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Lecture 34: Actor-critic deep reinforcement learning

Mark Hasegawa-Johnson
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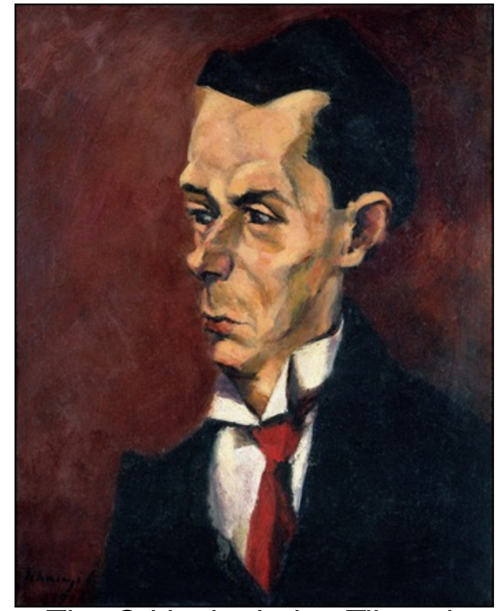


The Critic, by Lajos Tihanyi. Oil on canvas, 1916. Public Domain, https://commons.wikimedia.org/w/index.php?curid=17837438

Outline

- Two approaches to solving an MDP
- Two approaches to deep reinforcement learning
- Combining the two, in order to solve the problems with either

Solving an MDP

Remember that, if you know $P(s'|s,a)$, you can solve for the optimum policy $\pi(s)$. This is done by solving Bellman's equation:

$$U(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a)U(s') \qquad \forall s, s'$$

- Bellman's equation is N nonlinear equations in N unknowns (N is the number of states). In general, the only way to solve it is by exhaustively testing every possible policy ($O\{d^N\}$ computations where d is the number of possible actions).

Two approaches to solving an MDP

We've learned two practical algorithms for solving an MDP:

1. Value Iteration: focuses on finding $U(s)$
2. Policy Iteration: focus on finding $\pi(s)$

Two approaches to solving an MDP

Value Iteration: focuses on finding $U(s)$

- Initialize with the value of a length-0 path: $U_0(s) = 0$
- Iterate by finding the best value of a length-t path:

