UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

CS440/ECE448 Artificial Intelligence

Exam 2

Spring 2022

April 4, 2022

Your Name: ______________________________

Your NetID: ______________________________

Instructions

- Please write your NetID on the top of every page.
- This is a CLOSED BOOK exam. You will be permitted to bring one 8.5x11 page of handwritten notes (front & back).
- Calculators are not permitted. You need not simplify explicit numerical expressions.

Possibly Useful Formulas

Search:

Admissible Heuristic: $h(n) \leq d(n)$

Consistent Heuristic: $h(m) - h(n) \leq d(m) - d(n)$ if $d(m) - d(n) \geq 0$

Belief Propagation:

$$P(A,B,C) = P(A)P(B|A)P(C|A,B)$$
$$P(A,C) = \sum_b P(A,B=b,C)$$
$$P(A|C) = \frac{P(A,C)}{P(C)}$$

Expectation Maximization:

$$P(B=b|A=a) \leftarrow \frac{E\left[\#\text{ times } A=a, B=b\right]}{E\left[\#\text{ times } A=a\right]}$$
$$E\left[\#\text{ times } A=a, B=b\right] = \sum_t P\left(A_t=a, B_t=b|\text{observations on day } t\right)$$

HMM:

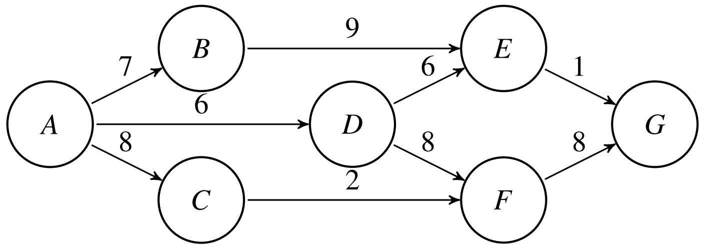
$$P(Y_1, X_1, \ldots, Y_T, X_T) = \prod_{t=1}^{T} P(Y_t|Y_{t-1})P(X_t|Y_t)$$

Viterbi Algorithm:

$$e_{i,j,t} = a_{j,i} b_{j,k}$$
$$v_{i,1} = \pi_i b_{i,x_1}$$
$$v_{j,t} = \max_i v_{i,t-1} e_{i,j,t}$$

Question 1 ***(7 points)***

Consider the following search graph. The starting state is A, the goal state is G, and the cost of each possible action is shown on the corresponding edge:



Suppose that the states shown have the following heuristics:

	A	B	C	D	E	F	G
$h(n)$:	8	8	4	4	1	4	0

A* search (with repetitions avoided using an explored set) is applied to this graph to find the shortest path.

- What states are expanded, and
- what is the shortest path?

Solution: Expanded states: A,D,C,E,G
Shortest path: A,D,E,G

Question 2 ***(7 points)***

A typical freight management problem seeks to deliver several large objects from point A to point B using a truck that can carry up to M kilograms. First, you weigh each of the objects, so that you know its mass. Then you use the following search problem to devise an optimal plan:

- State: $S =$ a list of the objects that have not yet been delivered.
- Action: Load a set of objects onto the truck, take them from A to B, then return the truck from B to A.
- Cost: Each trip from A to B has a cost of 1, regardless of the weight of the objects on the truck. Your goal is simply to minimize the number of trips.

Define a nonzero heuristic for this problem, and prove that your heuristic is admissible.

Solution: A reasonable heuristic is the sum of the weights of all objects that have not yet been delivered, divided by M.

$d(S)$ is the total number of trips remaining. The truck can only carry up to M kilograms per trip, therefore $d(S) \geq \frac{1}{M}$(mass of remaining objets).

Question 3 *(7 points)*

The figure below shows a map of the 7 provinces of Costa Rica, donated to Wikipedia by Pixeltoo.

Your goal is to color each of these 7 provinces Red, Green, or Blue, in such a way that no two neighboring provinces have the same color.

Answer the following three questions using the LRV, MCV, and LCV heuristics.

(a) Suppose that none of the regions have been colored yet. Which region should be colored first, and why?

> **Solution:** Region 6 should be colored firt, because it constrains the largest number of other provinces (MCV).

(b) Suppose that region 1 has been colored Red, and no other regions have been colored. Now you want to find a color for region 2. What color should region 2 be colored, and why?

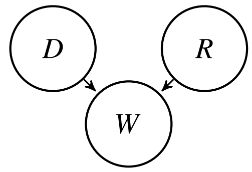
> **Solution:** Region 2 should be colored Red also, because that is the only choice that will leave region 7 with more than one possible color (LCV).

