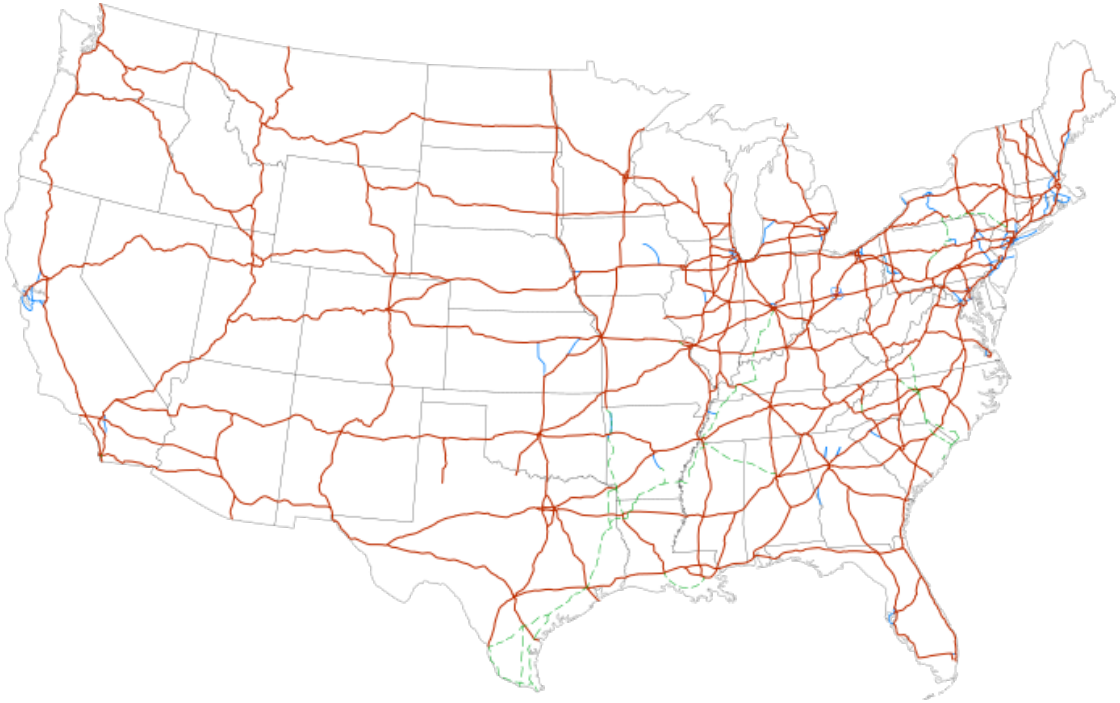
CS440/ECE 448 Lecture 2: Breadth-First Search

