UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

CS440/ECE448 Artificial Intelligence

Practice Exam 3

Spring 2026

Exam is April 6, 2026

Your Name: ____________________

Your NetID: ____________________

Instructions

- Please write your name on the top of every page.
- Have your ID ready; you will need to show it when you turn in your exam.
- This will be a CLOSED BOOK, CLOSED NOTES exam. You are permitted to bring and use only one 8.5x11 page of notes, front and back, handwritten.
- No electronic devices (phones, tablets, calculators, computers etc.) are allowed.
- Make sure that your answer includes only the variables that it should include, but DO NOT simplify explicit numerical expressions, including expressions involving standard functions (exp, log, sin, cos) and expressions involving the sum, product, maximum, or argmax of a complete list of alternatives. For example, the answer $x = \max\left(\sin(0.3), \frac{1}{1+\exp(-0.1)}\right)$ is MUCH preferred (much easier for us to grade) than the answer $x = 0.524979$.

Possibly Useful Formulas

$$\sigma(z)=\frac{1}{1+e^{-z}},\quad \sigma'(z)=\sigma(z)\left(1-\sigma(z)\right)$$

$$f=g\left(\mathbf{w}^T\mathbf{x}\right),\quad \frac{\partial f}{\partial \mathbf{w}}=g'\left(\mathbf{w}^T\mathbf{x}\right)\mathbf{x}$$

$$\text{softmax}(\mathbf{z})=\frac{\exp(\mathbf{z})}{\sum_i \exp(z_i)},\quad \frac{\partial\,\text{softmax}_k(\mathbf{z})}{\partial \mathbf{z}}=\underset{k}{\text{softmax}}(\mathbf{z})\left(\mathbb{1}_k-\text{softmax}(\mathbf{z})\right)$$

$$f[k]=\sum_i w[i]x[k-i],\quad \frac{\partial f[k]}{\partial w[i]}=x[k-i]$$

$$\mathbf{C}=\text{softmax}\left(\frac{\mathbf{QK}^T}{\sqrt{d}}\right)\mathbf{V}$$

$$S:\{\mathscr{V}_P,\mathscr{V}_Q\}\rightarrow\{\mathscr{V}_Q,\mathscr{C}\}\text{ s.t. }S(P)=S(Q)=U$$

$$\frac{x'}{x}=\frac{y'}{y}=\frac{-f}{z}$$

$$R_{c,d}=\frac{\frac{\partial z_c}{\partial x_d}x_d}{\sum_{d'}\frac{\partial z_c}{\partial x_{d'}}x_{d'}}R_c$$

$$\mathscr{C}_{\text{obs}}=\{\mathbf{c}:\exists\mathbf{b}:\phi(\mathbf{b},\mathbf{c})\in\mathscr{W}_{\text{obs}}\}$$

Admissible: $\hat{h}(n)\le h(n)$

Consistent: $\hat{h}(n)-\hat{h}(m)\le d(n,m)$

Bellman: $u(s)=r(s)+\gamma\max_a\sum_{s'}P(s'|s,a)u(s')$

Policy Eval: $u_i(s)=r(s)+\sum_{s'}P(s'|s,\pi_i(s))u_i(s')$

Policy Update: $\pi_{i+1}(s)=\arg\max_a\sum_{s'}P(s'|s,a)u_i(s')$

Value Iteration: $u_{i+1}(s)=r(s)+\max_a\sum_{s'}P(s'|s,a)u_i(s')$

Model-based: $P(s_{t+1}|s_t,a_t)=\frac{N(s_t,a_t,s_{t+1})+k}{\sum_{s'}\left(N(s_t,a_t,s')+k\right)}$

TD-Learning: $q(s_t,a_t)\leftarrow(1-\eta)q(s_t,a_t)+\eta\left(r_t+\gamma\max_a q_t(s_{t+1},a)\right)$

SARSA: $q(s_t,a_t)\leftarrow(1-\eta)q(s_t,a_t)+\eta\left(r_t+\gamma q_t(s_{t+1},a_{t+1})\right)$

Actor-Critic: $\mathscr{L}_{\text{actor}}=-\sum_a\pi_a(s)q(s,a)$

REINFORCE: $\Delta\mathbf{W}=\eta(r-\mu)\sum_t\frac{\partial\log\pi_{a_t}(s_t)}{\partial\mathbf{W}}$

Question 1 ***(28 points)***

Consider a neural network with the following architecture. The input is a scalar x, and there is a weight vector $\mathbf{w} = [w_1, \ldots, w_n]^T$. The vector outputs $\mathbf{f}$, $\mathbf{g}$ and $\mathbf{h}$ are defined as

$$\mathbf{f} = x\mathbf{w},$$
$$\mathbf{g} = \exp(\mathbf{f}),$$
$$\mathbf{h} = \frac{\mathbf{g}}{\sum_{i=1}^{n} g_i},$$

where $\exp(\cdot)$ is applied element-wise to its argument.

(a) (14 points) Suppose that you know $\frac{\partial \mathscr{L}}{\partial h_j} = 0$ for all j except $j = 2$, and suppose that $\frac{\partial \mathscr{L}}{\partial h_2}$ is known. In terms of $\frac{\partial \mathscr{L}}{\partial h_2}$ and any of the elements of $\mathbf{f}$, $\mathbf{g}$, $\mathbf{h}$, $\mathbf{w}$, find $\frac{\partial \mathscr{L}}{\partial g_2}$.

