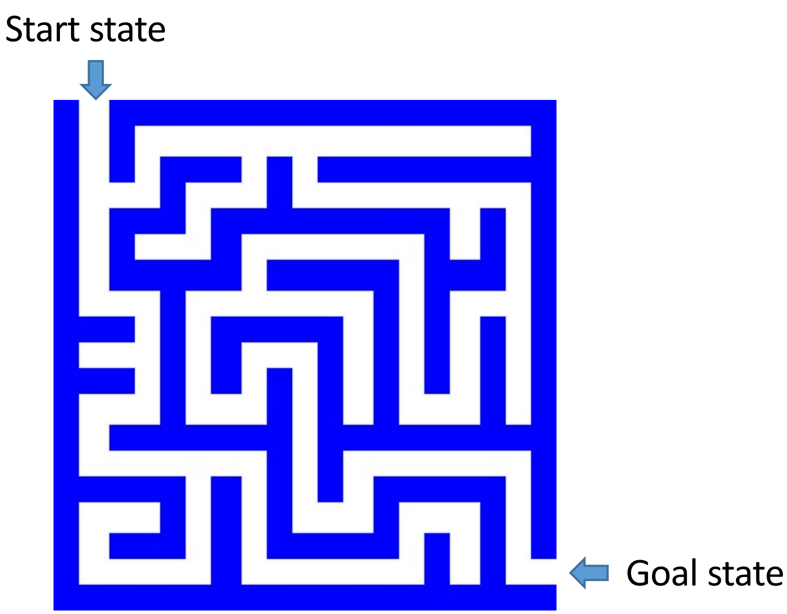
CS440/ECE 448 Lecture 15: Search

Outline of today's lecture

1. Initial state, goal state, transition model
2. General algorithm for solving search problems
    1. First data structure: a frontier queue
    2. Second data structure: a search tree
    3. Third data structure: explored set
    4. Fourth data structure: explored dict
3. Breadth-first search (BFS) and Depth-first search (DFS)
    1. Completeness
    2. Optimality
    3. Time Complexity
    4. Space Complexity

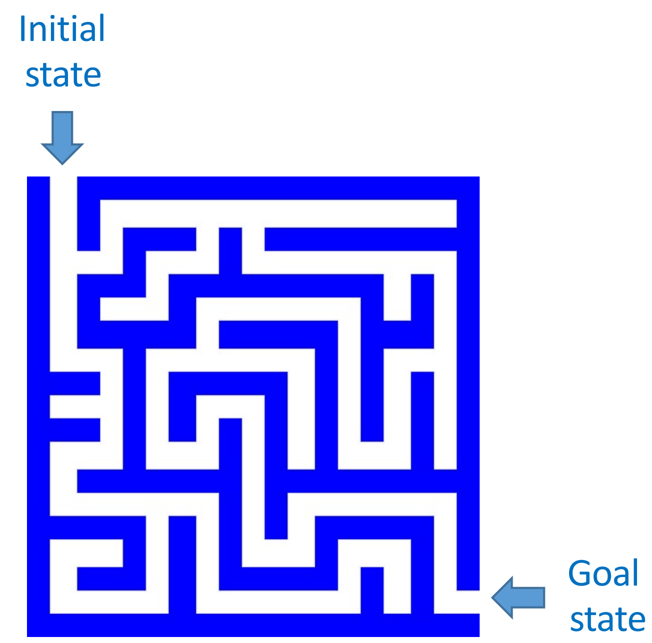
Search

- We will consider the problem of designing **goal-based agents** in **fully observable, deterministic, discrete, static, known** environments
- Environment is **sequential**: agent's action changes its state
- Agent must plan the best sequence of actions to achieve a goal



Search problem components

- **Initial state**
- **Actions**
- **Transition model**
    - What ***successor state*** results from performing a given ***action*** in a given ***predecessor state***?
- **Goal state**
- **Path cost**
    - Assume that it is a sum of nonnegative *step costs*



- The **optimal solution** is the sequence of actions that gives the *lowest* path cost for reaching the goal

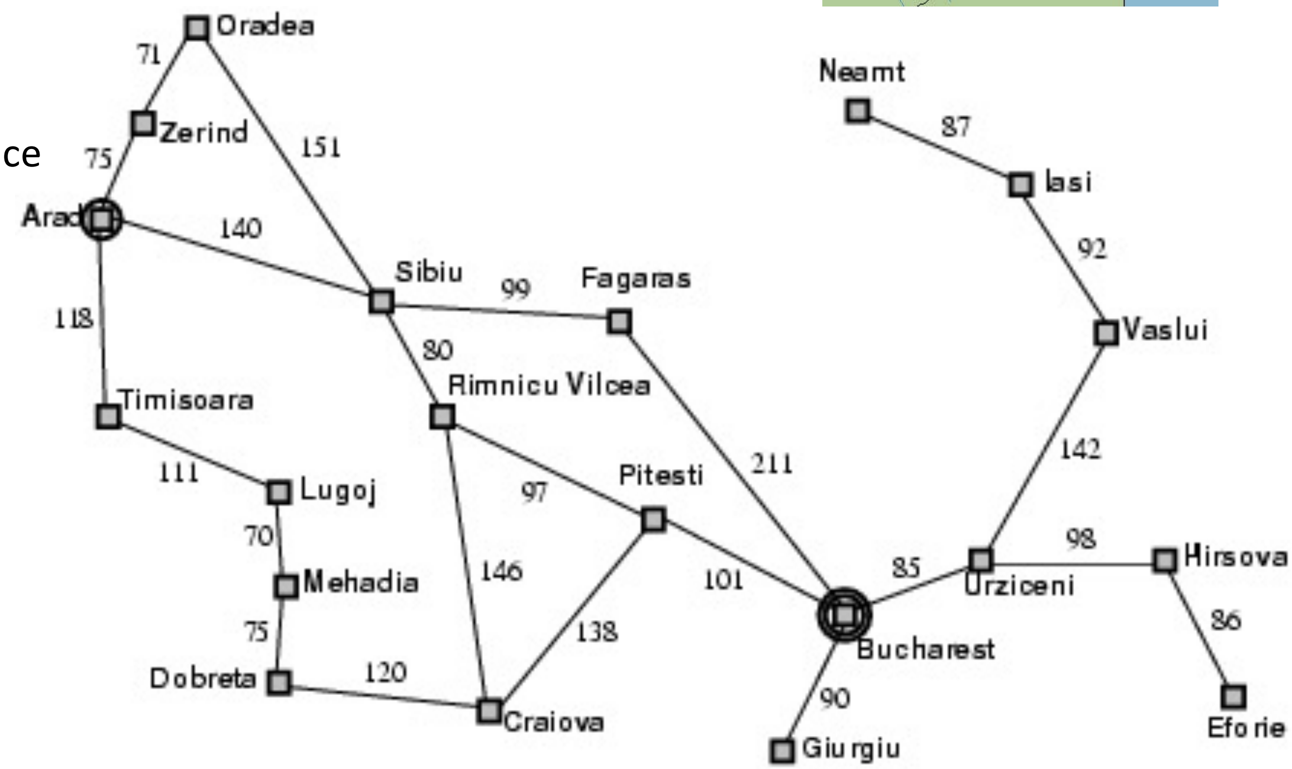
Knowledge Representation: State

- State = description of the world
  - Must have enough detail to decide whether or not you're currently in the **initial state**
  - Must have enough detail to decide whether or not you've reached the **goal state**
  - Often but not always: "defining the state" and "defining the transition model" are the same thing

Example of state definition: Romania

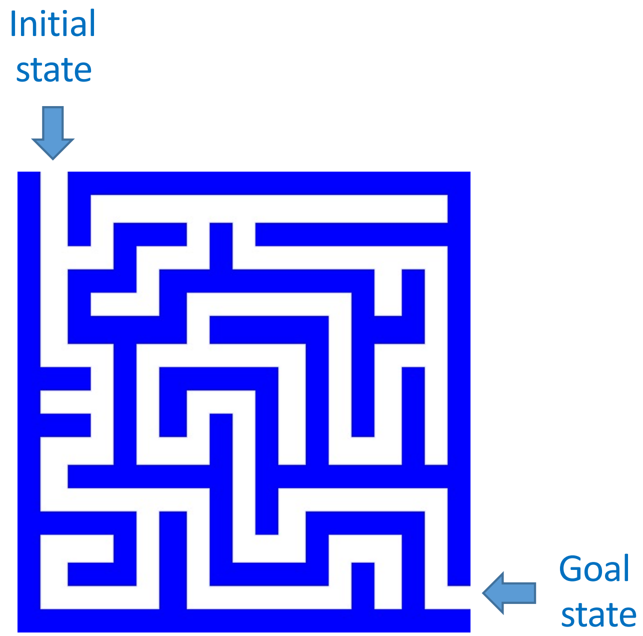
- On vacation in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

- **state** = name of the city
- **Path cost**
  - Sum of edge costs (total distance traveled)



Example of state definition: Maze solving

- **State = (x,y)**, current position of the agent

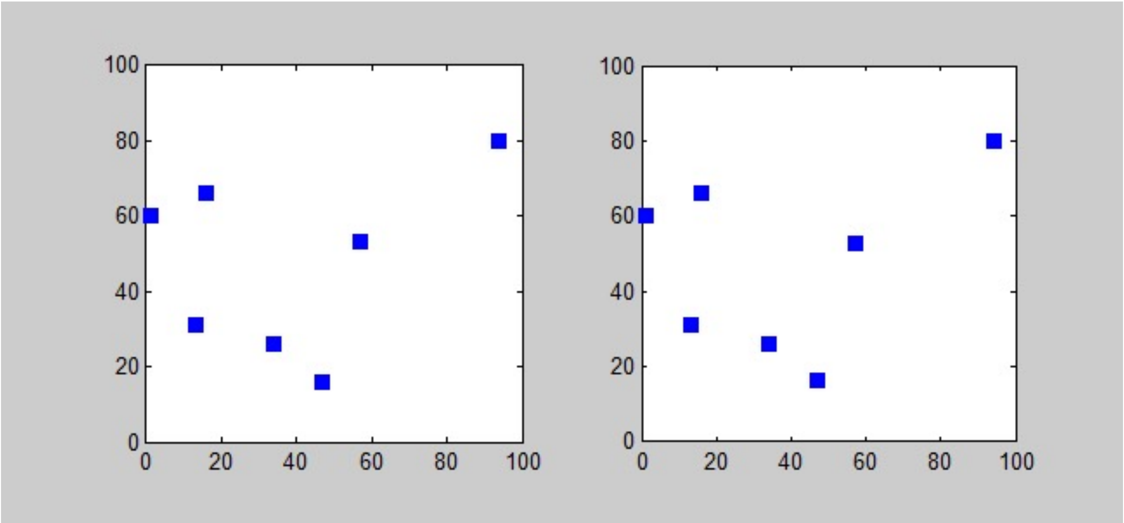


Example of state definition: Traveling salesman problem

- Goal: visit every city in the United States
- Path cost: total miles traveled
- Initial state: Champaign, IL
- Action: travel from one city to another
- Transition model: when you visit a city, mark it as “visited.”