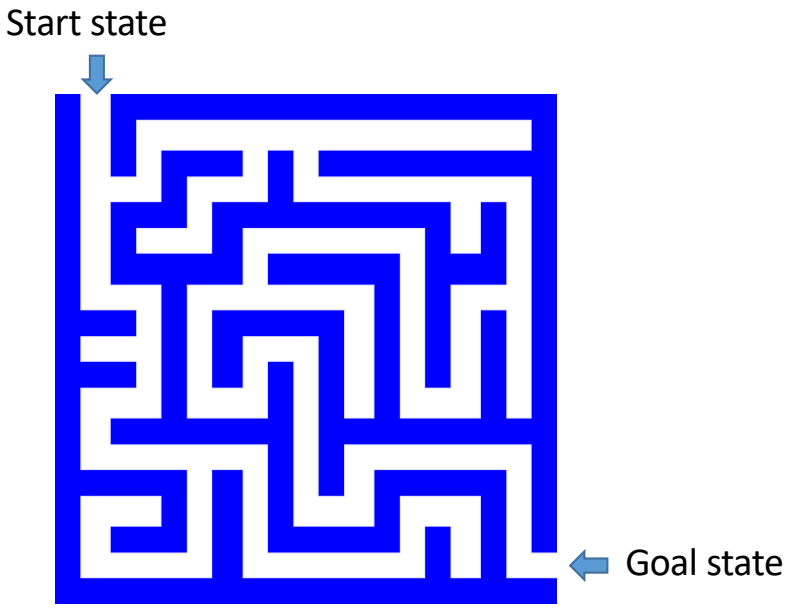


Outline of today's lecture

1. Initial state, goal state, transition model
2. General algorithm for solving search problems
    1. First data structure: a frontier queue
    2. Second data structure: a search tree
    3. Third data structure: a "visited states" dict
3. Breadth-first search (BFS) and Depth-first search (DFS)
4. Minimum spanning tree (MST)

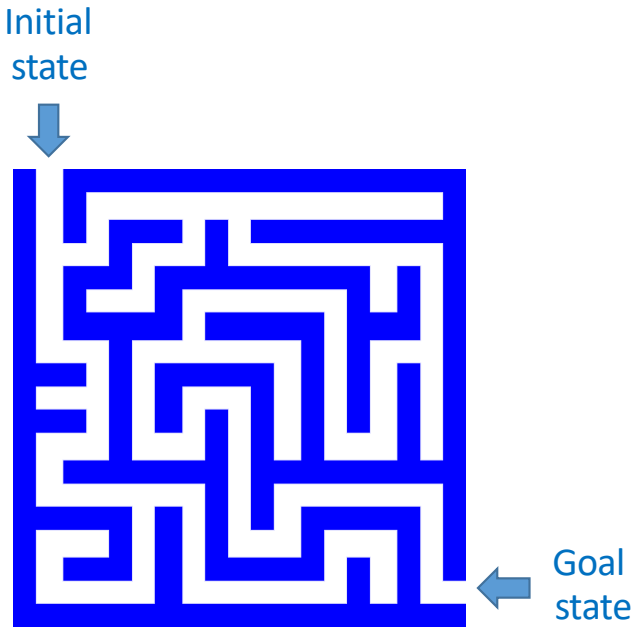
Search

- We will consider the problem of designing **goal-based agents** in **fully observable**, **deterministic**, **discrete**, **static**, **known** environments
- Environment is **sequential**: agent's action changes its state
- Agent must plan the best sequence of actions to achieve a goal



Search problem components

- **Initial state**
- **Actions**
- **Transition model**
  - What state results from performing a given action in a given state?
- **Goal state**
- **Path cost**
  - Assume that it is a sum of nonnegative *step costs*



- The **optimal solution** is the sequence of actions that gives the *lowest* path cost for reaching the goal

