Your Name: ____________________________________________

Your NetID: ____________________________________________

Instructions

• Please write your name on the top of every page.

• Have your ID ready; you will need to show it when you turn in your exam.

• This will be a CLOSED BOOK, CLOSED NOTES exam. You are permitted to bring
  and use only one 8.5x11 page of notes, front and back, handwritten or typed in a font size
  comparable to handwriting.

• No electronic devices (phones, tablets, calculators, computers etc.) are allowed.

• SHOW YOUR WORK. Correct answers derivation may not receive full credit if you don’t
  show your work.

• Make sure that your answer includes only the variables that it should include, but DO NOT
  simplify explicit numerical expressions. For example, the answer $x = \frac{1}{1+\exp(-0.1)}$ is MUCH
  preferred (much easier for us to grade) than the answer $x = 0.524979$. 
Possibly Useful Formulas

\[ P(X = x|Y = y)P(Y = y) = P(Y = y|X = x)P(X = x) \]
\[ P(X = x) = \sum_y P(X = x, Y = y) \]
\[ E[f(X, Y)] = \sum_{x,y} f(x,y)P(X = x, Y = y) \]

Precision, Recall: \[ \frac{TP}{TP + FP} \quad \text{TP} \]
MPE = MAP: \[ f(x) = \arg \max (\log P(Y = y) + \log P(X = x|Y = y)) \]
Naive Bayes: \[ P(X = x|Y = y) \approx \prod_{i=1}^n P(W = w_i|Y = y) \]

Laplace Smoothing: \[ P(X = x|Y = y) = \frac{\text{Count}(X = x, Y = y) + k}{\text{Count}(Y = y) + k|X|}, \quad |X| = \# \text{possible distinct values of } X \]
Fairness: \[ P(Y|A) = \frac{P(Y|\hat{Y}, A)P(\hat{Y}|A)}{P(\hat{Y}|Y, A)} \]

Linear Regression: \[ \epsilon_i = f(x_i) - y_i = b + w@x_i - y_i \]
Mean Squared Error: \[ \text{MSE} = \frac{1}{n} \sum_{i=1}^n \epsilon_i^2 \]
Linear Classifier: \[ f(x) = \arg \max_k w_k@x + b \]

Cross-Entropy: \[ \mathcal{L} = -\frac{1}{n} \sum_{i=1}^n \log f_{y_i}(x_i) \]
Softmax: \[ \text{softmax}(w@x + b) = \frac{\exp(w_c@x + b_c)}{\sum_{k=1}^{V-1} \exp(w_k@x + b_k)} \]
Softmax Error: \[ \epsilon_{i,c} = \begin{cases} f_c(x_i) - 1 & c = y_i \\ f_c(x_i) - 0 & \text{otherwise} \end{cases} \]
Gradient Descent: \[ w \leftarrow w - \eta \nabla_w \mathcal{L} \]
Neural Net: \[ h = \text{ReLU}(b_0 + w_0@x), \quad f = \text{softmax}(b_1 + w_1@h) \]

Back-Propagation: \[ \frac{\partial \mathcal{L}}{\partial h_j} = \sum_k \frac{\partial \mathcal{L}}{\partial f_k} \times \frac{\partial f_k}{\partial h_j}, \quad \frac{\partial \mathcal{L}}{\partial w_{0,k,j}} = \frac{\partial \mathcal{L}}{\partial h_k} \times \frac{\partial h_k}{\partial w_{0,k,j}} \]

Consistent Heuristic: \[ h(p) \leq d(p, r) + h(r) \]
Alpha-Beta Max Node: \[ v = \max(v, \text{child}); \quad \alpha = \max(\alpha, \text{child}) \]
Alpha-Beta Min Node: \[ v = \min(v, \text{child}); \quad \beta = \min(\beta, \text{child}) \]
Variance Network: \[ \mathcal{L} = \frac{1}{n-1} \sum_{i=1}^n \left(f_2(x_i) - (f_1(x_i) - x_i)^2\right)^2 \]
Unification: $U = S(P) = S(Q); U \Rightarrow \exists x: Q; U \Rightarrow \exists x: P$

Bayes Rule: $P(Y = y|X = x) = \frac{P(X = x|Y = y)P(Y = y)}{\sum_y P(X = x|Y = y')P(Y = y')}$

Unnormalized Relevance: $\tilde{R}(f_c, x_d) = \frac{\partial f_c}{\partial x_d} x_d f_c$

Normalized Relevance: $R(f_c, x_d) = \frac{\frac{\partial f_c}{\partial x_d} x_d f_c}{\sum_d' \frac{\partial f_c}{\partial x_d'} x_d' f_c}$

Softmax: $\text{softmax}(e_j) = \frac{\exp(e_j)}{\sum_k \exp(e_k)}$

Softmax Deriv: $\frac{\partial \text{softmax}(e_m)}{\partial e_n} = \text{softmax}(e) \delta[m - n] - \text{softmax}(e) \text{softmax}(e), \quad \delta[m - n] = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$

Viterbi: $v_t(j) = \max_i v_{t-1}(i) a_i' b_j(x_t)$

Transformer: $c_t = \text{softmax}(q_t @ k^T) @ v$

Pinhole Camera: $x' = \frac{x}{z}, \quad y' = \frac{y}{z}$

Convolution: $w_{k,l} * x_{k,l} = \sum_i \sum_j w_{k-i,l-j} x_{i,j}$

Kalman Prediction: $\mu_{t|t-1} = \mu_{t-1|t-1} + \mu_{\Delta}, \quad \sigma_{t|t-1}^2 = \sigma_{t-1|t-1}^2 + \sigma_{\Delta}^2$

Kalman Gain: $k_t = \frac{\sigma_{t|t-1}^2}{\sigma_{t|t-1}^2 + \sigma_e^2}, \quad \sigma_{t|t}^2 = \sigma_{t|t-1}^2 (1 - k_t)$

Kalman Update: $\mu_{t|t} = \mu_{t|t-1} + k_t (x_t - (\mu_{t|t-1} + \mu_e))$

Bellman Equation: $U(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s, a) U(s')$

Value Iteration: $U_t(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s, a) U_{t-1}(s')$

Policy Evaluation: $U_t(s) = R(s) + \gamma \sum_{s'} P(s'|s, \pi_t(s)) U_t(s')$

Policy Improvement: $\pi_{t+1}(s) = \arg \max_a R(s) + \gamma \sum_{s'} P(s'|s, a) U_t(s')$

Q-Learning: $Q_{t+1}(s, a) = Q_t(s, a) + \alpha (Q_{\text{local}}(s, a) - Q_t(s, a))$

TD Learning: $Q_{\text{local}}(s_t, a_t) = R(s_t) + \gamma \max_a Q_t(s_{t+1}, a')$

SARSA: $Q_{\text{local}}(s_t, a_t) = R(s_t) + \gamma Q_t(s_{t+1}, a_{t+1})$

Imitation Learning: $L = -\log \pi_a(s)$

Deep Q Learning: $L = \frac{1}{2} (Q_t(s, a) - Q_{\text{local}}(s, a))^2$

Actor-Critic: $L = -\sum_a \pi_a(s) Q(s, a)$

Inverse Kinematics: $\mathcal{C}_{\text{obs}} = \{ q : \exists b : \phi_b(q) \in \mathcal{W}_{\text{obs}} \}$