Example of state definition: Traveling salesman problem

- **state = (agent, goals)**
  - **agent = (agent_x,agent_y)** is current position of the agent
  - **goals = [goal[0], goal[1], …]** lists the goals that have not yet been reached
    - **goal[i] = (goal_x, goal_y)** tells the location of the i'th remaining goal



Solving TSP using a branch-and-bound algorithm. CC-BY-SA 3.0, Saurabh Harsh, 2012, https://commons.wikimedia.org/wiki/File:Branchbound.gif

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How does this problem differ from every problem you've ever seen before?

- Search differs from most Computer Science problems in that the state space might be infinite. We don't assume, in advance, that we can enumerate every possible configuration of the world.
- Traditional definition of Dijkstra's algorithm:
  - First, list all of the possible states in the "not explored" list
  - Then, move them to the "explored" list after we visit them
- Modifying Dijkstra's algorithm for the infinite-world assumption:
  - Instead of a list of all possible states, we have a method **(next_state,cost)=Transition_Model(current_state, action)**
  - Instead of an infinite "not explored" list, we have a finite "frontier."

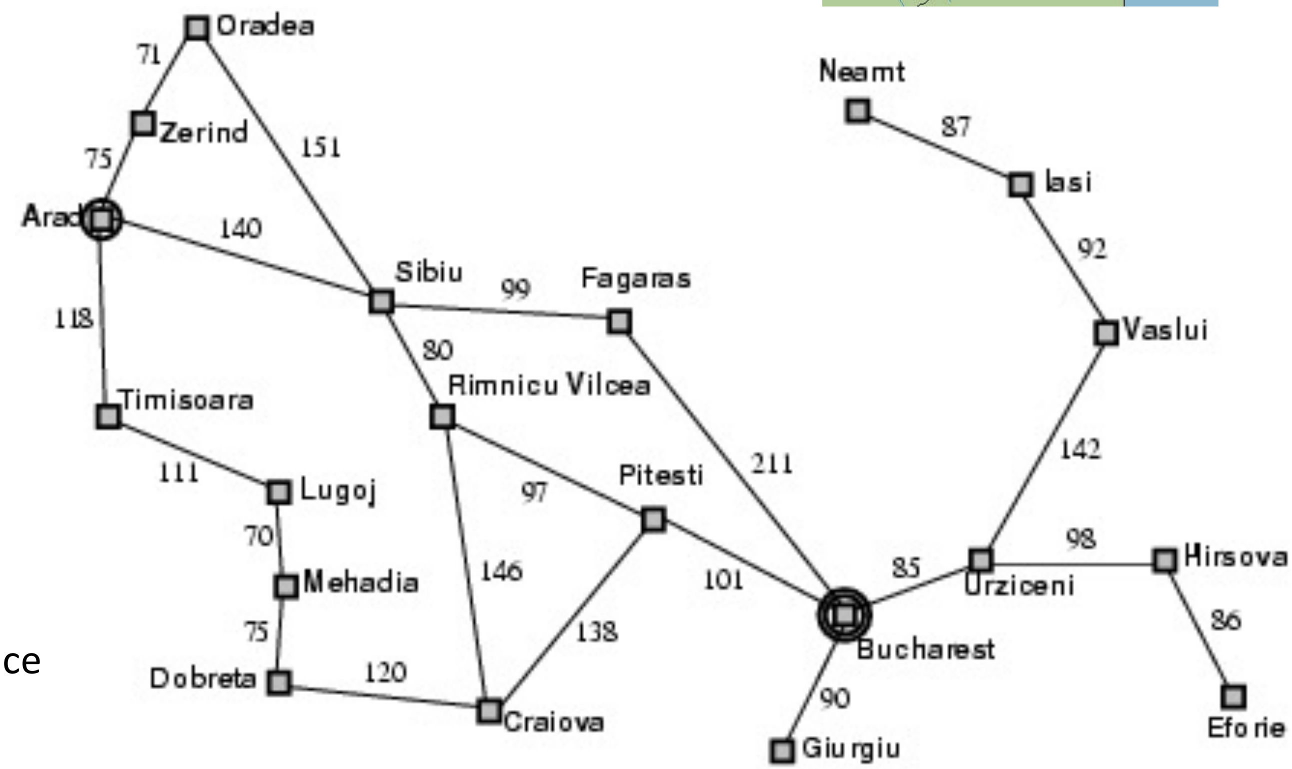
First data structure: Frontier

- Frontier = set of nodes that you know how to reach, but you haven't yet tested to see what comes next after those states
- **node = (state, parent_node, path_cost)**
- Initialize: **frontier = { (initial_state, None, 0) }**
- Iterate, until goal is reached:
  - Set **current_state** to some node from the frontier, remove it from the frontier.
  - Expand **current_state**:
    - If it's the goal, then you're done! Return the corresponding path.
    - If not, then find its children, and transition costs, using **(next_state,cost)=Transition_Model(current_state, action)**, and add them to the frontier.

Example: Romania

- On vacation in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

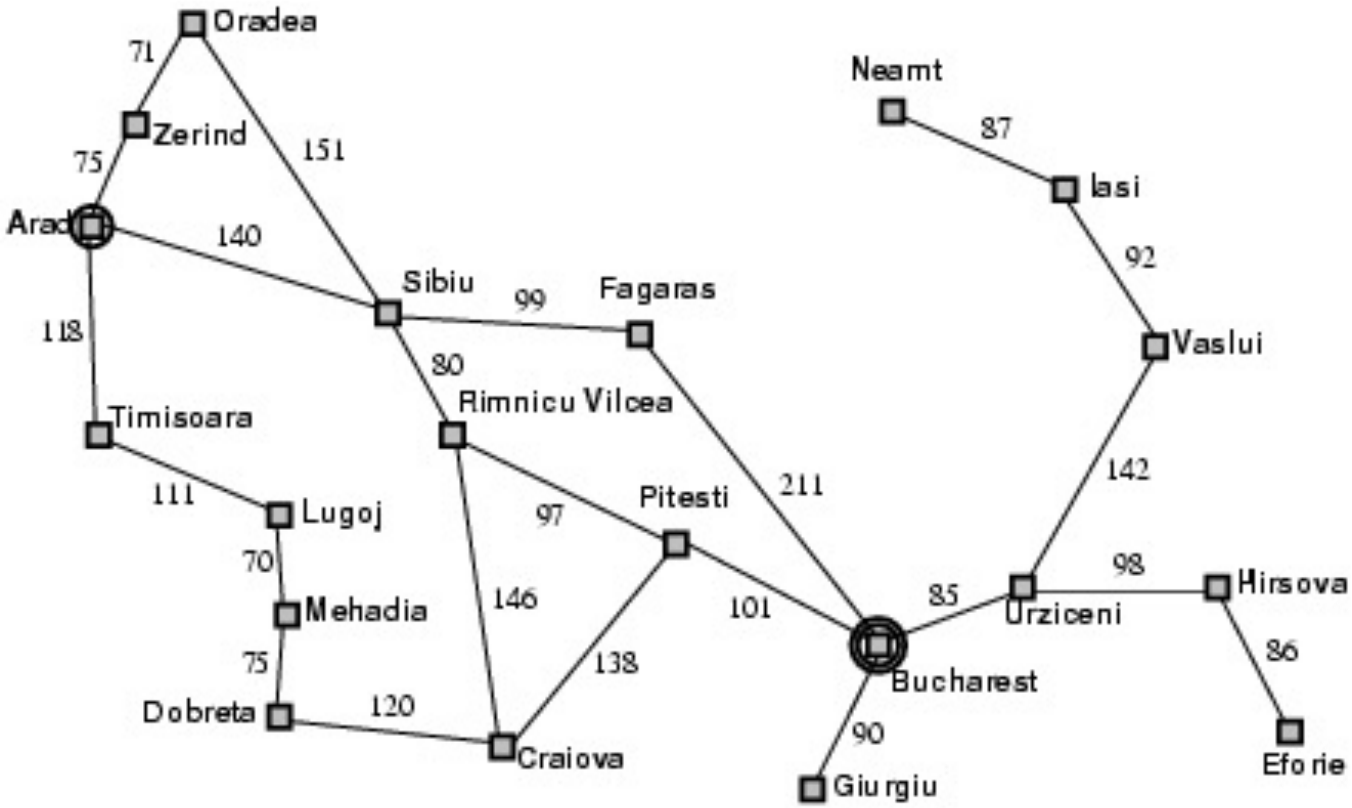
- **Initial state**
  - Arad
- **Actions**
  - Go from one city to another
- **Transition model**
  - If you go from city A to city B, you end up in city B
- **Goal state**
  - Bucharest
- **Path cost**
  - Sum of edge costs (total distance traveled)