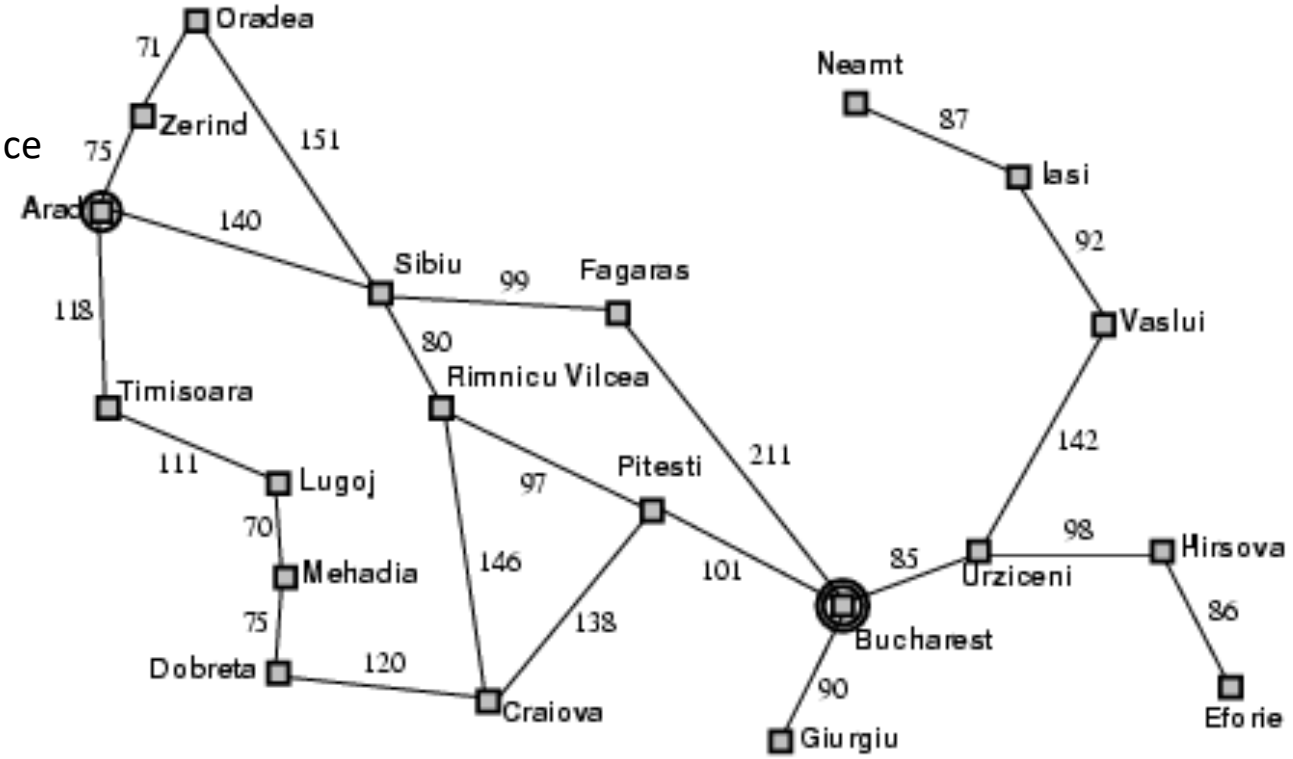
Knowledge Representation: State

- State = description of the world
  - Must have enough detail to decide whether or not you're currently in the **initial state**
  - Must have enough detail to decide whether or not you've reached the **goal state**
  - Often but not always: "defining the state" and "defining the transition model" are the same thing

Example of state definition: Romania

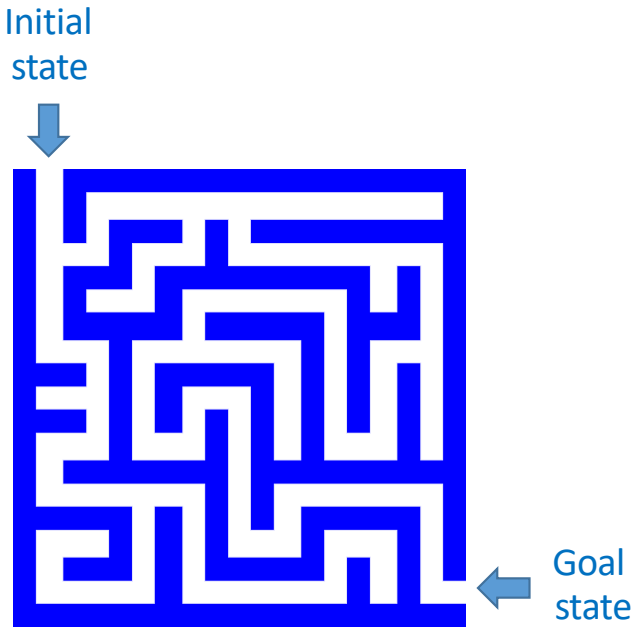
- On vacation in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

- **state** = name of the city
- **Path cost**
  - Sum of edge costs (total distance traveled)