$$U_t(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_{t-1}(s)$$

Value Iteration

$$U_2(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a)U_1(s')$$

$U_2(s)$ $(\gamma = 1)$

−0.08	−0.08	+0.75	
−0.08		−0.08	
−0.08	−0.08	−0.08	−0.08

$\sum_{s'} P(s'|s,\text{down})U_1(s')$

−0.04	−0.04	+0.06	
−0.04		−0.14	
−0.04	−0.04	−0.04	−0.04

$\sum_{s'} P(s'|s,\text{up})U_1(s')$

−0.04	−0.04	+0.06	
−0.04		−0.14	
−0.04	−0.04	−0.04	−0.81

$\sum_{s'} P(s'|s,\text{left})U_1(s')$

−0.04	−0.04	−0.04	
−0.04		−0.04	
−0.04	−0.04	−0.04	−0.14

$\sum_{s'} P(s'|s,\text{right})U_1(s')$

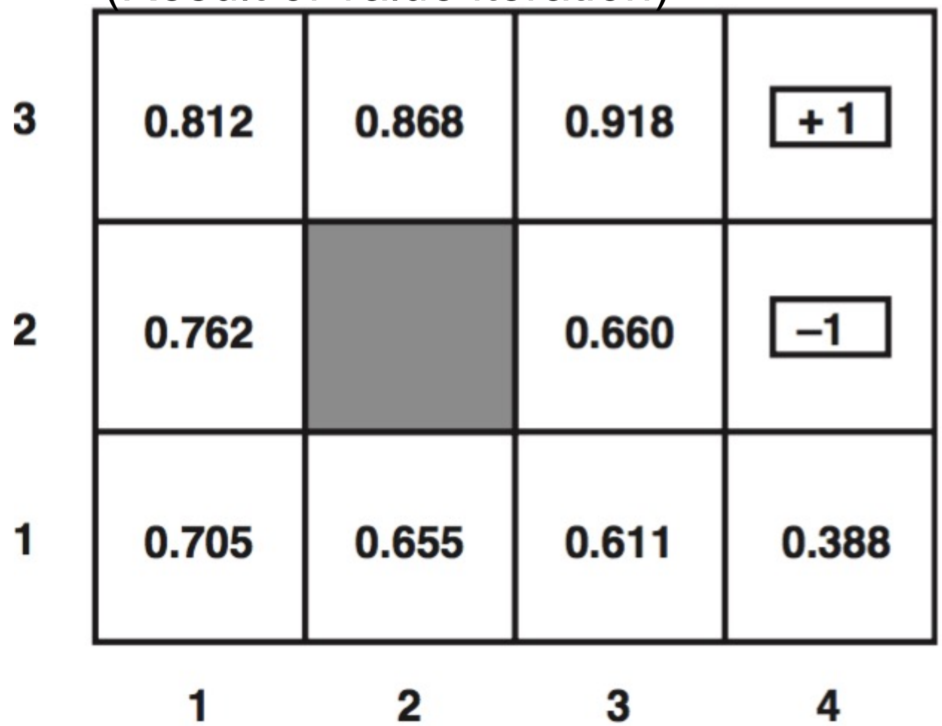
−0.04	−0.04	+0.79	
−0.04		−0.81	
−0.04	−0.04	−0.04	−0.14

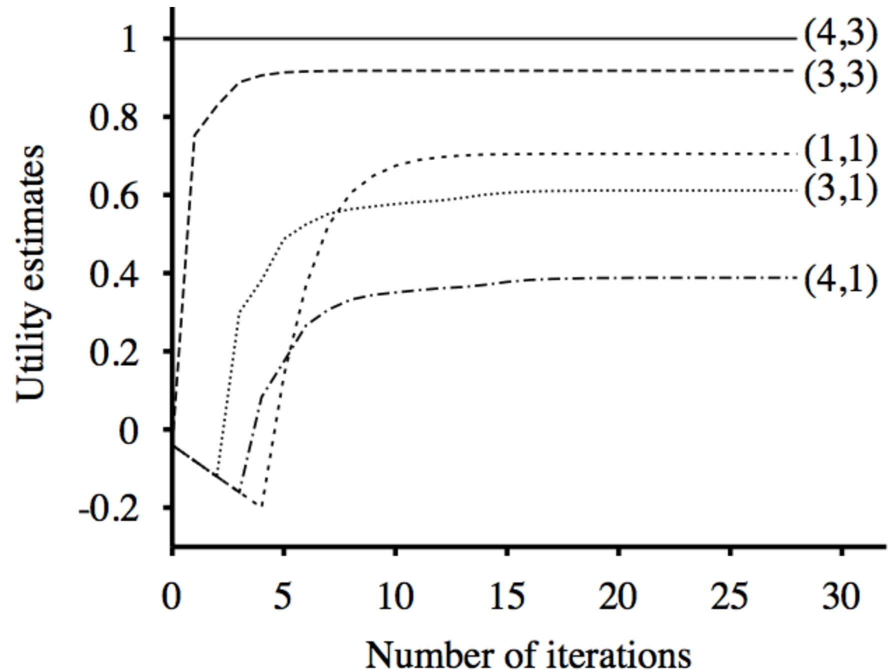
$U_1(s)$

−0.04	−0.04	−0.04	
−0.04		−0.04	
−0.04	−0.04	−0.04	−0.04

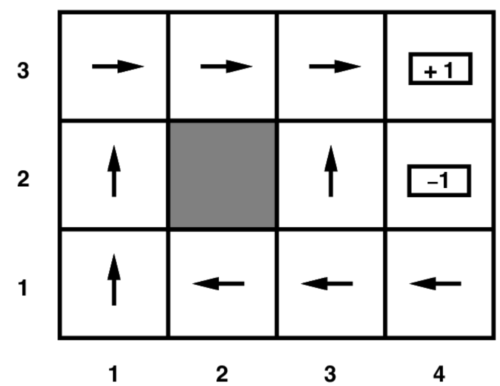
Value iteration

Optimal utilities with discount factor 1
(Result of value iteration)





Final policy



Two approaches to solving an MDP

Policy Iteration: focus on finding $\pi(s)$

- Initialize with a completely arbitrary initial policy, e.g.:

$$\pi_0(s) = \text{Left}$$

- Iterate:
  - Policy evaluation: find out the value of each state under current policy:

$$U^\pi(s) = R(s) + \gamma \sum_{s'} P(s'|s,\pi(s)) U^\pi(s')$$

  - Policy improvement: change the action, in each state, to improve value:

$$\pi(s) = \underset{a \in A(s)}{\operatorname{argmax}} \sum_{s'} P(s'|s,a) U^\pi(s)$$