Search step 0

Frontier: { Arad }

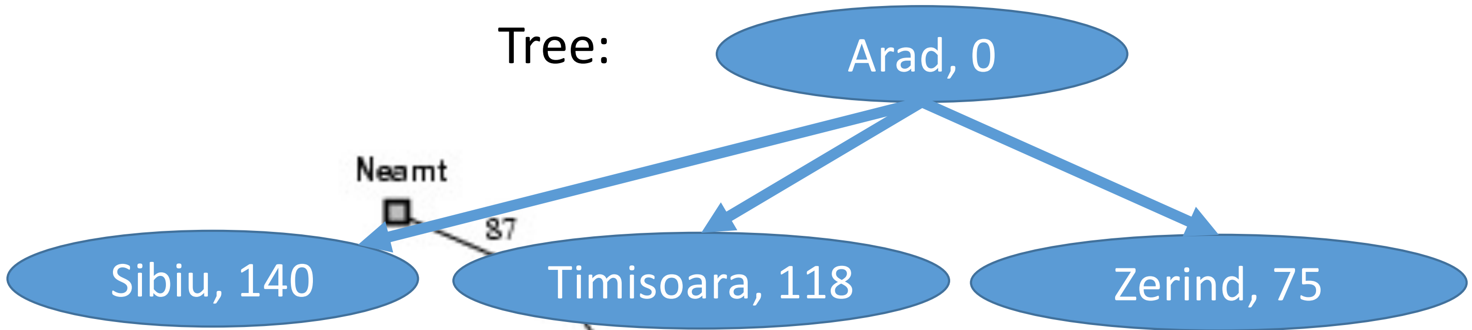
Tree:

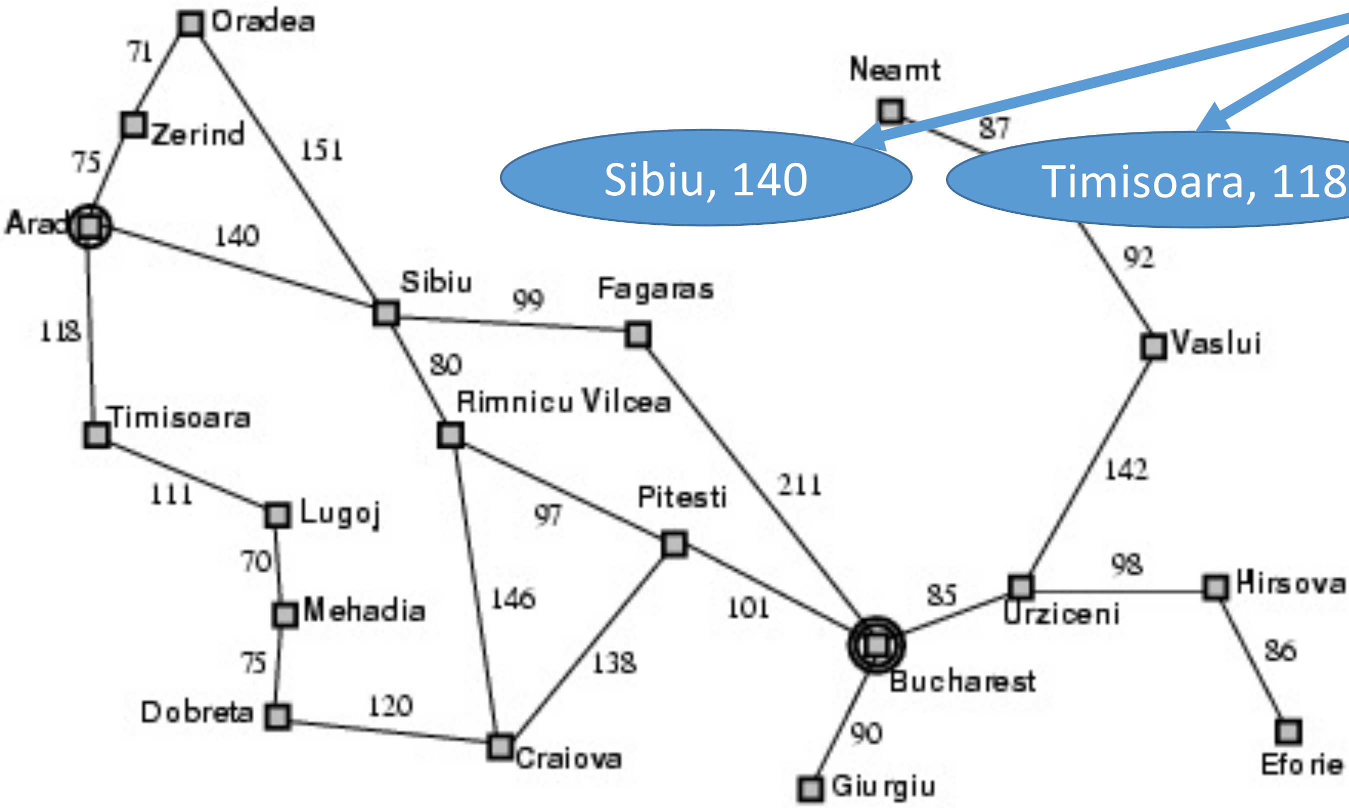


Search step 1

Frontier: { Sibiu, Zerind, Timisoara }

Tree:



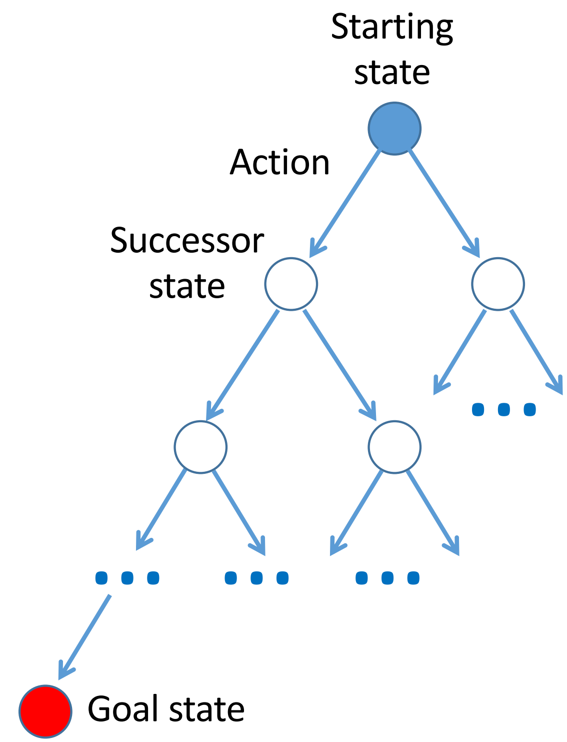


Tree Search: Basic idea

1. SEARCH for an optimal solution
   - Maintain a **frontier** of unexpanded states
   - At each step, pick a state from the frontier to **expand:**
     - Check to see whether or not this state is the goal state. If so, DONE!
     - If not, then list all of the states you can reach from this state, add them to the frontier, and add them to the tree
2. BACK-TRACE: go back up the tree; list, in reverse order, all of the actions you need to perform in order to reach the goal state.
3. ACT: the agent reads off the sequence of necessary actions, in order, and does them.

Nodes vs. States

- State = description of the world
  - Must have enough detail to decide whether or not you're currently in the initial state
  - Must have enough detail to decide whether or not you've reached the goal state
  - Often but not always: "defining the state" and "defining the transition model" are the same thing
- Node = a point in the search tree
  - Knows the ID of its STATE
  - Knows the ID of its PARENT NODE
  - Knows the COST of the path