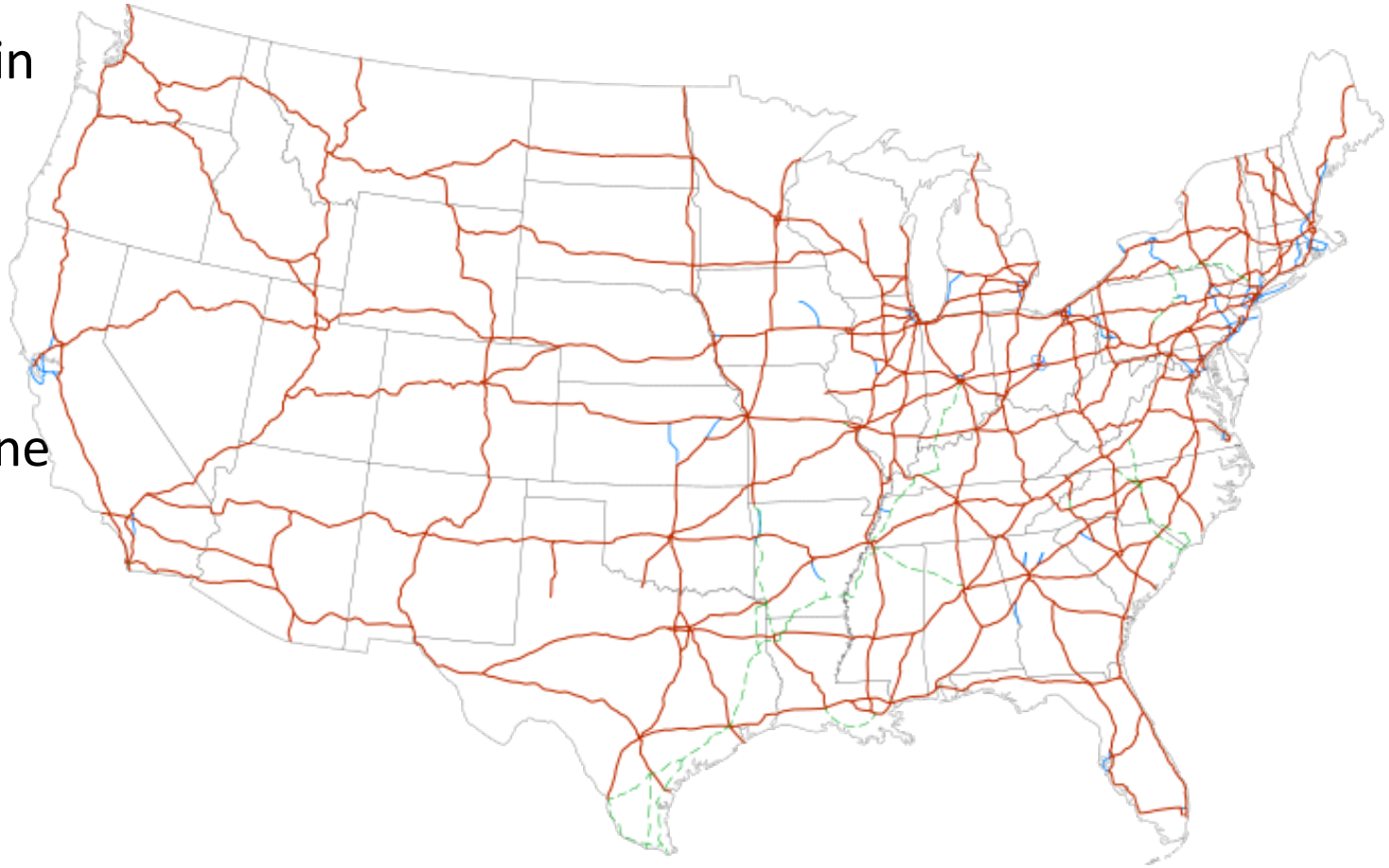
Example of state definition: Maze solving

