where $\tau_{21} \gg \tau_c$

$$\phi' = \phi_{ss} + \phi(t) \tag{15}$$

$$\Delta N' = \Delta N_{ss} + \Delta N(t) \tag{16}$$

$$\phi(t) \approx exp\left[-\frac{\sigma_{se} c \phi_{ss}}{2} t\right] sin\left[\sigma_{se} c (\phi_{ss} \Delta N_{ss})^{1/2} t\right] \tag{17}$$

Why Pulse a Laser?

- Physical parameters of gain medium
- Increase **peak** output power
    - Nonlinear optical processes scale as I^n, where n is the order of the nonlinearity and I is the intensity of the optical field.
- Extreme pumping requirements for threshold gain
- Increase laser bandwidth
- Time resolved spectroscopy

Example: Pumping Requirements of an Excimer

Problem: Predict the threshold pumping rate for a KrF laser. The laser has the following properties: $R_1 = 0.99$, $R_2 = 0.04$, $L = 1$ m, $A_{mode} = 5.25$ cm^2, $\lambda = 248$ nm, $\tau_2 = 5$ ns and $\sigma_{se} = 2.6$ $\AA^2$. Only 15% of pump energy will go into upper state formation.

Solution: The first step is to calculate the threshold gain:

$$\gamma_{th} = -\frac{1}{2L} ln(R_1 R_2) = 0.016 \text{ cm}^{-1}$$

This requires:

$$\Delta N_{th} = \frac{\gamma_{th}}{\sigma_{se}} = 6.21 \times 10^{13} \text{ cm}^{-3}$$

Example: Pumping Requirements of an Excimer

The required volumetric pumping rate at threshold is:

$$R = \frac{1}{\eta}\frac{hc}{\lambda}\frac{\Delta N_{th}}{\tau_2} = 66.3 \text{ kW-cm}^{-3}$$

which means the total pump power must be:

$$P = RV = (66.3 \text{ kW-cm}^{-3}) \cdot (100 \text{ cm}) \cdot (5.25 \text{ cm}^2) = 34.8 \text{ MW}$$

Characterizing Pulsed Lasers

Two parameters are commonly used to characterize pulsed lasers. The first is the average output power:

$$P_{ave} = E_{pulse} f_{rep} \tag{18}$$

The second is the peak power, which may be approximated as follows:

$$P_{peak} = \frac{E_{pulse}}{\Delta t} \tag{19}$$

Where E_{pulse} is the energy per pulse and Δt is the FWHM of the pulse.

Pulse Laser Characterization Example

Problem: LOPE's femtosecond laser produces 40 fs pulses containing up to 4.5 mJ of energy at a repetition rate of 1 kHz. Find the peak and average power of the laser.

Solution:

$$P_{ave} = (4.5 \text{ mJ}) \cdot (1 \text{ kHz}) = 4.5 \text{ W} \tag{20}$$

$$P_{peak} \approx \frac{4.5 \text{ mJ}}{40 \text{ fs}} = 112.5 \text{ GW !!!} \tag{21}$$

For comparison, summer electricity demand in the US is 783 GW.

The Idea

- In steady state the round trip gain must be equal to one for a laser.
- Therefore in steady state ΔN is locked to ΔN_{th}
- This limits the rate at which energy can be extracted
- Nothing prevents $\Delta N \gg \Delta N_{th}$ on a *transient* basis.

Q-Spoiling (Pump and Dump)

The process:

1. Create an extremely high Q cavity and pump continuously
2. Allow the CW intensity to build up inside the cavity
3. Suddenly lower the cavity Q by increasing the strength of the output coupling.

Discussion:

- Intracavity intensity is much greater than output-coupled light
- Minimum pulse duration limited by round trip time of cavity
- Maximum intensity limited by cavity losses
- Don't try this on Wall St.

Pulse the Pump

The process:

1. Rapidly switch on a high-powered pumping mechanism
2. Wait for a pulse to come out

Discussion:

- Minimum pulse duration is limited by cavity build up time or speed of pump
- This approach requires fast, high power electronics.
- This is the most primitive method. It is commonly used in conjunction with Q-Switching.