Search step 1

Frontier: { Sibiu, Zerind, Timisoara }

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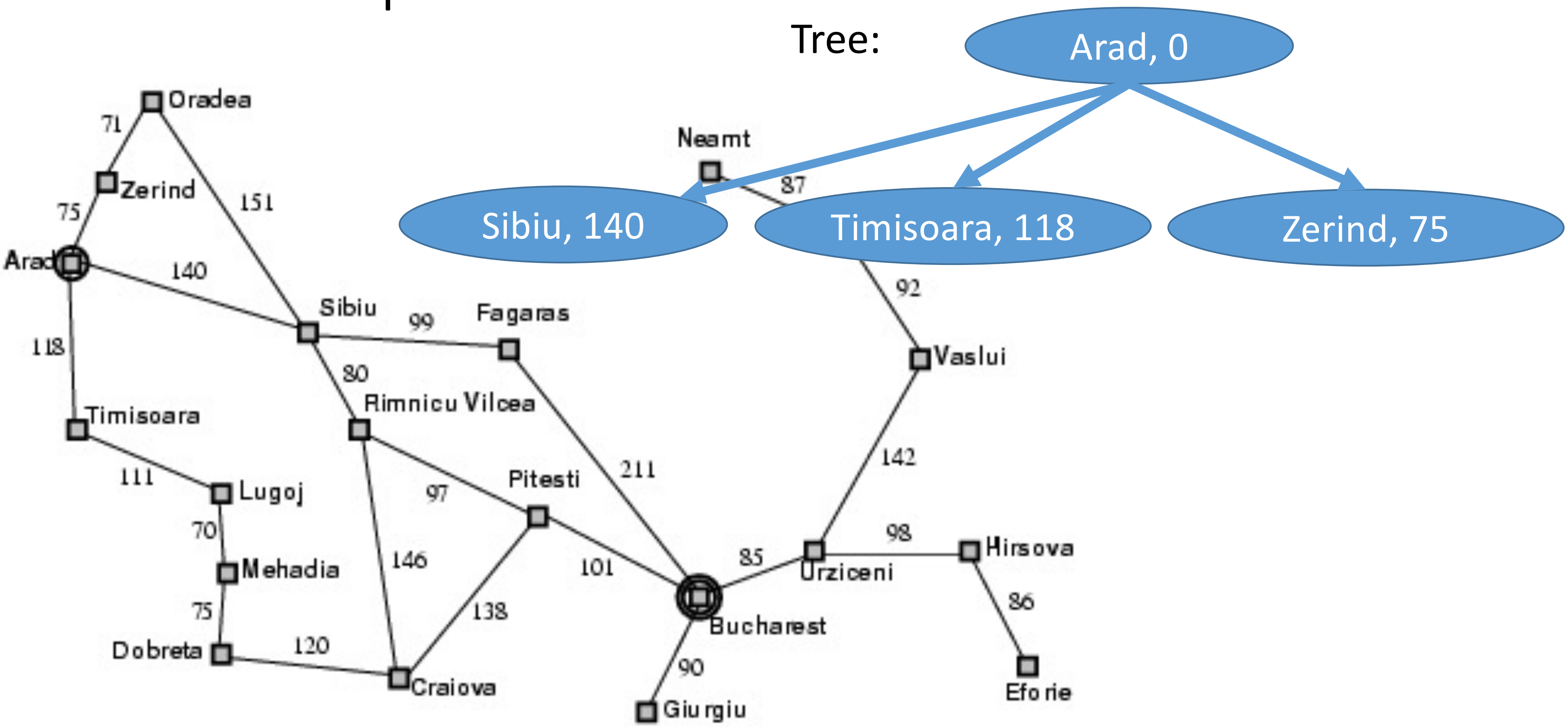
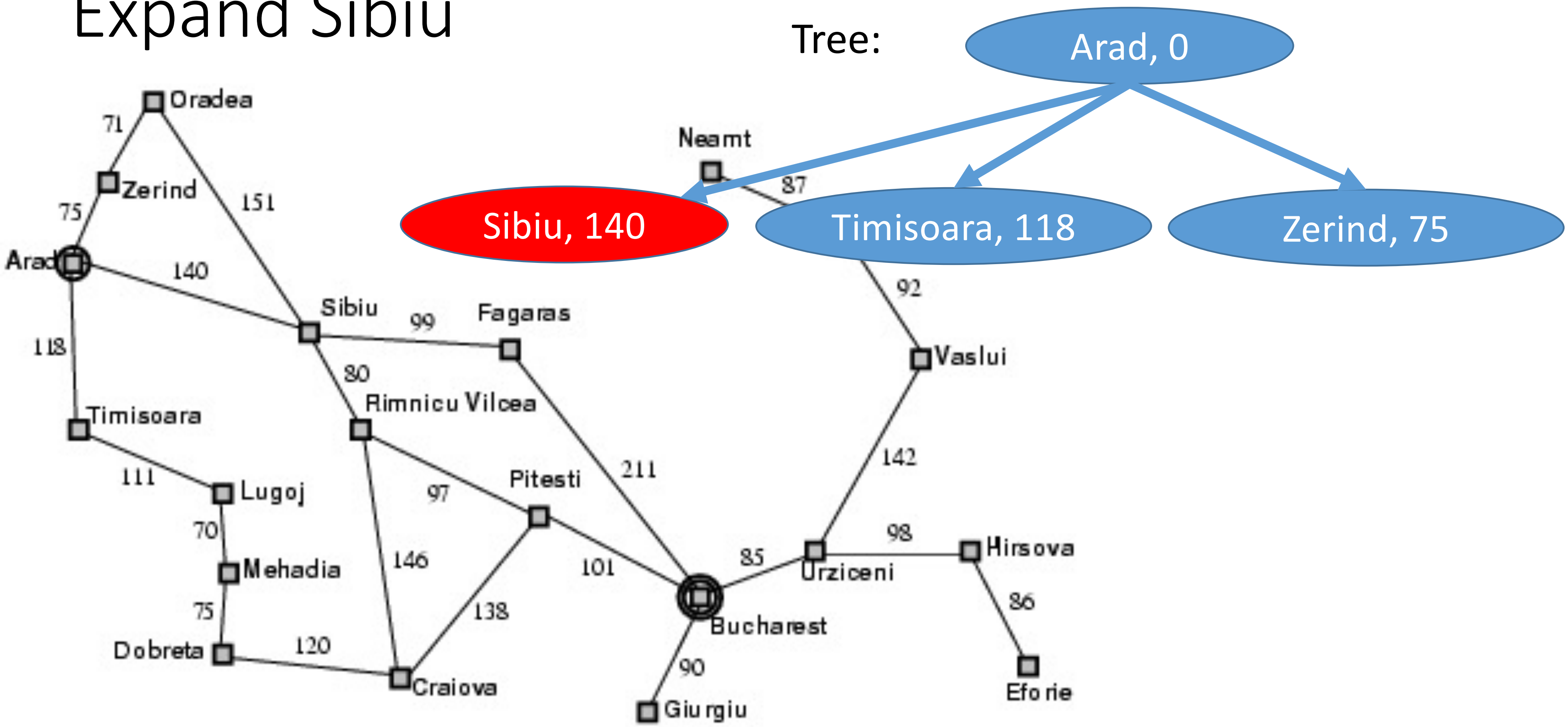


Search step 2
Expand Sibiu

Frontier: { Sibiu, Zerind, Timisoara }

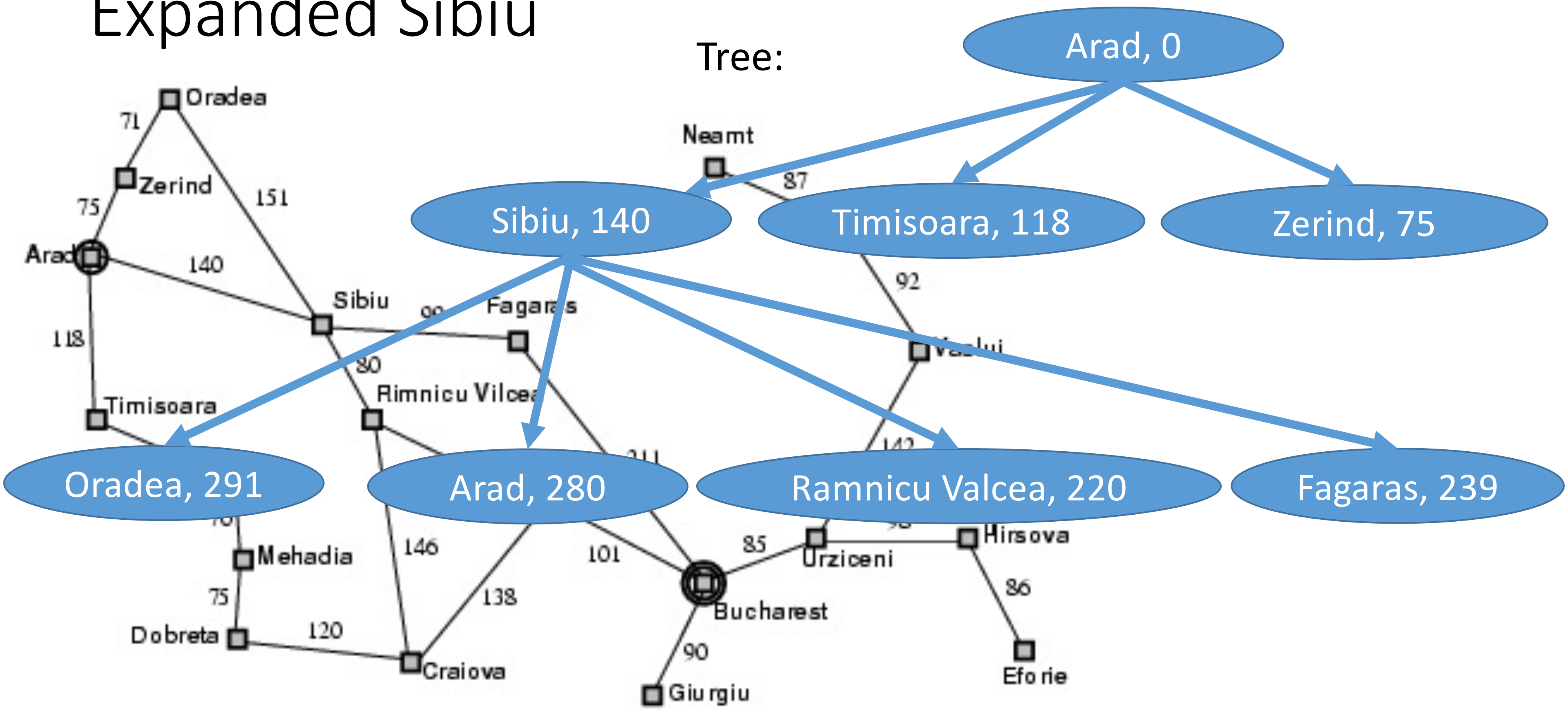
Tree:



Search step 2
Expanded Sibiu

Frontier: { Zerind, Timisoara, Oradea, Arad, Rimnicu Vilcea, Fagaras }

Tree:



Tree Search: Computational Complexity

- b = “branching factor” = max # states you can reach from any given state
- d = “depth” = # layers in the tree (# moves that you have made)
- Complexity of Tree Search = $O\{b^d\}$

If the world is infinite (there are an infinite number of possible states), then $O\{b^d\}$ is a reasonable cost to pay.

But what if (as in the Romania example) the world is finite? What if there are only N cities, where $O\{N\} < O\{b^d\}$? … it’s foolish to suffer $O\{b^d\}$ complexity for a tree search, when an exhaustive search would be only $O\{N\}$.

Third data structure: Explored set

How to limit complexity to $O\{\min(N, b^d)\}$: use an explored set

When you expand a state, do the following for each child state.