- **State = (x,y)**, current position of the agent



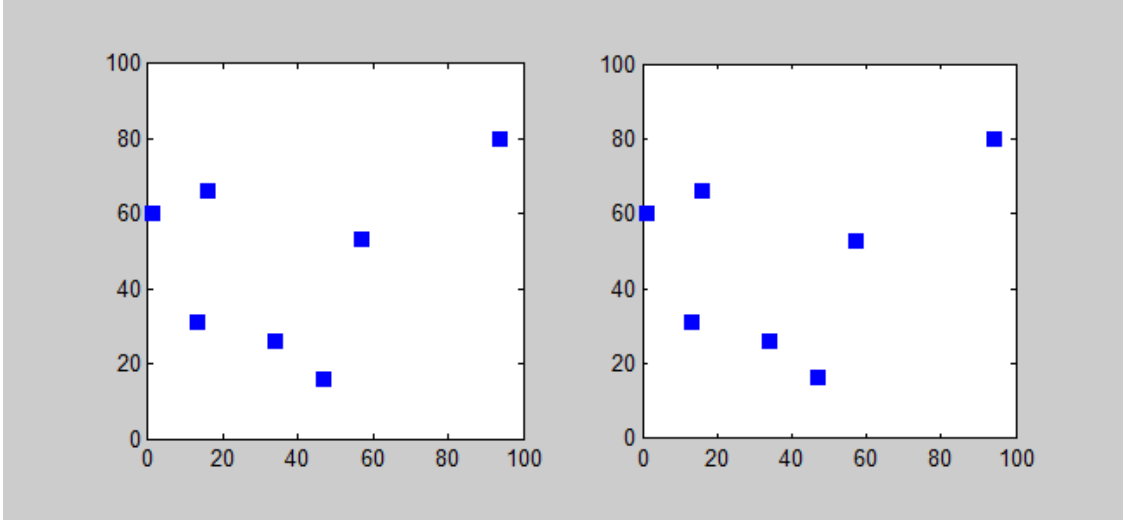
Example of state definition: Traveling salesman problem

- Goal: visit every city in the United States
- Path cost: total miles traveled
- Initial state: Champaign, IL
- Action: travel from one city to another
- Transition model: when you visit a city, mark it as “visited.”



Example of state definition: Traveling salesman problem

- **state = (agent, goals)**
  - **agent = (agent_x,agent_y)** is current position of the agent
  - **goals = [goal[0], goal[1], …]** lists the goals that have not yet been reached
    - **goal[i] = (goal_x, goal_y)** tells the location of the i'th remaining goal



Solving TSP using a branch-and-bound algorithm. CC-BY-SA 3.0, Saurabh Harsh, 2012, https://commons.wikimedia.org/wiki/File:Branchbound.gif

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    1. First data structure: a frontier queue
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How does this problem differ from every problem you've ever seen before?

- Search differs from most Computer Science problems in that the state space might be infinite. We don't assume, in advance, that we can enumerate every possible configuration of the world.
- Traditional definition of Dijkstra's algorithm:
  - First, list all of the possible states in the "not explored" list
  - Then, move them to the "explored" list after we visit them
- Modifying Dijkstra's algorithm for the infinite-world assumption:
  - Instead of a list of all possible states, we have a method **(next_state,cost)=Transition_Model(current_state, action)**
  - Instead of an infinite "not explored" list, we have a finite "frontier."