Q-Switching

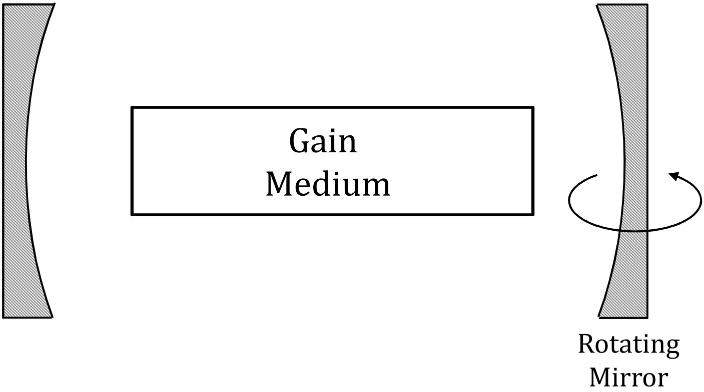
The process:

1. Pump at low rate with cavity Q spoiled to prevent oscillation
2. Generate a large population inversion $\Delta N > \Delta N_{th}$
3. Quickly restore Q to a high value to allow laser oscillation
4. Laser pulse extracts energy from inversion, driving $\Delta N < \Delta N_{th}$ (absorption)
5. Laser pulse terminates
6. Turn off Q Switch and repeat

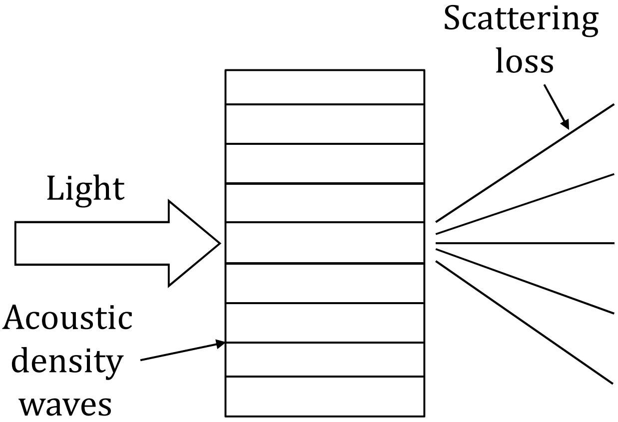
Two types of Q-Switches are:

- Rotating mirror
- Pockells cell

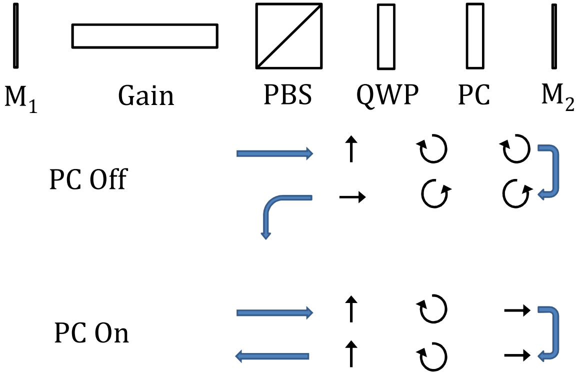
Q-Switching Methods: Rotating Mirror



Q-Switching Methods: Acousto-Optic Modulator (AOM)



Q-Switching Methods: Pockels Cell



PBS = Polarizing Beam Splitter; QWP = Quarter Wave Plate; PC = Pockels Cell

Q-Switch: Energy Storage

- The length of time that energy can be stored is limited by the lifetime of the of upper state. The lifetime sets an upper limit on the maximum useful pump duration.
- Energy storage in an amplifier or laser is limited by the onset of parasitic oscillations
- Small stimulated emission cross sections keep the and allow a large population in the upper state
- An ideal amplifier material has a high fluorescence lifetime and a small stimulated emission cross section

Energy Storage Example

Problem: Nd:YAG and Nd:YLF both lase at similar wavelengths (1064 nm and 1053 nm respectively). Calculate the maximum amount of energy that can be stored in both a Nd:YAG and Nd:YLF amplifier before parasitic oscillations begin. Assume the amplifier crystal is 8 cm long and that 1% of radiation is scattered back at each interface.

For Nd:YAG

$$\sigma_{se} = 2.8 \times 10^{-19}\ \text{cm}^2$$

For Nd:YLF

$$\sigma_{se} = 1.8 \times 10^{-19}\ \text{cm}^2$$

Solution: The first step is to calculate the threshold gain:

$$\gamma_{th} = -\frac{1}{2L} ln(R_1 R_2) = -\frac{1}{2 \cdot 8cm} ln(.01 \cdot 0.1) = 0.576\ \text{cm}^{-1}$$

Energy Storage Example Continued

From the threshold gain, we can calculate the threshold inversions of both of these lasers. From the inversion density, we can easily calculate the energy storage density.

