Collab Worksheet 2 Solutions

CS440/ECE448, Spring 2021

Week of 2/10 - 2/15, 2021

Question 1

Consider three events, A, B, and C, with probabilities given by $P(A) = 0.7$, $P(B) = 0.4$, and $P(C) = 0.3$.

(a) What's the largest possible $P(B \wedge C)$?

Solution:

$$P(B \wedge C) \leq \min(P(B), P(C)) = 0.3$$

(b) If A and B are independent, what's $P(A \wedge \neg B)$?

Solution: If A and B are independent, then

$$P(A \wedge \neg B) = P(A)P(\neg B) = (0.7)(0.6) = 0.42$$

Question 2

Consider three events, A, B, and C, with probabilities given by $P(A) = 0.7$, $P(B) = 0.4$, and $P(C) = 0.3$.

(a) If B and C are mutually exclusive, what's $P(B \wedge \neg C)$?

Solution: If B and C are mutually exclusive, then

$$P(B \wedge \neg C) = P(B) = 0.4$$

(b) What's the largest possible $P(\neg(A \wedge B))$?

Solution:

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B),$$

but since $P(A \vee B) \leq 1$, we see that the smallest possible value of $P(A \wedge B)$ is $0.7 + 0.4 - 1.0 = 0.1$, therefore the largest possible value of $P(\neg(A \wedge B))$ is 0.9.

Question 3

Laplace invented "Laplace smoothing" in order to estimate the probability that the sun will rise tomorrow. Suppose he had historical records indicating that the sun had been observed to rise on 1,826,200 consecutive days (and the event "the sun did not rise today" has never been observed). What probability would Laplace smoothing estimate for the event "The sun wil rise tomorrow"?

Solution:

$$P(R) = \frac{1826200 + 1}{1826200 + 2}$$

Question 4

Y is a random variable denoting the class of a newspaper title: $Y = 0$ means the article is about sports, $Y = 1$ means the article is about science. The title is only three words long; its three words are the random variables W_0, W_1, and W_2. Depending on whether the article is about sports or science, the title may contain any word from the following vocabulary: $\{\text{Illini}, \text{win}, \text{discover}, \text{everything}\}$. The prior probability of an article about science is $P(Y = 1) = 0.4$. Assume a naïve Bayes model, with word likelihoods of

$$\begin{aligned}
P(W_i = \text{Illini}|Y = 0) &= 0.3\\
P(W_i = \text{Illini}|Y = 1) &= 0.3\\
P(W_i = \text{win}|Y = 0) &= 0.3\\
P(W_i = \text{win}|Y = 1) &= 0.1\\
P(W_i = \text{discover}|Y = 0) &= 0.1\\
P(W_i = \text{discover}|Y = 1) &= 0.4
\end{aligned}$$

Now you download the article, and discover that its title is $X =$ Illini discover everything. What is $P(Y = 1|W_1, W_2, W_3)$? Leave your answer in the form of an expression composed of numbers; do not simplify.

Solution:

$$P(Y = 1|X) = \frac{P(Y = 1, X)}{P(Y = 0, X) + P(Y = 1, X)}$$

$$P(Y = 1, X) = P(Y = 1)P(W_1|Y = 1)P(W_1|Y = 1)P(W_2|Y = 1) = (0.4)(0.3)(0.4)(0.2)$$

$$P(Y = 0, X) = P(Y = 0)P(W_1|Y = 0)P(W_1|Y = 0)P(W_2|Y = 0) = (0.6)(0.3)(0.1)(0.3)$$

$$P(Y = 1|X) = \frac{(0.4)(0.3)(0.4)(0.2)}{(0.6)(0.3)(0.1)(0.3) + (0.4)(0.3)(0.4)(0.2)}$$