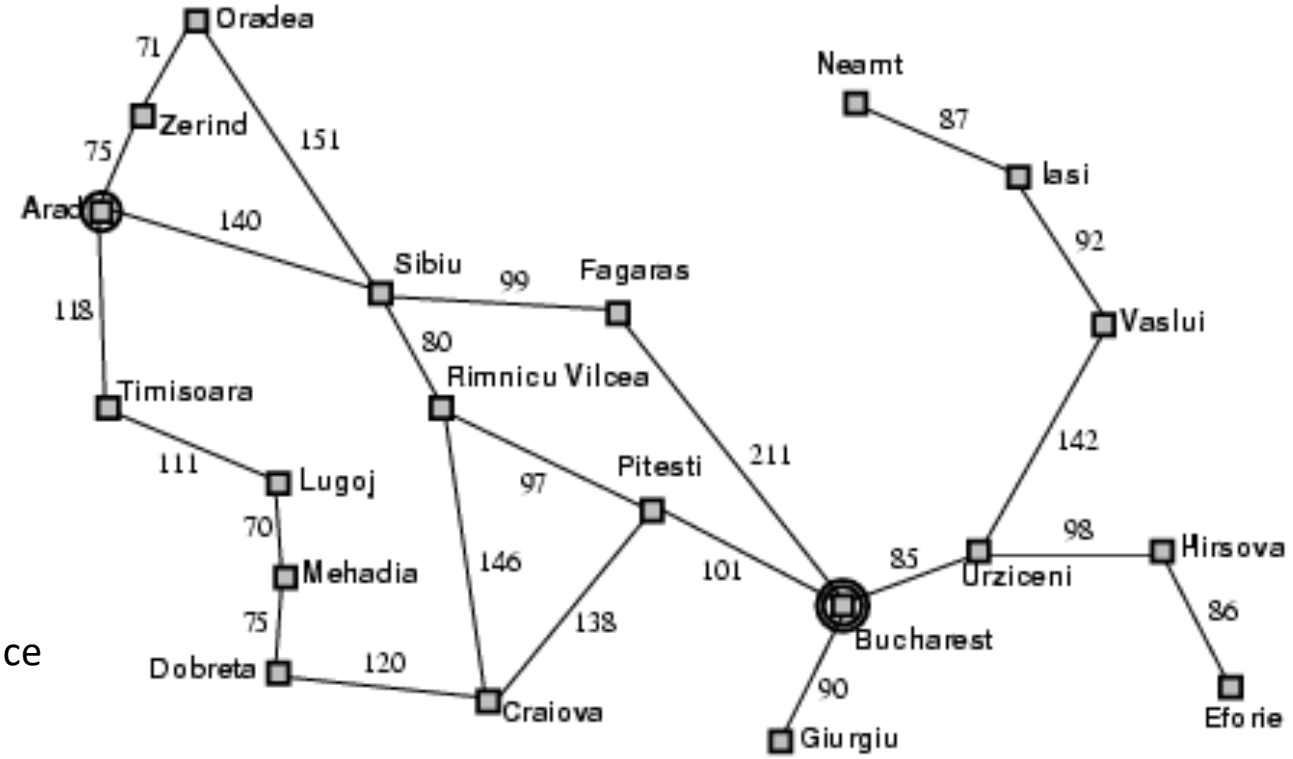
First data structure: Frontier

- Frontier = set of nodes that you know how to reach, but you haven't yet tested to see what comes next after those states
- **node = (state, parent_node, path_cost)**
- Initialize: **frontier = { (initial_state, None, 0) }**
- Iterate, until goal is reached:
  - Set **current_state** to some node from the frontier, remove it from the frontier.
  - Expand **current_state**:
    - If it's the goal, then you're done! Return the corresponding path.
    - If not, then find its children, and transition costs, using **(next_state,cost)=Transition_Model(current_state, action)**, and add them to the frontier.