1. Policy Evaluation:

$\pi_0(s)$:

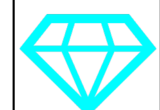
$\rightarrow$	$\rightarrow$	$\rightarrow$	
$\rightarrow$		$\rightarrow$	
$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$

$$U^{\pi_0}(s) = R(s) + \gamma \sum_{s'} P(s'|s,\pi_0(s))U^{\pi_0}(s')$$

…write it in matrix form:

$$\begin{bmatrix} U^{\pi_0}(1) \\ \vdots \\ U^{\pi_0}(N) \end{bmatrix} = \begin{bmatrix} R\ (1) \\ \vdots \\ R\ (N) \end{bmatrix} + \gamma \begin{bmatrix} P(1|1,\pi(1)) & \cdots & P(N|1,\pi(N)) \\ \vdots & \ddots & \vdots \\ P(1|N,\pi(1)) & \cdots & P(N|N,\pi(N)) \end{bmatrix} \begin{bmatrix} U^{\pi_0}(1) \\ \vdots \\ U^{\pi_0}(N) \end{bmatrix}$$

…and solve it: $U^{\pi_0(s)}(s)$:

$+0.50$	$+0.69$	$+0.74$	
-0.65		-0.90	
-1.40	-1.44	-1.39	-1.40

2. Policy Improvement:

$$U^{\pi_0}(s) = R(s) + \gamma \sum_{s'} P(s'|s, \pi_0(s)) U^{\pi_0}(s')$$

$$\pi_1(s) = \underset{a}{\operatorname{argmax}} \sum_{s'} P(s'|s, \pi_0(s)) U^{\pi_0}(s')$$

$\pi_0(s)$:

$\rightarrow$	$\rightarrow$	$\rightarrow$	
$\rightarrow$		$\rightarrow$	
$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$

$U^{\pi_0(s)}(s)$:

$+0.50$	$+0.69$	$+0.74$	
-0.65		-0.90	
-1.40	-1.44	-1.39	-1.40

$\pi_1(s)$

$\rightarrow$	$\rightarrow$	$\rightarrow$	
$\uparrow$		$\uparrow$	
$\uparrow$	$\rightarrow$	$\uparrow$	$\uparrow$

Outline

- Two approaches to solving an MDP
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- Combining the two, in order to solve the problems with either

Two approaches to deep reinforcement learning

- Deep Q learning: train a network to estimate Q(s,a)
  - Like value iteration: we focus on Q(s,a), which is closely related to U(s)
  - Big problem: Q(s,a) is very noisy, needs lots of smoothing
- Imitation learning: train a network to imitate a human being
  - Like policy iteration: focus directly on estimating $\pi(s)$
  - Big problem: the only way to train this is by imitating a human!