Solution:

$$\frac{\partial \mathscr{L}}{\partial g_2} = \left(\frac{1}{\sum_{i=1}^{n} g_i} - \frac{g_2}{\left(\sum_{i=1}^{n} g_i\right)^2} \right) \frac{\partial \mathscr{L}}{\partial h_2}$$

(b) (14 points) Suppose that you know $\frac{\partial \mathscr{L}}{\partial g_j}$ for all j. In terms of $\frac{\partial \mathscr{L}}{\partial g_j}$, and in terms of x and/or any of the elements of $\mathbf{f}$, $\mathbf{g}$, $\mathbf{h}$, $\mathbf{w}$, find $\frac{\partial \mathscr{L}}{\partial x}$.

Solution:

$$\frac{\partial \mathscr{L}}{\partial x} = \sum_i \sum_j \frac{\partial \mathscr{L}}{\partial g_i} \frac{\partial g_i}{\partial f_j} \frac{\partial f_j}{\partial x} = \sum_j \frac{\partial \mathscr{L}}{\partial g_j} g_j w_j$$

Question 2 ***(15 points)***

Consider an unusual convolutional neural network with input image $x[i,j]$, weights $w[i]$, and output image $f[i,j]$ related as:

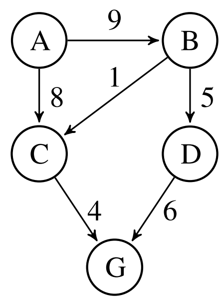
$$f[k,l] = \sum_i x[k-i, l-i] w[i]$$

In terms of the elements of x, w, and/or f, what is $\frac{\partial f[k,l]}{\partial x[i,j]}$?

Solution:

$$\frac{\partial f[k,l]}{\partial x[i,j]} = \begin{cases} w[k-i] & k-i = l-j \\ 0 & \text{otherwise} \end{cases}$$

Question 3 *(28 points)*



(a) (14 points) The search graph above starts at node A, and ends at node G. Create a table showing, in the first column, the node that is expanded by uniform cost search at each step of the search process (starting with A, ending with G), and in the second column, the set of nodes that are in the frontier after the node in the first column has been expanded. Optionally, you may list a priority next to each node in the frontier if you wish. Assume that, if a node has already been expanded, it need not be placed back in the frontier. Ties are broken in alphabetical order.

Solution:

Expand	Frontier
A	B:9, C:8
C	B:9, G:12
B	G:12, D:14
G	D:14

(b) (14 points) Suppose you're given the following heuristic, and asked to search the graph using an A* search: $\hat{h}(A) = 5, \hat{h}(B) = 5, \hat{h}(C) = 2, \hat{h}(D) = 2, \hat{h}(G) = 0$. Create a table showing, in the first column, the node that is expanded by A* search at each step of the search process, and in the second column, the set of nodes that are in the frontier after the node in the first column has been expanded. Optionally, you may list a priority next to each node in the frontier if you wish. Note that, since this heuristic is not consistent, you may need to expand a node more than once. Ties are broken in alphabetical order.

Solution:

Expand	Frontier
A	B(9,5), C(8,2)
C	B(9,5), G(12,0)
G	B(9,5)

Question 4 *(29 points)*

Consider an MDP with two states, $s \in \{0,1\}$, and two actions, $a \in \{0,1\}$. The states have rewards $r(0) = 9$ and $r(1) = 5$, and the transition probabilities are:

s,a	$P(s'=0\|s,a)$	$P(s'=1\|s,a)$
0,0	0.8	0.2
0,1	0.3	0.7
1,0	0.4	0.6
1,1	0.1	0.9

(a) (15 points) Suppose you start out with an initial policy that always tries action $a = 0$, i.e., $\pi_1(s) = 0$ for $s \in \{0,1\}$. The policy-dependent utility vector $\mathbf{u}_1 = [u_1(0), u_1(1)]^T$ can be computed by solving a linear equation of the form $\mathbf{Au} = \mathbf{b}$. Find the numerical values of the matrix $\mathbf{A}$ and the vector $\mathbf{b}$ assuming the discount factor $\gamma = \frac{1}{2}$.

Solution:

$$u_1(s) = r(s) + \gamma \sum_{s'} P(s'|s,\pi(s))u_1(s')$$

$$u_1(0) = 9 + \frac{1}{2}\left(0.8u_1(0) + 0.2u_1(1)\right)$$

$$u_1(1) = 5 + \frac{1}{2}\left(0.4u_1(0) + 0.6u_1(1)\right)$$

$$0.6u_1(0) - 0.1u_1(1) = 9$$

$$-0.2u_1(0) + 0.7u_1(1) = 5$$

$$\mathbf{A} = \begin{bmatrix} 0.6 & -0.1 \\ -0.2 & 0.7 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

(b) (14 points) The actual state utilities in this problem are approximately $u(0) = 17$, $u(1) = 12$. Assuming $\gamma = \frac{1}{2}$, what is the q-function, $q(s,a)$, that corresponds to these utilities?

Solution:

$$q(s,a) = r(s) + \gamma \sum_{s'} P(s'|s,a)u(s')$$

$$q(0,0) = 9 + \frac{1}{2}\left(0.8 \times 17 + 0.2 \times 12\right)$$

$$q(0,1) = 9 + \frac{1}{2}\left(0.3 \times 17 + 0.7 \times 12\right)$$

$$q(1,0) = 5 + \frac{1}{2}\left(0.4 \times 17 + 0.6 \times 12\right)$$

$$q(1,1) = 5 + \frac{1}{2}\left(0.1 \times 17 + 0.9 \times 12\right)$$

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