Example: Romania

- On vacation in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

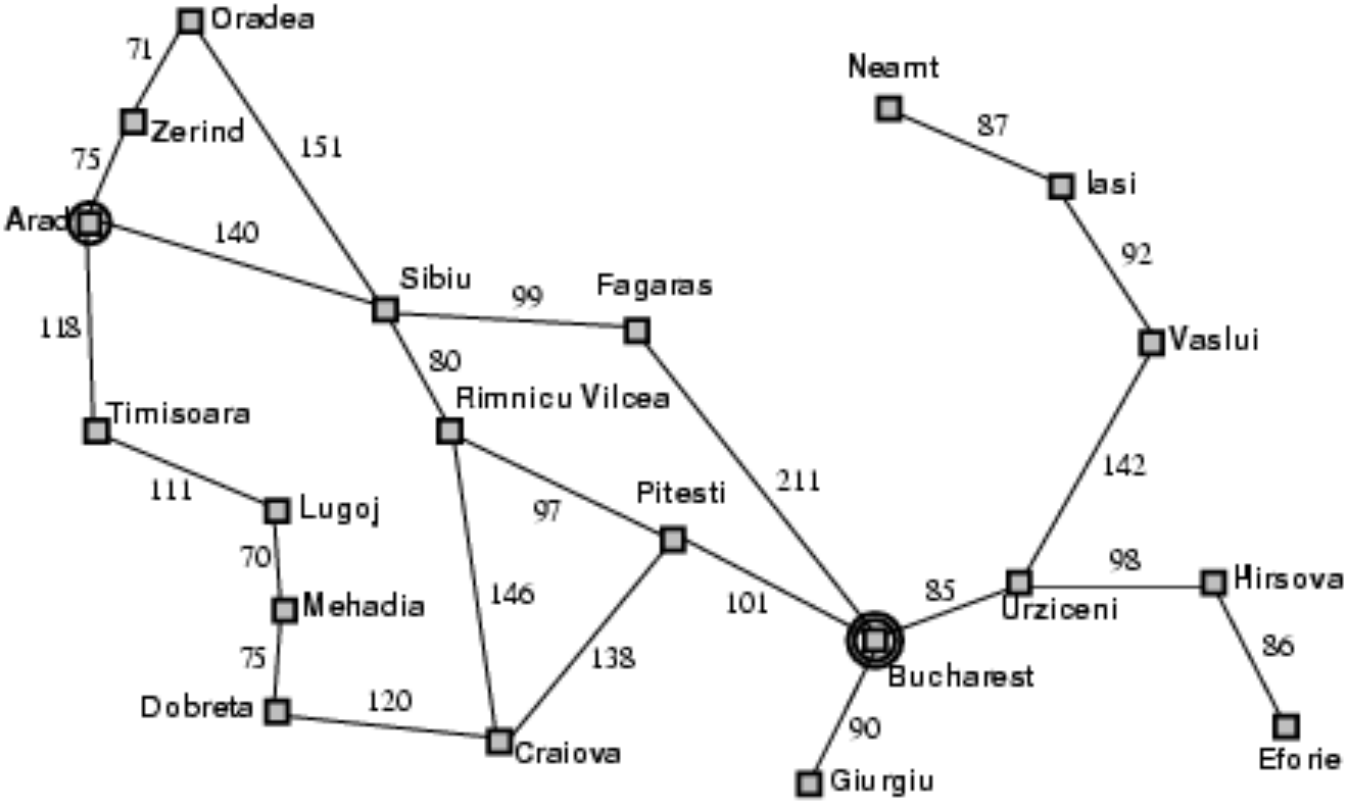
- **Initial state**
  - Arad
- **Actions**
  - Go from one city to another
- **Transition model**
  - If you go from city A to city B, you end up in city B
- **Goal state**
  - Bucharest
- **Path cost**
  - Sum of edge costs (total distance traveled)