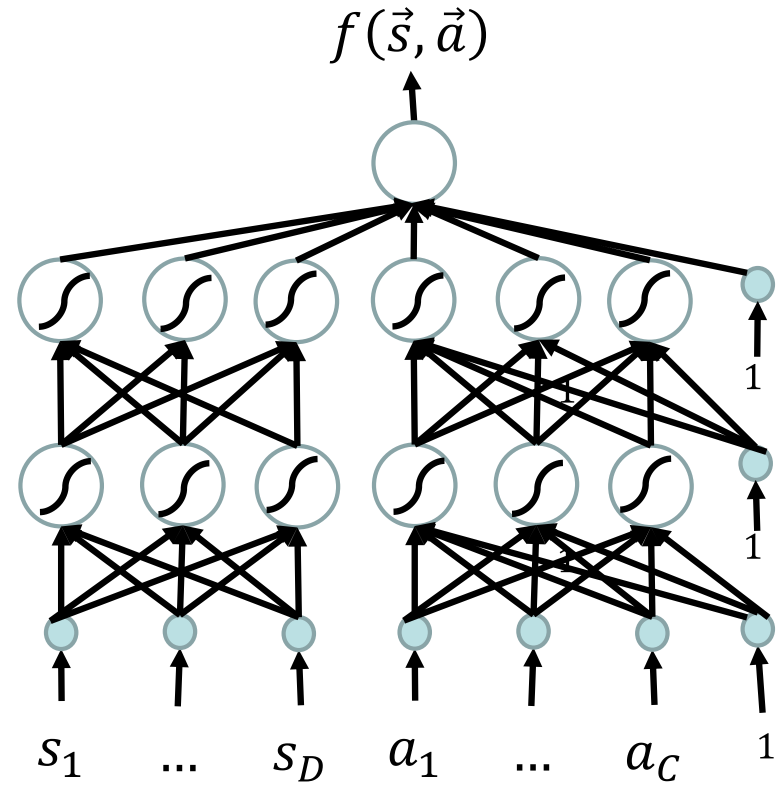
Deep Q learning

Train the neural network weights in order to minimize the mean-squared error:

$$\mathcal{L} = \frac{1}{2} E[(f(\vec{s},\vec{a}) - Q_{local}(\vec{s},\vec{a}))^2]$$

$Q_{local}(\vec{s},\vec{a})$ is the estimated value of the current action:

$$Q_{local}(\vec{s}_t,\vec{a}_t) = R_t(\vec{s}_t) + \gamma \max_{\vec{a}'} f(\vec{s}_{t+1},\vec{a}')$$



Imitation Learning

If we have $|A|$ possible, actions, $1 \leq a \leq |A|$, we could train the network to learn a hidden layer $h(s)$ so that:

$$\pi_a(s) = \frac{\exp(w_a^T h(s))}{\sum_{k=1}^{|A|} \exp(w_k^T h(s))} = P(A = a|S = s)$$

Meaning “the probability that the best action is a.”

Two approaches to deep reinforcement learning

- Deep Q learning: train a network to estimate Q(s,a)
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The Actor-Critic Algorithm

- Deep Q-learning gives us a network Q(s,a) which is very noisy, so we don’t really want to trust it
- A policy network can directly estimate $\pi(s)$. The only problem is that we have no way to train it, unless we imitate human behavior.

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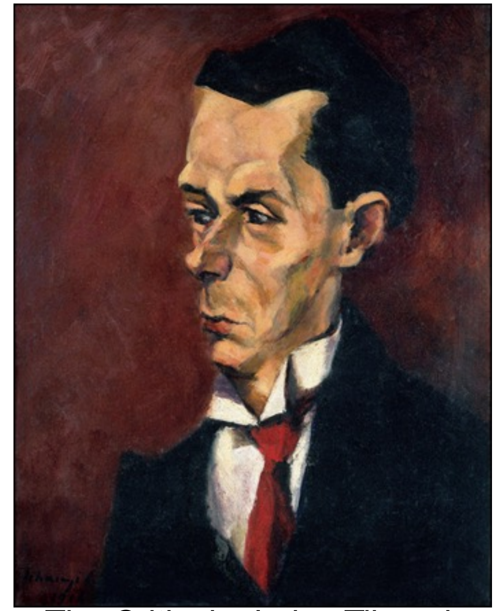
Actor-critic algorithm

So let's train two neural nets!

- $Q_t(s,a)$ is the **critic**, and is trained according to the deep Q-learning algorithm (MMSE).
- $\pi_a(s)$ is the **actor**, and is trained to satisfy the critic



The Critic, by Lajos Tihanyi. Oil on canvas, 1916. Public Domain, https://commons.wikimedia.org/w/index.php?curid=17837438

The Actor-Critic Algorithm

Main idea:

- The **actor** is a policy network that decides what action to perform:

  $\pi_a(s) =$ Probability that a is the best action in state s

- The **critic** is a deep Q-learning network that estimates the quality of that action ($Q(s,a)$).

  $Q(s,a) =$ Expected sum of future rewards if (s,a)

- The critic is noisy, so they don't get to decide the action. Instead, we only use the critic to help us to train the actor.

