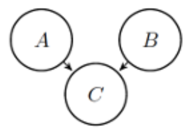
Collab Worksheet 6

CS440/ECE448, Spring 2021

Week of 3/17 - 3/22, 2021

Question 1

Consider a Bayes network with three binary random variables, A, B, and C, with the relationship and model parameters shown below:



$$P(A) = 0.4$$
$$P(B) = 0.1$$
$$P(C|A,B) = \begin{cases} 0.7 & A = \text{False}, B = \text{False} \\ 0.7 & A = \text{False}, B = \text{True} \\ 0.1 & A = \text{True}, B = \text{False} \\ 0.9 & A = \text{True}, B = \text{True} \end{cases}$$

(a) What is $P(C)$? Write your answer in numerical form, but you don't need to simplify.

Solution:

$$P(C) = (0.6)(0.9)(0.7) + (0.6)(0.1)(0.7) + (0.4)(0.9)(0.1) + (0.4)(0.1)(0.9)$$

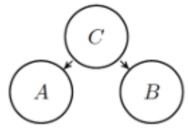
(b) What is $P(A = \text{True}|B = \text{True}, C = \text{True})$? Write your answer in numerical form, but you don't need to simplify.

Solution:

$$P(A = \text{True}|B = \text{True}, C = \text{True}) = \frac{(0.4)(0.1)(0.9)}{(0.4)(0.1)(0.9) + (0.6)(0.1)(0.7)}$$

Question 2

Consider a Bayes network with three binary random variables, A, B, and C, with the relationship shown below:



You've been asked to re-estimate the parameters of the network based on the following observations:

Token	A	B	C
1	False	True	False
2	True	True	False
3	False	False	True
4	False	False	True

(a) Given the data in the table, what are the maximum likelihood estimates of the model parameters?

Solution:

$$P(C) = 1/2$$
$$P(A|C) = \begin{cases} 0 & C = \text{True} \\ 1/2 & C = \text{False} \end{cases}$$
$$P(B|C) = \begin{cases} 0 & C = \text{True} \\ 1 & C = \text{False} \end{cases}$$

(b) Your roommate discovers two extra training tokens, scrawled on a half-burned piece of notebook paper. Unfortunately, the two new training tokens are incomplete: they only contain measurements of B and C, but no measurements of A. Including the original four training tokens plus the two new ones, your dataset is now:

Token	A	B	C
1	False	True	False
2	True	True	False
3	False	False	True
4	False	False	True
5	?	True	True
6	?	False	False

Using the model parameters that you estimated in part (a) as input to the EM algorithm, what is the expected number of observations of the event $A = \text{True}$?

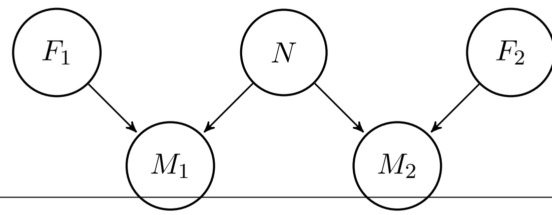
Solution: There is one visible observations of $A = \text{True}$. There are also two training tokens with hidden values of A. The two hidden values co-occur with values $C = \text{True}$ and $C = \text{False}$, respectively. The corresponding probabilities of A are currently estimated at $P(A|C = \text{True}) = 0$ and $P(A|C = \text{False} = 1/2$, so the total expected number of occurrences of $A = \text{True}$ is $1 + 0 + 0.5 = 1.5$.

Question 3

Consider an astronomy problem with five variables: N, M_1, M_2, F_1, and F_2. N is the true number of stars in a particular small patch of sky. Two astronomers in different parts of the world are trying to measure the value of N. Unfortunately, their telescopes sometimes suffer a hardware fault: denote the occurrence of a hardware fault using the binary variables F_1 and F_2, respectively, and specify that the probability of a hardware fault is f. If F_1 = True, then the measurement obtained by astronomer #1 is too small by at least three stars, $M_1 \leq \max(0, N-3)$. If F_1 = False, then $M_1 \approx N$, but it might be too large or too small by one star (it might be $N-1$ or $N+1$). Suppose that $P(M_1 = N-1) = e$, and $P(M_1 = N+1) = e$. Similar arguments relate the variables F_2, M_2 and N, with exactly the same parameters e and f.

(a) Draw a Bayesian network for this problem.

Solution:



(b) Write out a conditional distribution for $P(M_1|N)$ for the case where $N \in \{1, 2, 3\}$ and $M_1 \in \{0, 1, 2, 3, 4\}$. Each entry in the conditional distribution table should be expressed as a function of the parameters e and/or f.

Solution:

	M_1				
N	0	1	2	3	4
1	$f+e$	$1-f-2e$	e	0	0
2	f	e	$1-f-2e$	e	0
3	f	0	e	$1-f-2e$	e

(c) Suppose $M_1 = 1$ and $M_2 = 3$. What are the possible numbers of stars if you assume no prior constraint on the values of N?

Solution: $N = 2$ is possible, if both made small mistakes. $N = 4$ is possible, if M_2 made a small and M_1 a big mistake. $N \geq 6$ is possible, if both M_1 and M_2 made big mistakes.

(d) What is the most likely number of stars, given the observations $M_1 = 1, M_2 = 3$? Explain how to compute this, or if it is not possible to compute, explain what additional information is needed and how it would affect the result.

Solution: You can't solve this problem unless you know the distribution $P(N)$.