$$\Delta N_{th}(YAG) = \frac{\gamma_{th}}{\sigma_{se}} = \frac{0.576 \text{ cm}^{-1}}{2.8 \times 10^{-19} \text{ cm}^2} = 2.06 \times 10^{18} \text{ cm}^{-3}$$

$$\rho(YAG) = \frac{hc\Delta N_{th}}{\lambda} = 0.384 \text{ J-cm}^{-3}$$

$$\Delta N_{th}(YLF) = \frac{\gamma_{th}}{\sigma_{se}} = \frac{0.576 cm^{-1}}{1.8 \times 10^{-19} \text{ cm}^2} = 3.20 \times 10^{18} \text{ cm}^{-3}$$

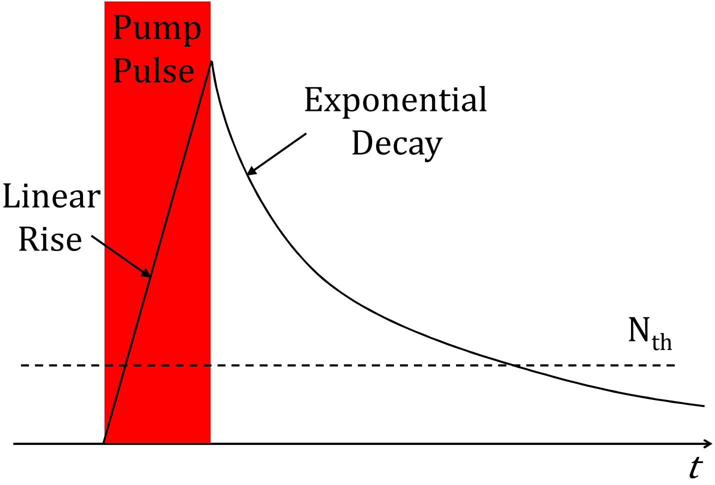
$$\rho(YLF) = \frac{hc\Delta N_{th}}{\lambda} = 0.597 \text{ J-cm}^{-3}$$

This analysis is of course approximate.

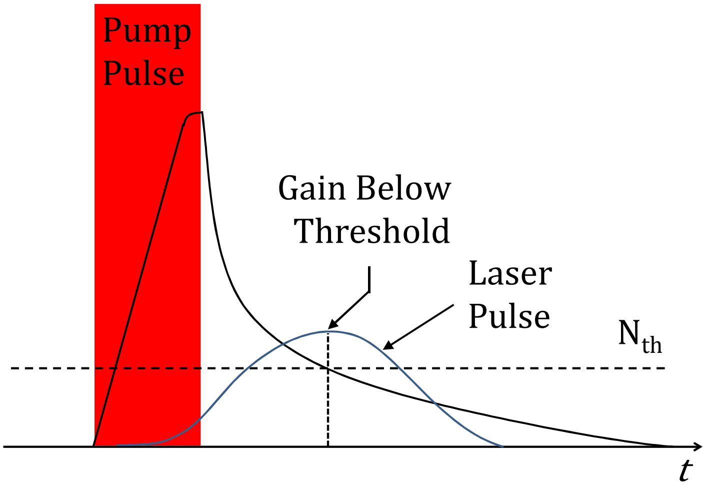
Q-Switch Discussion

- Minimum pulse duration limited by cavity build up time
- Maximum energy of pulse is limited by energy storage density inside medium and how efficiently it can be extracted

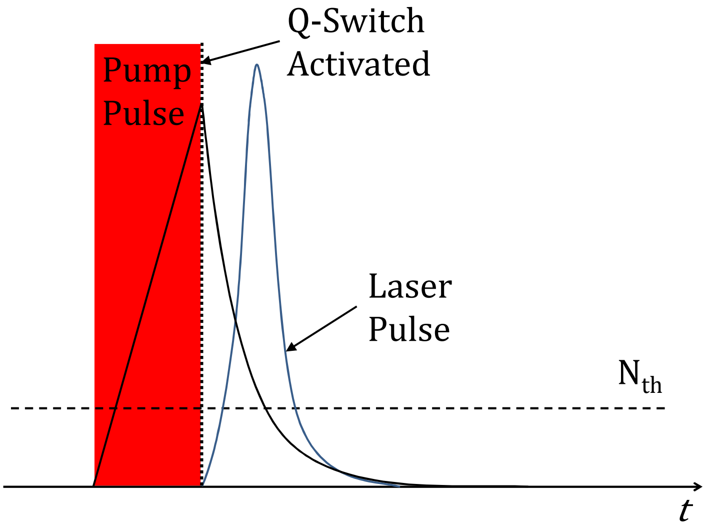
Pulsing Methods: Medium Behavior Outside Cavity