The Actor-Critic Algorithm

$$\pi_a(s) = \text{Probability that } a \text{ is the best action in state } s$$

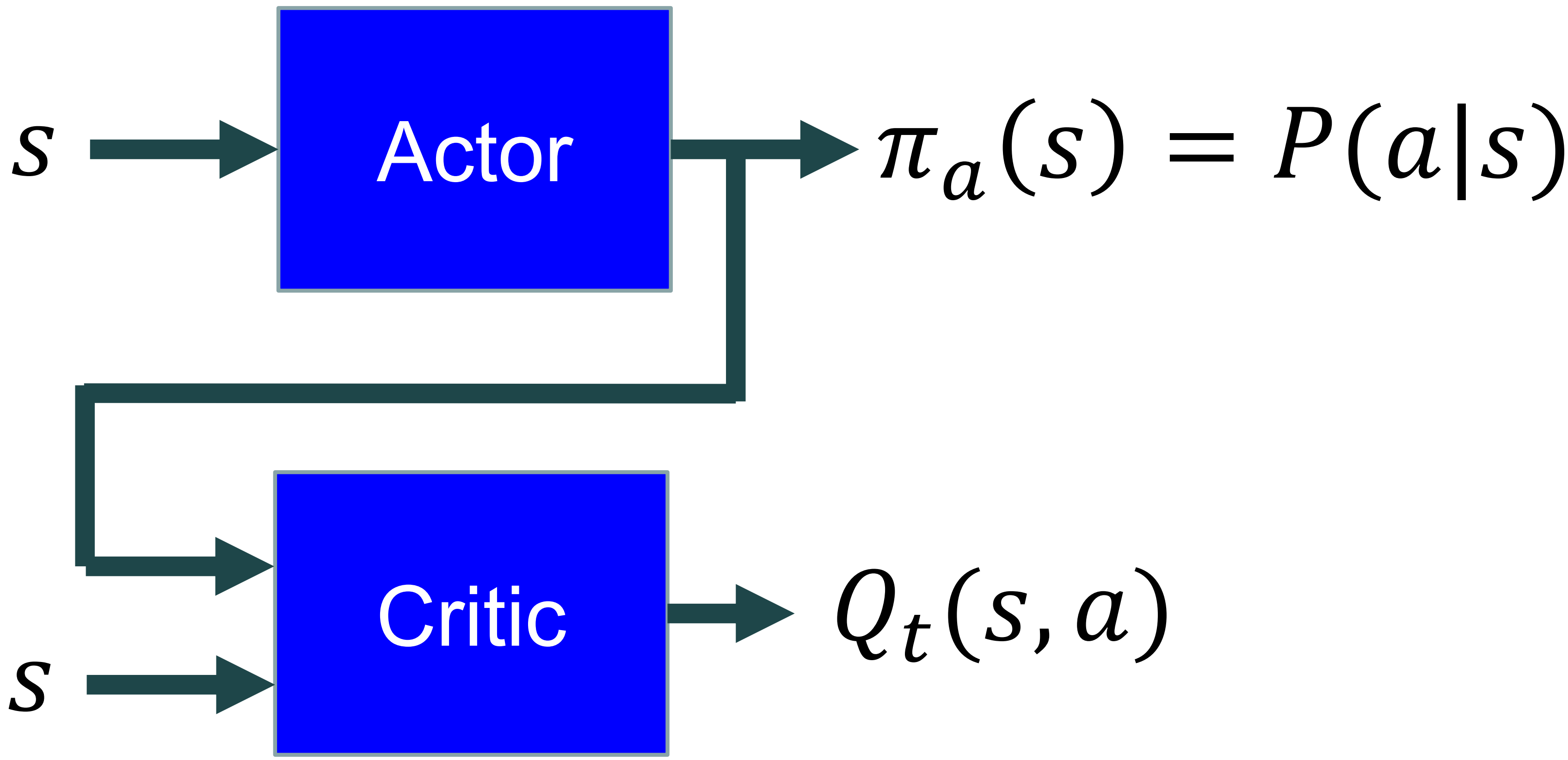
$$Q(s,a) = \text{Expected sum of future rewards if } (s,a)$$

- The critic is noisy, so they don't get to decide the action. Instead, we only use the critic to help us to train the actor.