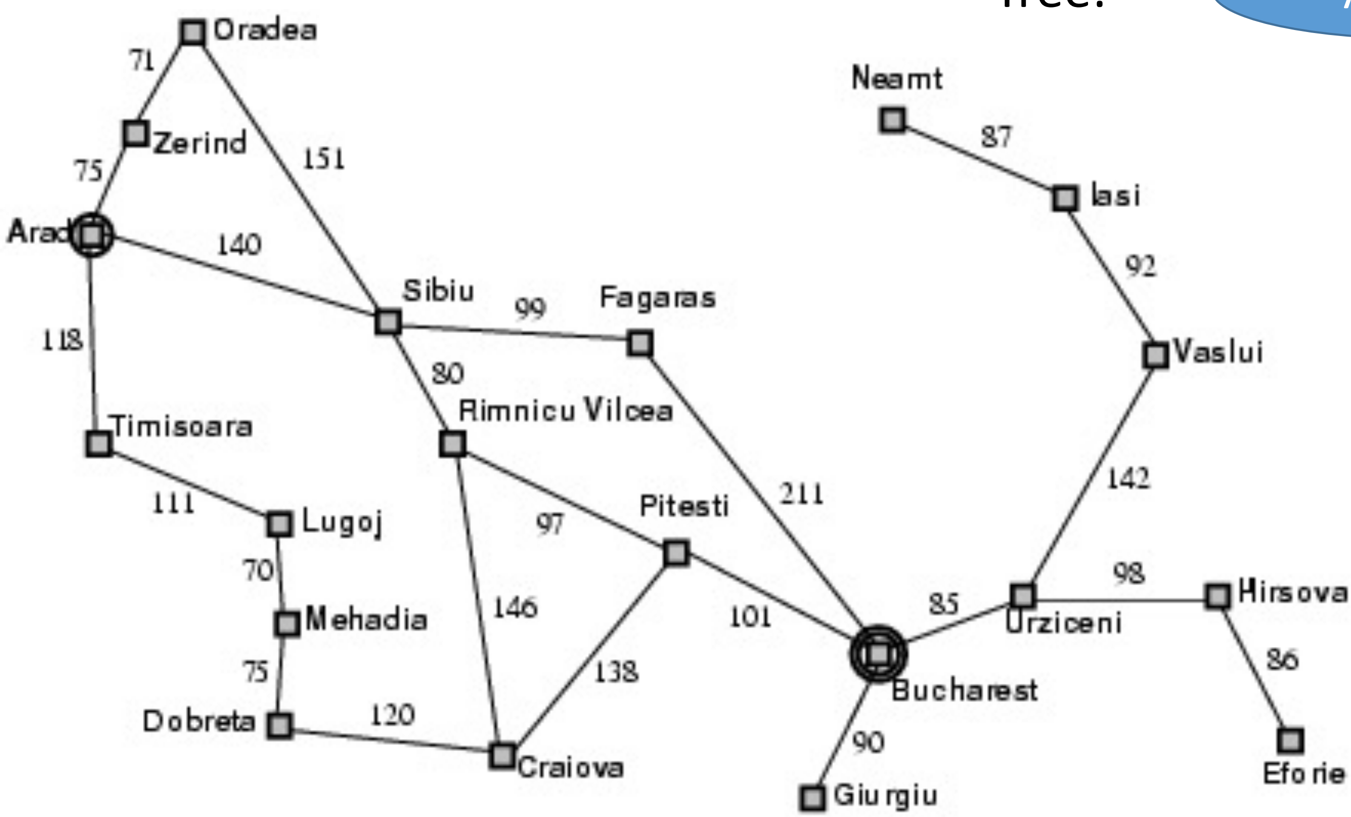
- Check to see whether it's already been explored.
- If so:
  - Skip it.
- If not:
  - Add it to the frontier
  - Add it to the tree
  - Add it to the explored set

Search step 0

Frontier: { Arad }

Explored: { Arad }

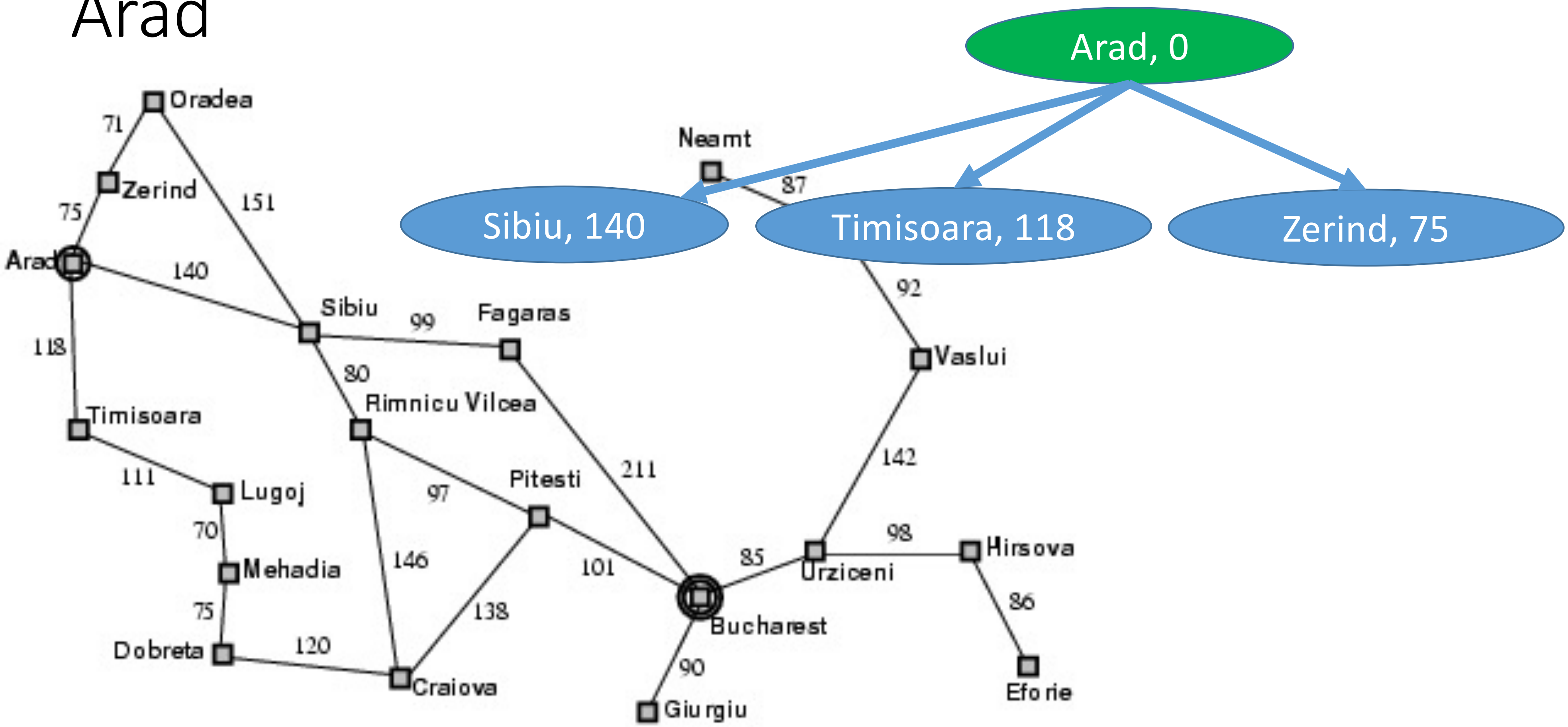
Tree: Arad, 0



Search step 1: expand Arad

Frontier: { Sibiu, Zerind, Timisoara }

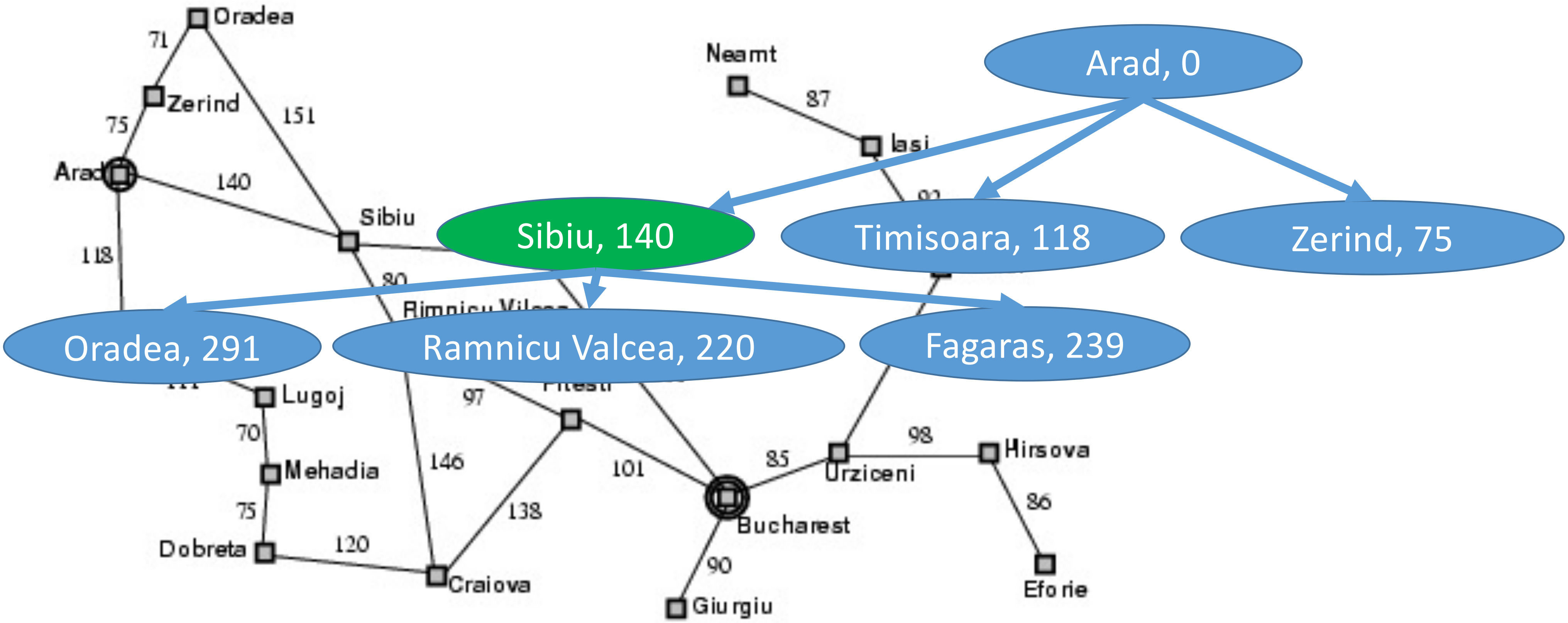
Explored: { **Arad**, Sibiu, Zerind, Timisoara }