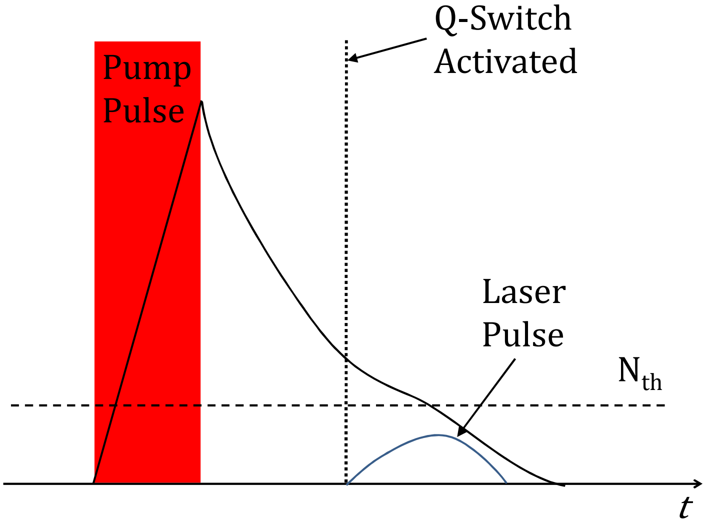
Pulsing Methods: Pulsing the Pump



Pulsing Methods: Q-Switch



Pulsing Methods: Poorly Timed Q-Switch



Keep Track of Assumptions

- Pump pulse much faster than relaxation time
- Pumping rate will not have a nice flat-top
- Saturation of the pumping process has been ignored
- Spatial effects (transverse and longitudinal) have been ignored

Mode Locking

In our discussion of homogeneously broadened media, the mode with the highest net gain would oscillate to the exclusion of other modes. What if a coherent superposition of cavity modes could have lower loss (and thus higher net gain) than any individual longitudinal mode? This is the idea behind the technique known as modelocking.

First let us review Fourier series.

Fourier Series

Recall that any periodic function may be represented as a weighted sum of complex exponentials

$$a(t) = \sum_{n=-\infty}^{\infty} c_n \cdot exp\left[\imath n \frac{2\pi}{T} t\right] \tag{22}$$

where

$$c_n = \frac{1}{2T} \int_0^T a(t) exp\left[-\imath n \frac{2\pi}{T} t\right] dt \tag{23}$$

Properties of Fourier Series

- The fundamental frequency of $a(t)$ is $f_{rep} = \frac{1}{T}$
- The Fourier spectrum of $a(t)$ comprises delta functions separated by f_{rep}
- The more rapid the variations in $a(t)$, the more terms will be needed in the Fourier series to approximate $a(t)$

Optical Fourier Series

Recall that the frequencies of the longitudinal modes of a cavity are

$$\nu_q = q\frac{c}{2L_{opt}} \tag{24}$$

If several of these modes are oscillating simultaneously, the electric field may be written

$$\begin{aligned} E(t) &= \sum_q |E_q| exp\left[\imath\phi_q\right] \cdot exp\left[\imath q 2\pi\nu_q t\right] && (25) \\ &= exp\left[\imath 2\pi\nu_0 t\right] \sum_q |E_q| exp\left[\imath\phi_q\right] \cdot exp\left[\imath q 2\pi\nu'_q t\right] && (26) \\ &= exp\left[\imath 2\pi\nu_0 t\right] a(t) && (27) \end{aligned}$$

where $\nu'_q \equiv \nu_q - \nu_0$ and $a(t)$ is a periodic function.

Properties of Optical Fourier Series

- A relatively slowly-varying envelope, $a(t)$, is modulated by optical carrier of frequency ν_0
- The fundamental frequency of the envelope is $f_{rep} = \frac{c}{2L_{opt}}$
- The Fourier spectrum $E(t)$ comprises delta functions separated by f_{rep} and offset from the axes by ν_0
- The more rapid the variations in the envelope, the more terms will be needed in the Fourier series to approximate the pulse

Consider envelope functions of the form:

$$a(t) = \sum_{q=1}^{N} e^{\imath\phi_q} e^{\imath q \omega_0 t} \tag{28}$$

Properties of Optical Fourier Series (cont.)

