# Collab Worksheet 10 

CS440/ECE448, Spring 2021
Week of 4/19-4/23, 2021

## Question 1

Consider a simultaneous game in which each player has three possible moves: A, B, or C. The payoff matrix is shown below. In each square, the first number refers to payoff for Alice (the player whose moves are shown on the row-label), the second number refers to payoff for Bob (the player shown on the column label).

| Alice's Move | Bob's Move |  |  |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
| A | $0 \backslash 0$ | $25 \backslash 40$ | $5 \backslash 10$ |
| B | $40 \backslash 25$ | $0 \backslash 0$ | $5 \backslash 15$ |
| C | $10 \backslash 5$ | $15 \backslash 5$ | $10 \backslash 10$ |

(a) Are there any dominant strategies? If so, what are they? If not, why not?

Solution: No. The best move, for each player, depends on what the other player does.
(b) Are there any pure-strategy Nash equilibria? If so, what are they? If not, why not?

Solution: (A,B), (B,A), and (C,C)
(c) Are there any Pareto-optimal solutions? If so, what are they? If not, why not?

Solution: $(\mathrm{A}, \mathrm{B})$ and $(\mathrm{B}, \mathrm{A})$.

## Question 2

Alice and Bob are playing a game called "exes and ohs". First, Alice draws either an X or an O, without showing it to Bob. Then Bob does the same. Then they show each other their moves, and they each receive the number of points shown in the following payoff matrix. In each square, Alice's payoff is below the diagonal line, Bob's is above it:

| Alice's Move | Bob's Move |  |
| :---: | :---: | :---: |
|  | X | O |
| X | $6 \backslash 6$ | $4 \backslash 10$ |
| O | $10 \backslash 4$ | $2 \backslash 2$ |

(a) What are the Pareto-optimal outcomes for this game?

Solution: Pareto optimal solutions are ( $\mathrm{X}, \mathrm{O}$ ), ( $\mathrm{O}, \mathrm{X}$ ), and ( $\mathrm{X}, \mathrm{X}$ ).
(b) Are there fixed-strategy Nash equilibria for this game? If so, what are they? If not, why not?

Solution: Yes: $(\mathrm{X}, \mathrm{O})$ and ( $\mathrm{O}, \mathrm{X})$.
(c) Is there a mixed-strategy Nash equilibrium for this game? If so, what is it? If not, why not?

Solution: In the mixed equilibrium, both Alice and Bob choose "O" with probability $p=0.67$.