Search step 2: expand Sibiu

Frontier: { Zerind, Timisoara, Oradea, Ramnicu Valcea, Fagaras }

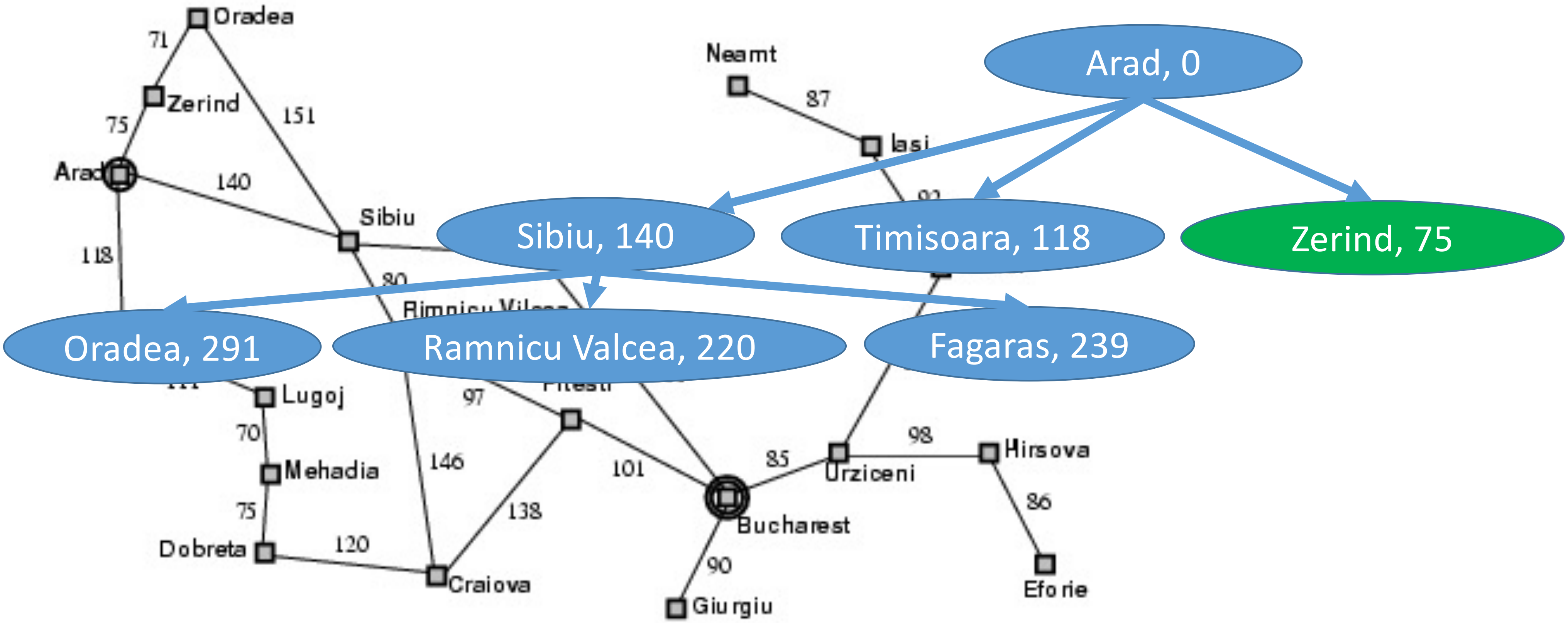
Explored: { Arad, **Sibiu**, Zerind, Timisoara, Oradea, Ramnicu Valcea, Fagaras }



Search step 3: expand Zerind

Frontier: { Timisoara, Oradea, Ramnicu Valcea, Fagaras }

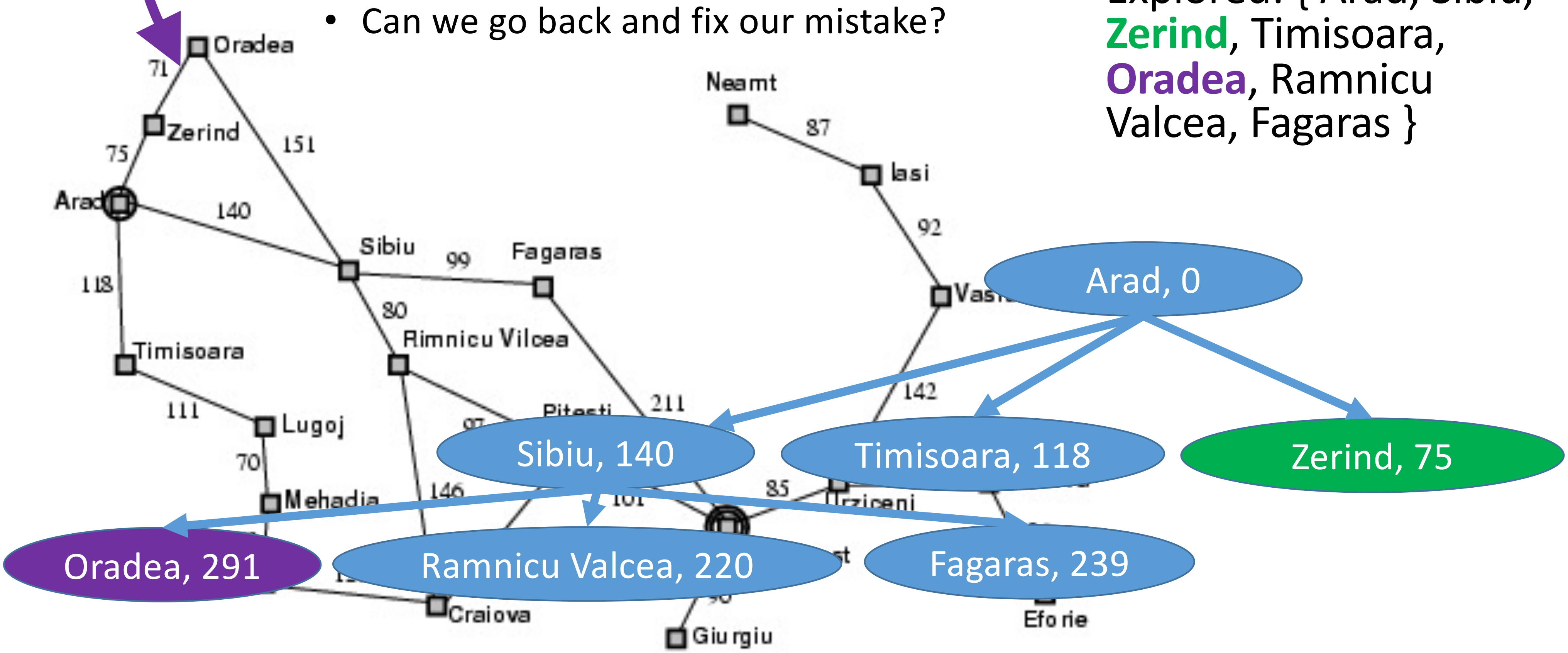
Explored: { Arad, Sibiu, **Zerind**, Timisoara, Oradea, Ramnicu Valcea, Fagaras }



- We don’t add **Oradea** to the frontier again, b/c already in explored set
- But our new path to Oradea is shorter – only **146km**, instead of **291km**!
- Can we go back and fix our mistake?

Frontier: { Timisoara, Oradea, Ramnicu Valcea, Fagaras }

Explored: { Arad, Sibiu, **Zerind**, Timisoara, **Oradea**, Ramnicu Valcea, Fagaras }



Fourth data structure: Explored Dictionary

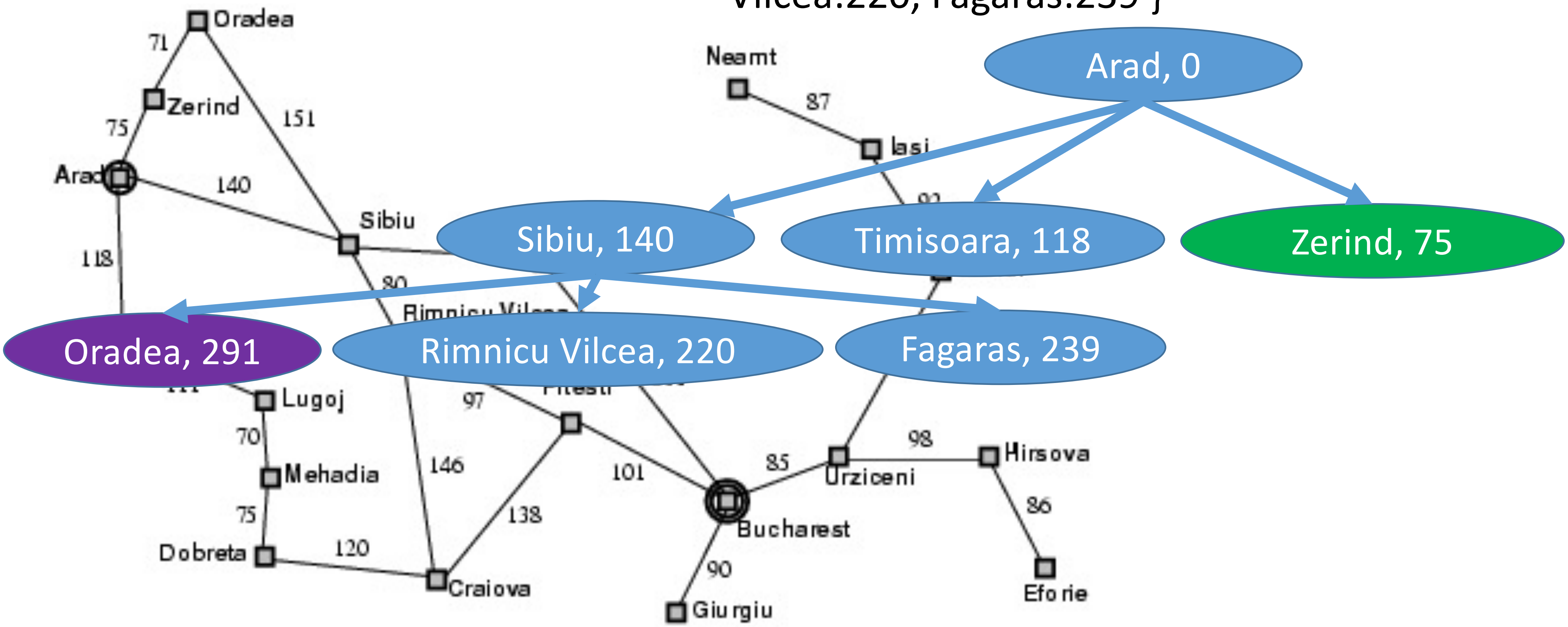
Explored = dictionary mapping from state ID to path cost