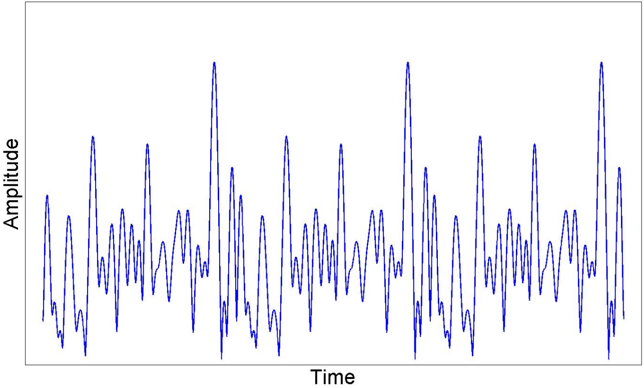
From Verdeyen, section 9.5.2, the electric field of a mode-locked laser can be written as

$$E(t) = E_0 e^{i\omega_0 t} \left[\frac{sin(N\omega_c t/2)}{sin(\omega_c t/2)} \right] \tag{29}$$

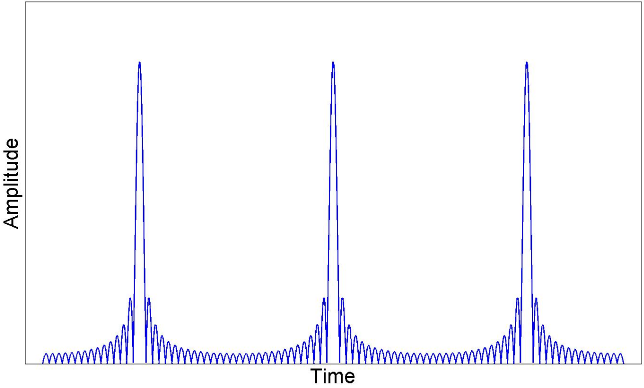
where ω_0 is the optical frequency, $\omega_c = 2\pi/\tau_{RT}$, and N is the number of longitudinal modes supported by the cavity and gain medium. This results in an intensity of the form

$$I(t) = \frac{E_0^2}{2\eta_0} \left[\frac{sin(N\omega_c t/2)}{sin(\omega_c t/2)} \right]^2 . \tag{30}$$

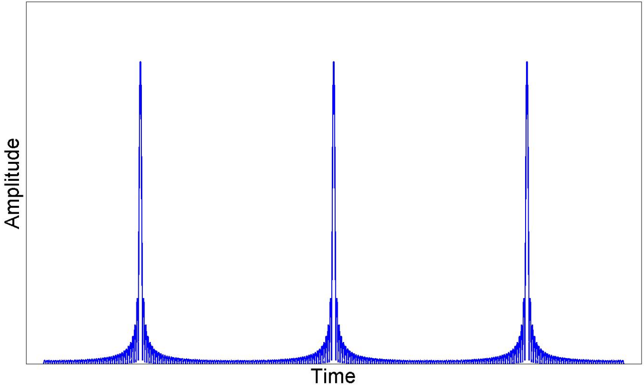
Multiple Longitudinal Modes: Random Phase N = 30



Multiple Longitudinal Modes: Zero Phase $N = 30$



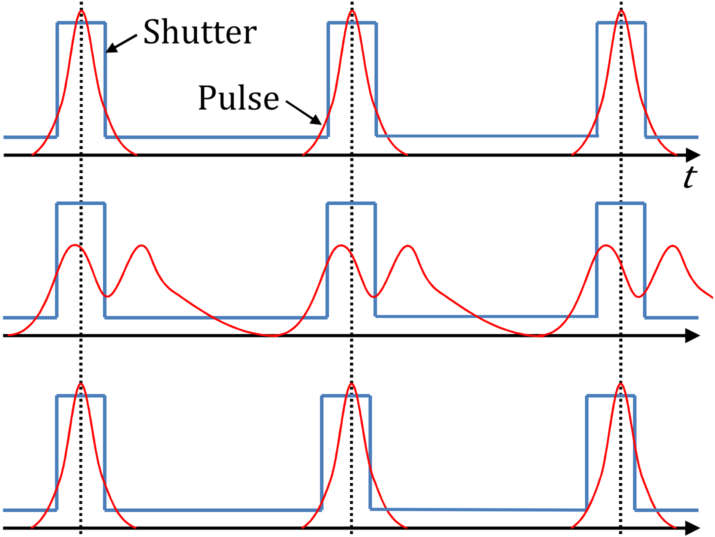
Multiple Longitudinal Modes: Zero Phase N = 90



Lessons

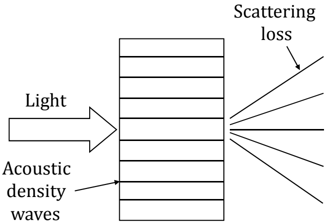
- Shortest pulses produces when phases locked together
- Greater number of locked modes produce shorter pulses
- Need a method to lock the phases
  - Make the pulsed mode lower loss than the CW mode
  - Introduce a shutter to modulate loss of cavity
- On the next slide you will see
  - Top: Pulses with modes phased so that the envelope fits in the low loss window of the shutter
  - Middle: Pulses with modes phased so that the envelope does not fit in the low loss window of the shutter
  - Bottom: A shutter which is not synchronous with the repetition rate
  - Which has the lowest loss?