(c) Suppose that region 1 has been colored Red, and region 2 has been colored Green. What region should be colored next, and why?

> **Solution:** Region 7 should be colored next, because it has only one possible color (LRV).

Question 4 ***(7 points)***

Consider the following Bayesian network:



Suppose that the model parameters are as follows:

$$P(D=d) = \frac{1}{2} \text{ for } d \in \{1,2\}$$
$$P(R=r) = \frac{1}{2} \text{ for } r \in \{1,2\}$$
$$P(W=T|D=d,R=r) = \begin{cases} \frac{2}{3} & d \geq r \\ \frac{1}{3} & d < r \end{cases}$$

What is $P(D=2|W=T)$?

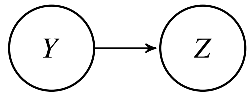
Solution:

$$P(D=2|W=T) = \frac{\sum_{r=1}^{2} P(D=2,R=r,W=T)}{\sum_{d=1}^{2}\sum_{r=1}^{2} P(D=d,R=r,W=T)}$$
$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$$

If you wish, you can simplify the last formula to $4/7$, but it's not required.

Question 5 *(7 points)*

Consider the following Bayesian network, showing the relationship between two binary variables Y and Z:



Suppose that you've been given the following initial estimates of the model parameters, where a, b, and c are some arbitrary constants:

$$P(Y=T)=a, \quad P(Z=T|Y=F)=b, \quad P(Z=T|Y=T)=c$$

You are now trying to re-estimate the values of these model parameters. You have observed the values of the variables on five consecutive days, but on the fifth day, the value of Y was unobserved (labeled with a "?" in the table below:

	Day 1	Day 2	Day 3	Day 4	Day 5
Value of Y:	T	F	T	F	?
Value of Z:	F	F	F	T	T

For this table, in terms of the current model parameters a, b, and c, what is the expected number of days on which $Y=T$?

Solution:

$$\begin{aligned} E\left[\#\text{ days } Y=T\right] &= 2+P(Y=T|Z=T) \\ &= 2+\frac{ac}{ac+(1-a)b} \end{aligned}$$

Question 6 ***(7 points)***

Your apartment is haunted by a ghost. Like most ghosts, your ghost tends to sleep for several days at a time. Let $Y_t = T$ if the ghost is awake on day t. You can't see the ghost, but if the ghost is awake, your cat tends to hide under the bed; let $X_t = T$ if your cat is hiding under the bed on day t. Suppose that these probabilities are given by the following distribution, where a, b, c and d are arbitrary constants:

Y_{t-1}	$P(Y_t = T \vert Y_{t-1})$
F	a
T	b

Y_t	$P(X_t = T \vert Y_t)$
F	c
T	d

Suppose you know that the ghost was asleep on day 0 ($Y_0 = F$). You don't know whether or not it was awake on day 1, but you know that your cat hid under the bed ($X_1 = T$). In terms of a, b, c and/or d, find $P(Y_1 = T|Y_0 = F, X_1 = T)$.

Solution:

$$\begin{aligned} P(Y_1 = T|Y_0 = F, X_1 = T) &= \frac{P(Y_1 = T, X_1 = T|Y_0 = F)}{P(X_1 = T|Y_0 = F)} \\ &= \frac{ad}{ad + (1-a)c} \end{aligned}$$

Question 7 ***(7 points)***

Consider an HMM with state variables $Y_1,\ldots,Y_T$ and observations $X_1,\ldots,X_T$. Suppose that the model has the following parameters, where a, b, c and d are some arbitrary constants:

Y_{t-1}	$P(Y_t=2\|Y_{t-1})$
1	a
2	b

Y_t	$P(X_t=2\|Y_t)$
1	c
2	d

For a particular observation sequence $X_1=x_1,\ldots,X_T=x_T$, define the Viterbi vertex probability to be

$$v_{j,t}=\max_{y_1,\ldots,y_{t-1}} P(Y_1=y_1,X_1=x_1,\ldots,Y_{t-1}=y_{t-1},X_{t-1}=x_{t-1},Y_t=j,X_t=x_t)$$

Suppose that $v_{j,t}$ has been calculated, and has been found to have the following values, where e and f are some arbitrary constants:

$$v_{1,t}=e$$
$$v_{2,t}=f$$

Furthermore, suppose that $x_{t+1}=2$. In terms of the constants a, b, c, d, e, and/or f, what are $v_{1,t+1}$ and $v_{2,t+1}$?

Solution:

$$v_{1,t+1}=\max\left(e(1-a)c, f(1-b)c\right)$$
$$v_{2,t+1}=\max\left(ead, fbd\right)$$