- If a child state is in the explored dict, and our new path has HIGHER COST, then
  - Skip it.
- If a child state is in the explored dict, but our new path has LOWER COST, then:
  - Update the dict: explored[state] = new_cost
  - Put the new (state, parent, cost) tuple into the frontier and the tree

Search step 3: expanded Zerind

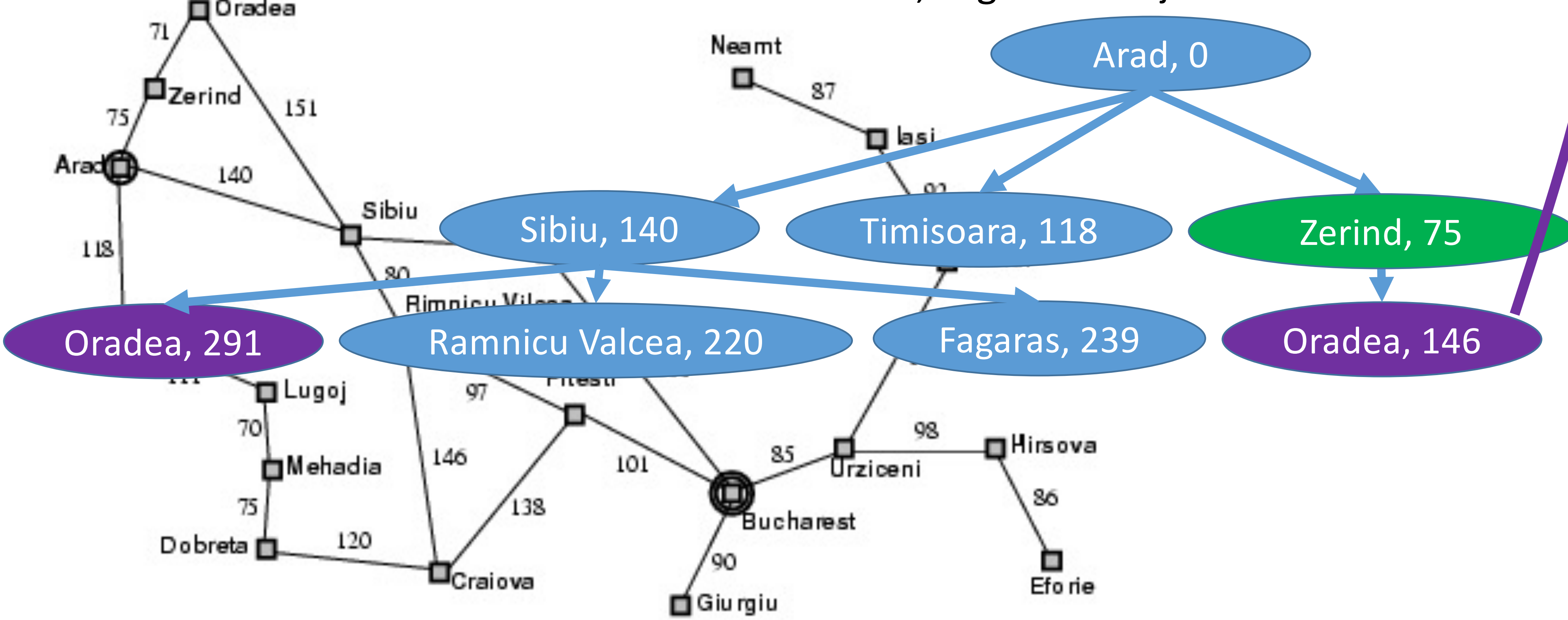
Frontier: { Timisoara, Oradea, Rimnicu Vilcea, Fagaras }

Explored: { Arad:0, Sibiu:140, **Zerind:75**, Timisoara:118, **Oradea:291**, Rimnicu Vilcea:220, Fagaras:239 }



Frontier: { Timisoara, Oradea, Ramnicu Valcea, Fagaras }

Explored: { Arad:0, Sibiu:140, **Zerind:75**, Timisoara:118, **Oradea:146**, Rimnicu Vilcea:220, Fagaras:239 }



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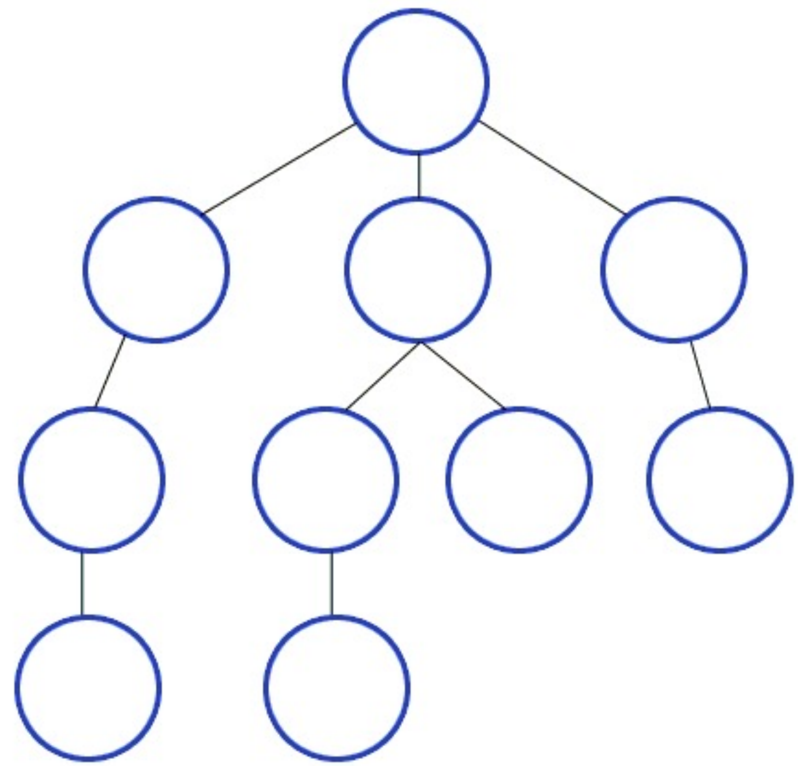
In which order should you pick nodes from the frontier?

- LIFO (last-in, first-out) = Depth-First Search (DFS):
  - the next node you expand will always be the one **most recently** added to the frontier.
- FIFO (first-in, first-out) = Breadth-First Search (BFS):
  - the next node you expand will always be the one **least recently** added to the frontier.

Depth-first search (DFS)

Expand frontier in LIFO order (last in, first out).

Result: most recently discovered path is pursued, all the way to the end.



Depth-first-search.
https://commons.wikimedia.org/wiki/File:Depth-First-Search.gif

Analysis of search strategies

- Strategies are evaluated along the following criteria:
  - **Completeness:** does it always find a solution if one exists?
  - **Optimality:** does it always find a least-cost solution?
  - **Time complexity:** number of nodes generated
  - **Space complexity:** maximum number of nodes in memory
- Time and space complexity are measured in terms of
  - ***b****:* maximum branching factor of the search tree
  - ***d****:* depth of the optimal solution
  - ***m***: maximum length of any path in the state space (may be infinite)

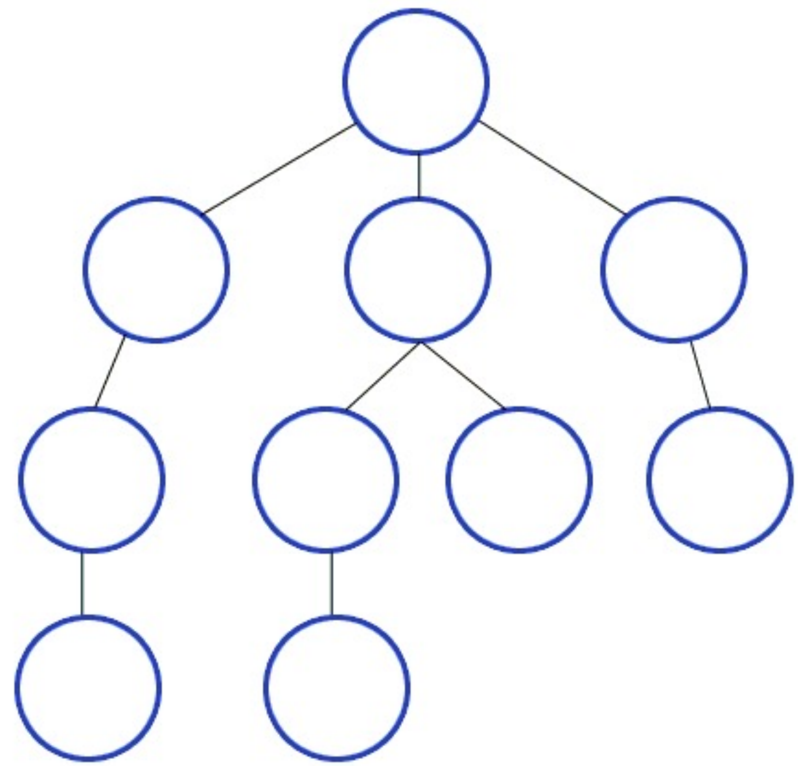
Depth-first search (DFS)

Incomplete: If there are an infinite number of states, DFS might go down a path of infinite length, and might never find a solution.