Modelocking Shutter Picture



Active Mode Locking: Acousto-Optic Modulator

- An acoustic transducer is coupled to a crystal
- The acoustic waves forms a standing wave pattern
- The spatially periodic variation in density forms a grating, which scatters the laser beam
- This can be modulated rapidly. The modulation must be synchronized with the cavity round trip time.



Passive Mode Locking: Saturable Absorber

- An absorber is placed inside the laser cavity
- Low intensity light, such as noise is absorbed by the absorber
- High intensity 'bleaches' the medium, making it transparent.
- It only works if the recovery time of the medium is much less than the round trip time.

Passive Mode Locking: Kerr Lens

- Relies on a third order nonlinearity known as self focusing
    - $n = n_1 + n_2 I$
- The higher peak intensity (pulsed) mode suffers lower diffraction losses than any individual longitudinal mode.
- Kerr lensing is a nonlinear optical process. Hence it is (approximately) instantaneous
- Discovered by graduate students who forgot to turn the AOM on

Pulse Propagation in a Material Medium I

- The actual condition for resonance of the qth longitudinal cavity resonance is

$$\phi_{RT} = q \cdot 2\pi \tag{31}$$

- Accounting for the crystal in the cavity, the previous condition becomes:

$$\nu_q = \frac{q \cdot c}{2(L - L_g + n(\nu)L_g)} \tag{32}$$

- Because n is a function of ν, the modes are not exactly evenly spaced
- Adjacent longitudinal modes must be evenly spaced to create a periodic envelope (recall Fourier series)

Pulse Propagation in a Material Medium II

The phase shift from propagating through the gain medium once is

$$\begin{aligned} \phi(\omega) &= \frac{2\pi}{\lambda} L_g n(\omega) \qquad (33) \\ &= \frac{2\pi}{\lambda} L_g \left[n(\omega_0) + \frac{\partial n}{\partial \omega}(\omega - \omega_0) + \frac{1}{2}\frac{\partial^2 n}{\partial \omega^2}(\omega - \omega_0)^2 + \ldots \right] \end{aligned}$$

Which terms are important?

- $n(\omega_0)$ - constant phase
- $\frac{\partial n}{\partial \omega}(\omega - \omega_0)$ - linear phase ⇔ group delay
- $\frac{1}{2}\frac{\partial^2 n}{\partial \omega^2}(\omega - \omega_0)^2$ - quadratic phase ⇔ group delay dispersion (GDD)
- $\frac{1}{6}\frac{\partial^3 n}{\partial \omega^3}(\omega - \omega_0)^3$ - cubic phase ⇔ third order dispersion (TOD)

Pulse Propagation in a Material Medium III

The Fourier transform of a pulse is

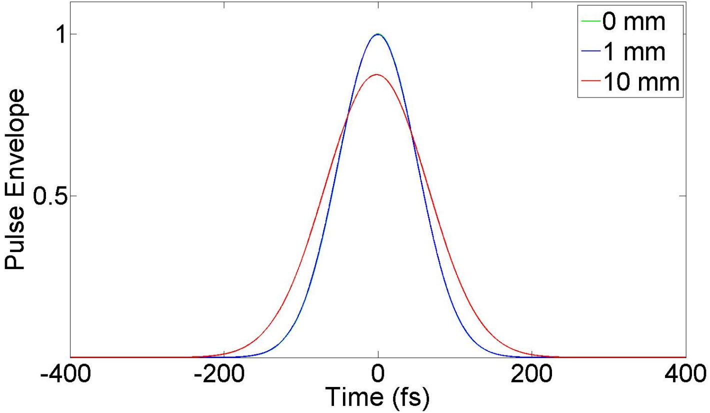
$$E(\omega) = A(\omega)e^{\imath\omega_0 t} \tag{34}$$

where $A(\omega)$ is the Fourier transform of the envelope and $e^{\imath\omega_0 t}$ is the optical carrier frequency.