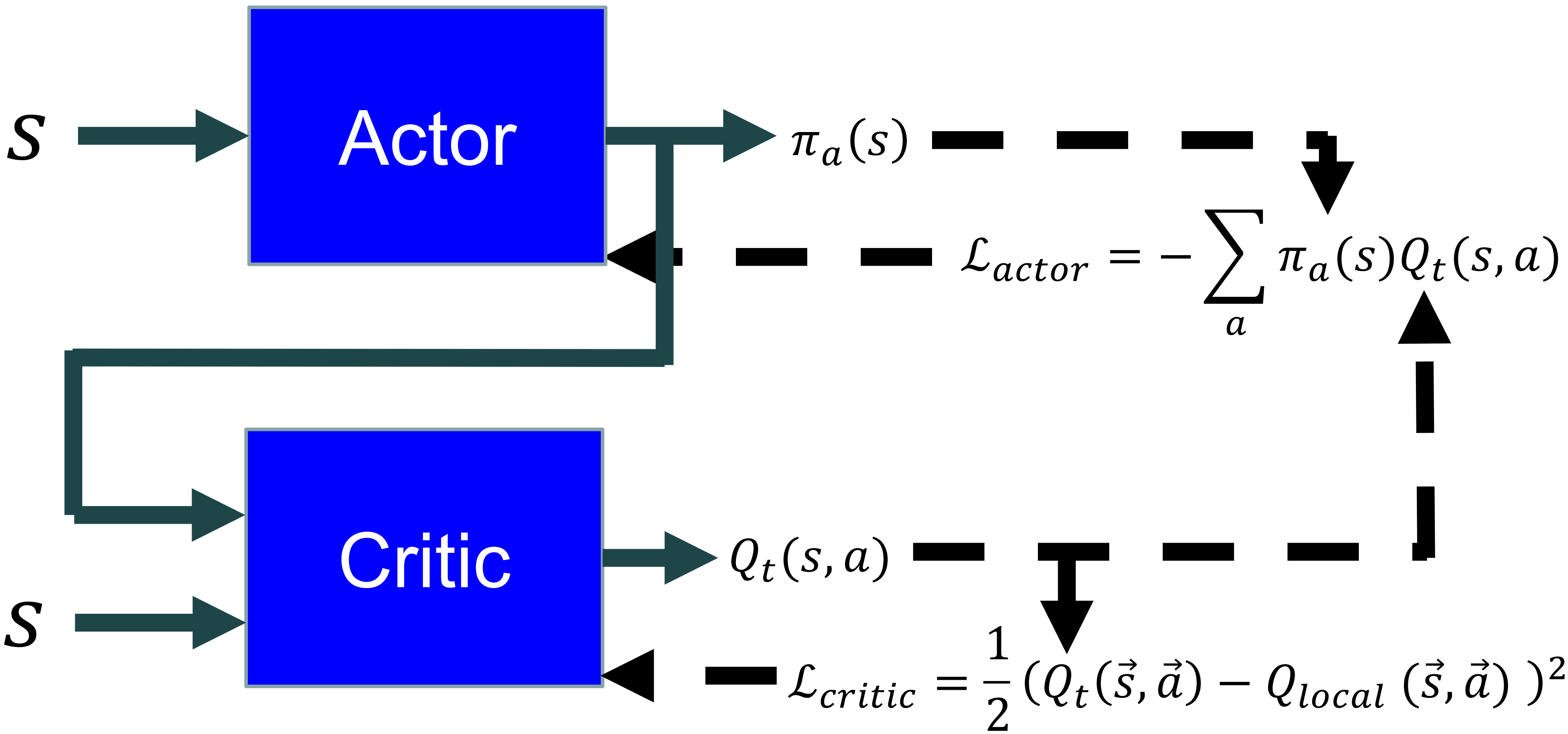
$$\mathcal{L} = -\sum_a \pi_a(s) Q(s,a)$$

- The training loss = negative expected sum of future rewards given action a, averaged over the probability with which the actor chooses action a.

The Actor-Critic Algorithm: Forward-Prop



The Actor-Critic Algorithm: Back-Prop



Quiz

- Try the quiz!

https://us.prairielearn.com/pl/course_instance/129874/assessment/2342329

Asynchronous advantage actor-critic (A3C)

[TORCS car racing simulation video]

Mnih et al. [Asynchronous Methods for Deep Reinforcement Learning]. ICML 2016

Overview: All of the Model-Free Reinforcement Learning Algorithms You've Learned

- Policy learning: learn $\pi(s)$ directly
  - Imitation learning
- Q-learning: learn $Q(s,a) = R(s) + \gamma \sum_{s'} P(s'|s,a)U(s')$
  - Table-based: TD, SARSA
  - Deep Q-learning
- Actor-Critic: learn both