Suboptimal: DFS returns the first path it finds, which might not be the shortest path.

Time Complexity: $O\{b^m\}$, where m is the longest possible path length.

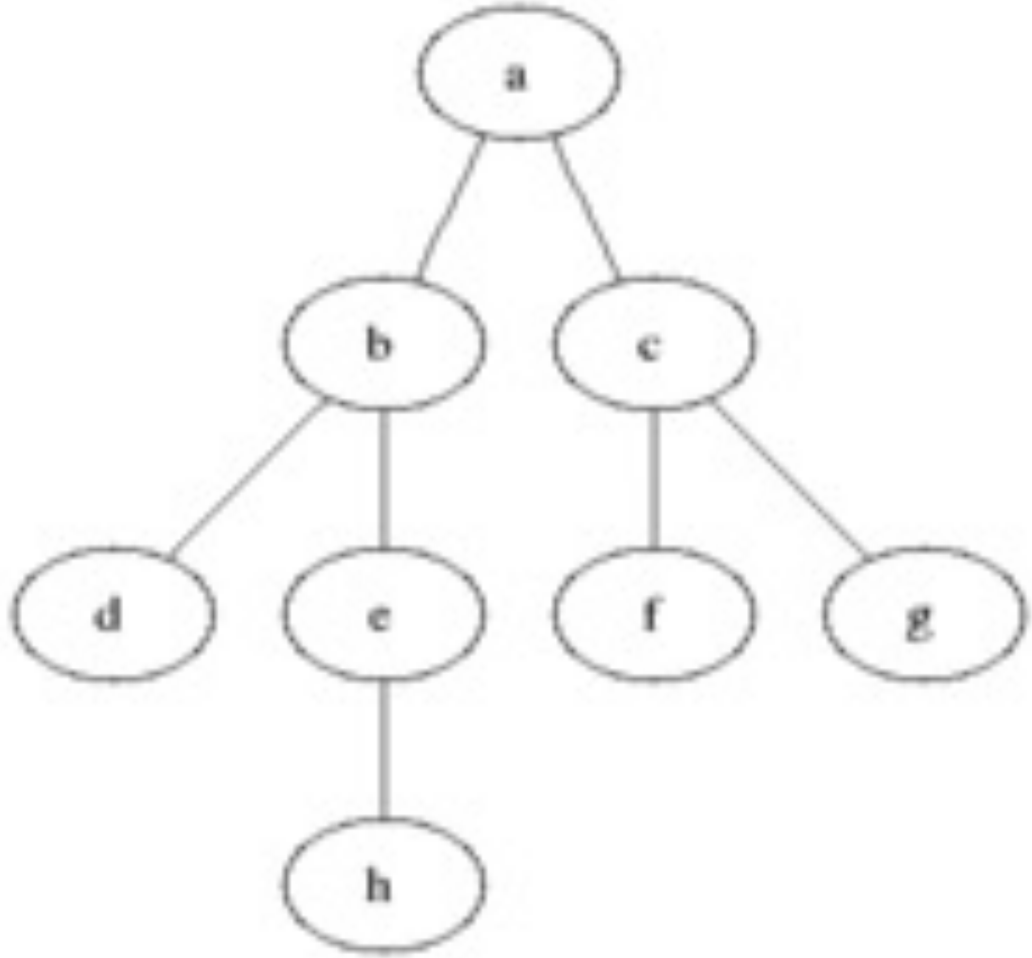
Space Complexity: only $O\{m\}$! Once you've finished a path, you can delete it from the tree!



Breadth-first search (BFS)

Expand the frontier in FIFO order (first-in, first-out).

Result: all paths of length d are explored, then all paths of length d+1, and so on.



Animated-BFS. CC-SA 3.0, Blake Matheny, 2007
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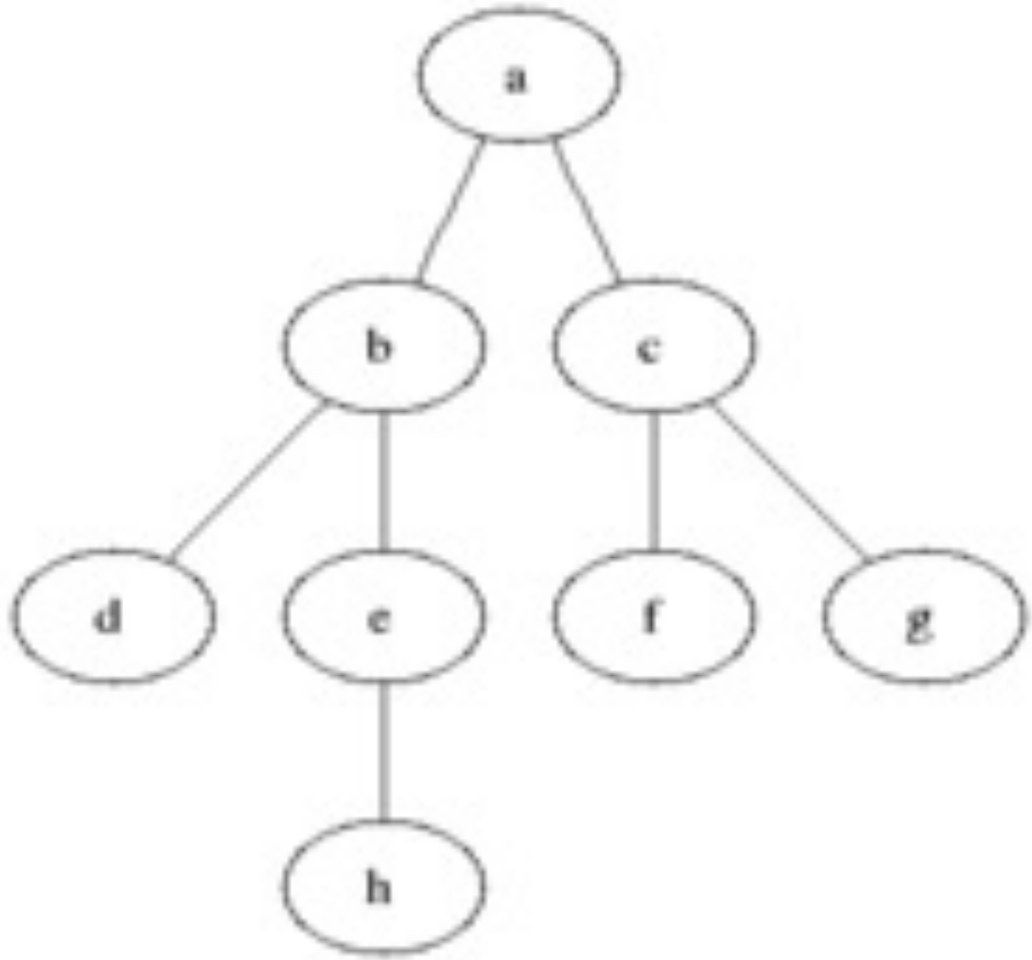
Breadth-first search (BFS)

Complete: if a finite-length path exists, BFS will find it.

Optimal: BFS returns the first solution it finds, which is always the shortest path.

Time Complexity: $O\{b^d\}$, where d is the length of the best path. This is usually much less than $O\{b^m\}$, because $d < m$.

Space Complexity: $O\{b^d\}$. No part of the tree can be deleted until you've found the solution.

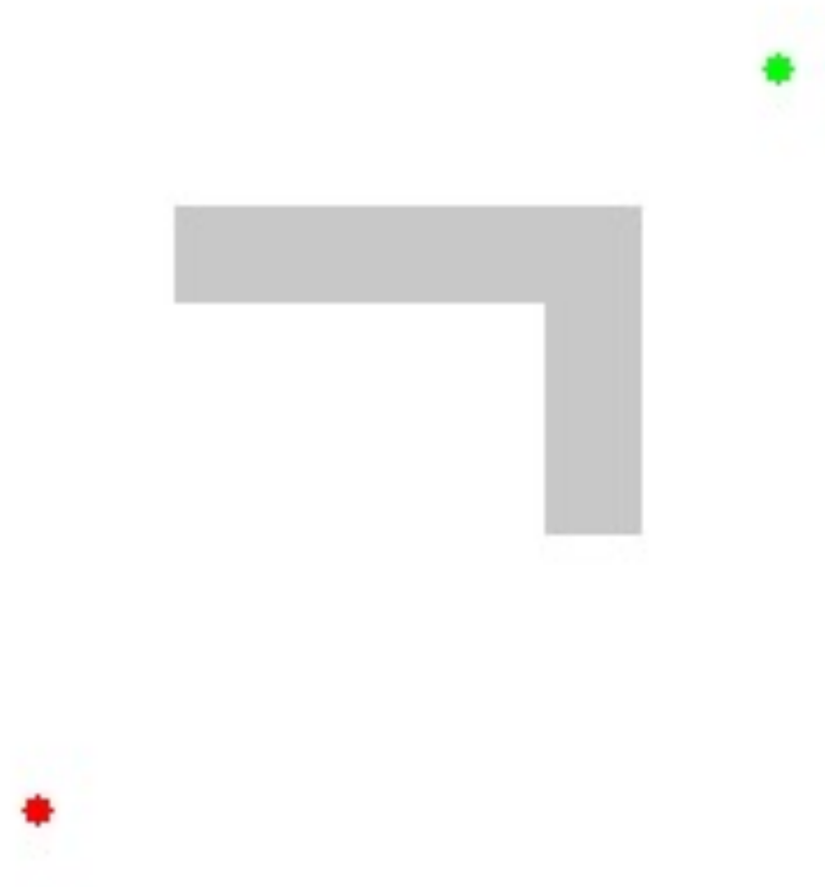


Animated-BFS. CC-SA 3.0, Blake Matheny, 2007
https://commons.wikimedia.org/wiki/File:Animated_BFS.gif

BFS: How to do it

- Notice that BFS searches in exactly the same order as Dijkstra's algorithm.

- BFS is the normal way you would implement Dijkstra's algorithm for a possibly-infinite search space.



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