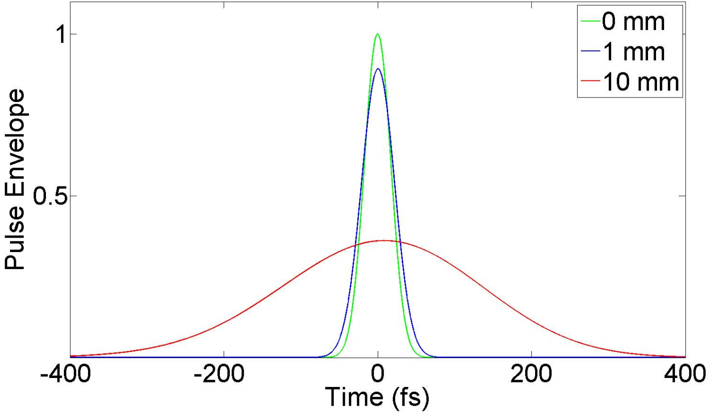
The Fourier transform of the pulse after is has propagated through a material medium is

$$E'(\omega) = A(\omega)e^{\imath\omega_0 t}e^{\imath\phi(\omega)} \tag{35}$$

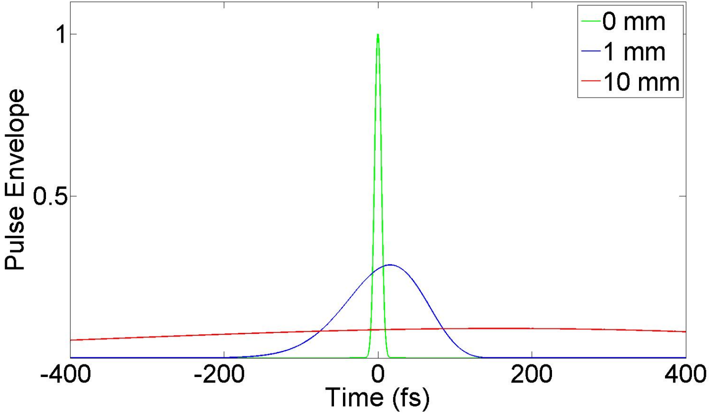
Propagation Through Sapphire 120 fs Pulse



Propagation Through Sapphire 40 fs Pulse



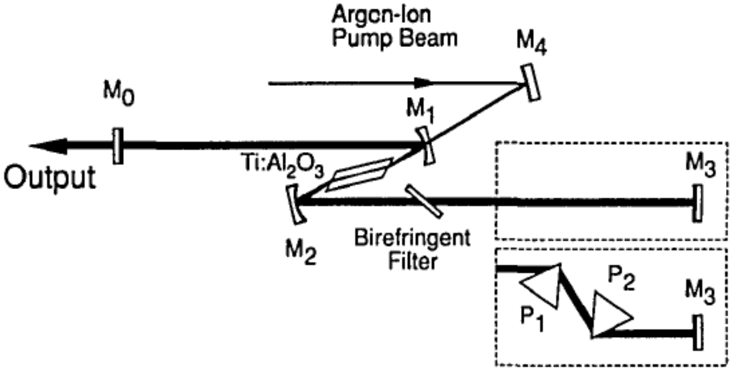
Propagation Through Sapphire 10 fs Pulse



Lessons

- We need a mechanism to counteract the dispersion present in the cavity
    - Prisms
    - Chirped mirrors
    - Photonic Crystals
- Pulse dispersion is more severe for shorter pulses
- Propagation through materials is bad. Use reflective optics.

A Mode-Locked Oscillator



D. E. Spence et al. Optics Letters **16**, 42 (1991)

CW and Modelocked Operation

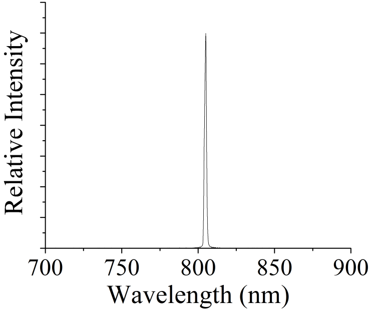


Figure: A Ti:Sapphire laser operating CW.

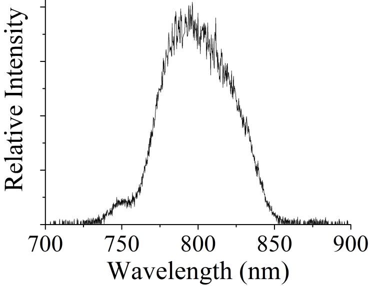


Figure: The same laser modelocked.

One More Option

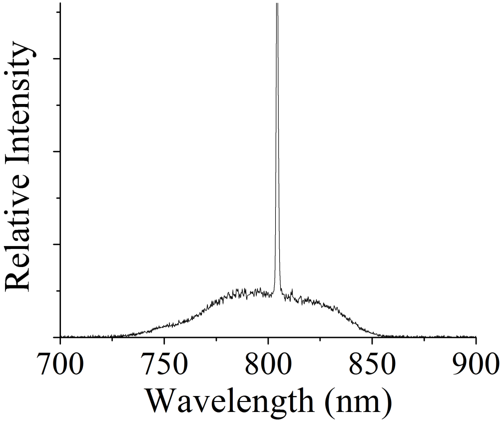


Figure: A Ti:Sapphire laser in modelocked operation with continuous wave breakthrough

Example: Shortest Possible Pulses

Of all pulsing techniques, mode locking produces the shortest pulses. As a first estimate, the shortest which can be made with using an optical transition is:

$$\Delta t \approx \frac{1}{\Delta f} \tag{36}$$

where Δf is the FWHM bandwidth of the pulse.

Example Shortest Pulse

Problem: As a gain medium, Ti:Sapphire exhibits gain from roughly 650 nm to 1100 nm. What is the shortest possible pulse which a Ti:Sapphire laser can generate?

Solution:

$$\Delta f = \frac{c}{650 \text{ nm}} - \frac{c}{1100 \text{ nm}} = 1.89 \times 10^{14} \text{ Hz}$$

Therefore, an estimate for the shortest pulses possible is:

$$\Delta t \approx \frac{1}{\Delta f} = 5.29 \text{ fs}$$

For comparison, a single optical cycle at 800 nm (the peak of Ti:Sapphire gain spectrum) is

$$T = \frac{800 \text{ nm}}{c} = 2.67 \text{ fs}$$

Example: Finding Number of Modes Locked Together

Problem: A mode-locked Ti:Sapphire oscillator has mirrors which are 1.8 m apart. Consider the spectrum in Figure 2, how many longitudinal modes are oscillating simultaneously?

Solution: The separation between longitudinal modes is simply the free spectral range of the cavity.

$$FSR = \frac{c}{2nL} = 83.3 \text{ MHz}$$

To determine the number of modes oscillating simultaneously, we take the entire spectrum bandwidth, not the FWHM. This bandwidth is:

$$\Delta f = \frac{c}{730 \text{ nm}} - \frac{c}{855 \text{ nm}} = 6 \times 10^{13} \text{ Hz}$$

The number of modes oscillating simultaneously is therefore:

$$\frac{\Delta f}{FSR} = 721000$$

Mode Locking Conclusions

In order to create a modelocked laser:

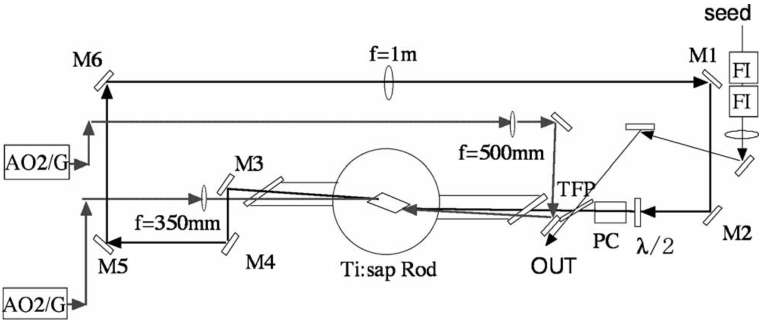
- There must be a mechanism to lock the phases of the various longitudinal modes.
- The cavity dispersion must go to zero.
- The shortest pulse possible is limited by one of four factors:
  - The recovery time of the mode locking mechanism
  - The bandwidth of the lasing transition
  - The reflectivity and dispersion of the cavity optics
  - The frequency of the optical carrier wave

Regenerative Amplification I

The pulse energy possible with mode locking is limited because the high repetition rate of oscillators would require a very high power pump laser. In order to generate high-powered short pulses, a technique known as regenerative amplification is used. The strategy is as follows:

1. Create low-energy high-repetition rate pulses from an oscillator
2. Use an optical switch to lower the repetition rate
3. Temporally stretch pulses by using diffraction gratings to 'chirp' the pulses
4. Amplify the pulse by passing it through another crystal multiple times.
5. Use another diffraction grating to remove the chirp introduced by the stretcher and cavity dispersion

A Regenerative Amplifier



A02/G - Pump Laser; FI - Faraday Isolator; PC - Pockels Cell; TFP - Thin Film Polarizer

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From the examples given, you may get the impression that Ti:Sapphire is the only medium for mode locking, not so!

- Dye Lasers
- Nd:YAG
- Cr:LiCAF and Cr:LiSAF
- Er and Yb doped fiber
- Semiconductor Lasers

Summary

- Lasers are 'seeded' from spontaneous emission
- Full models of laser dynamics are quite complicated
- Pulsing lasers can result in a dramatic increase in peak intensity
- Mode-locking produces the shortest laser pulses, but due to the high rep rate, the energy per pulse is low