Search step 0

Frontier: { Arad }

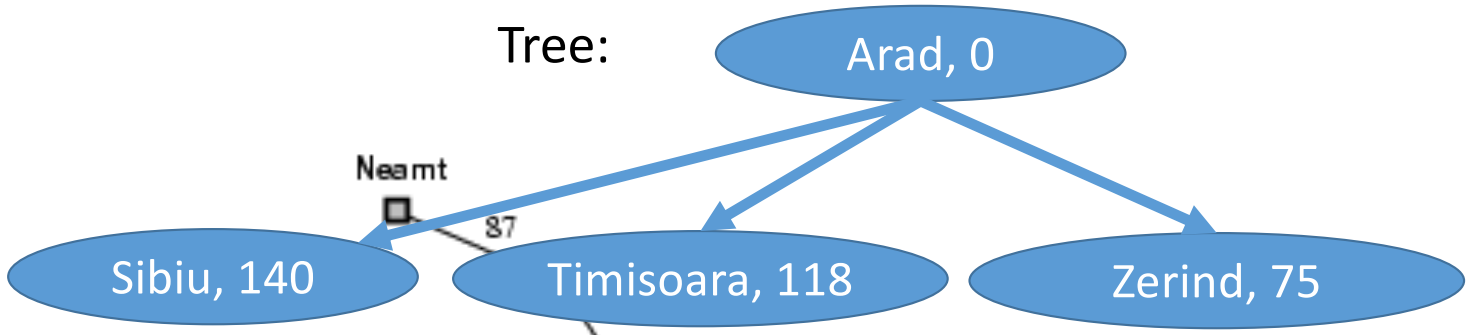
Tree: Arad, 0

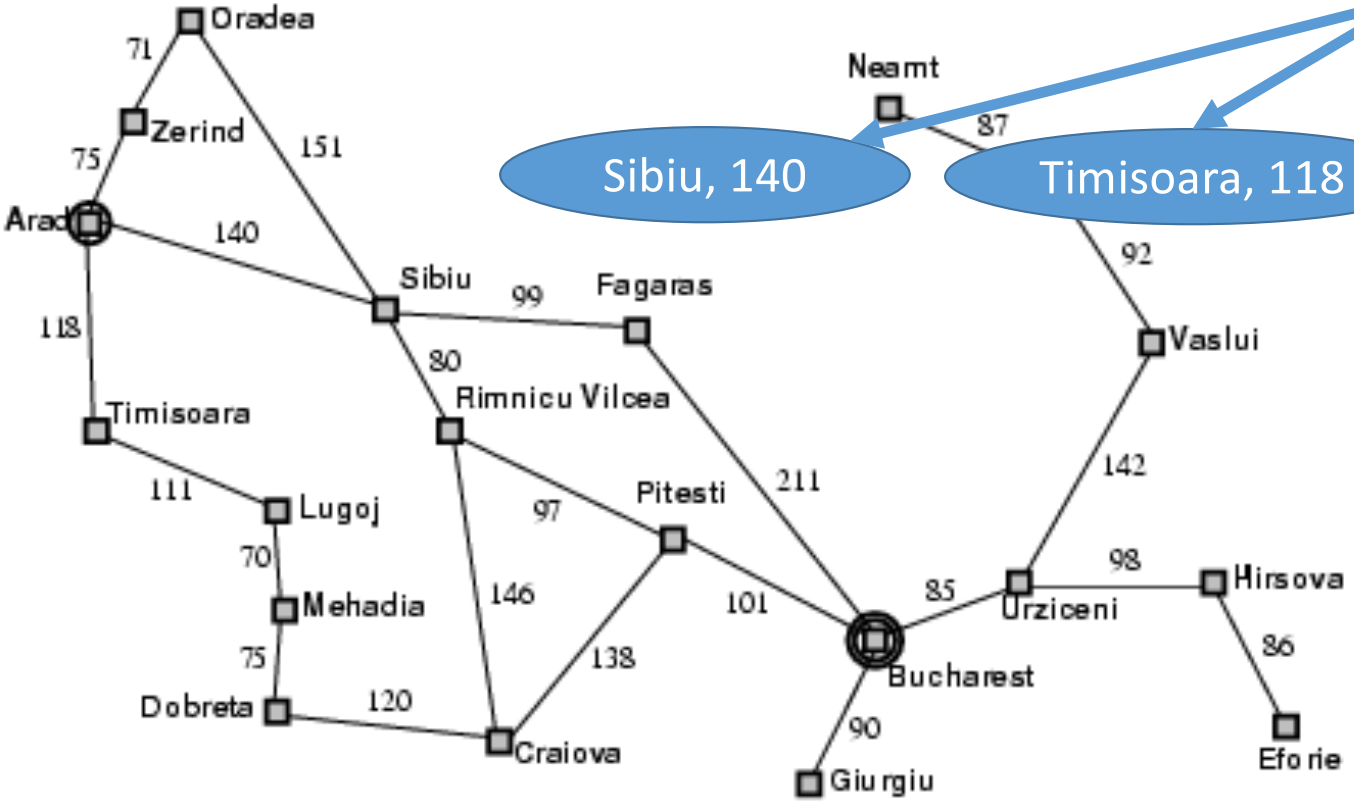


Search step 1

Frontier: { Sibiu, Zerind, Timisoara }

Tree:



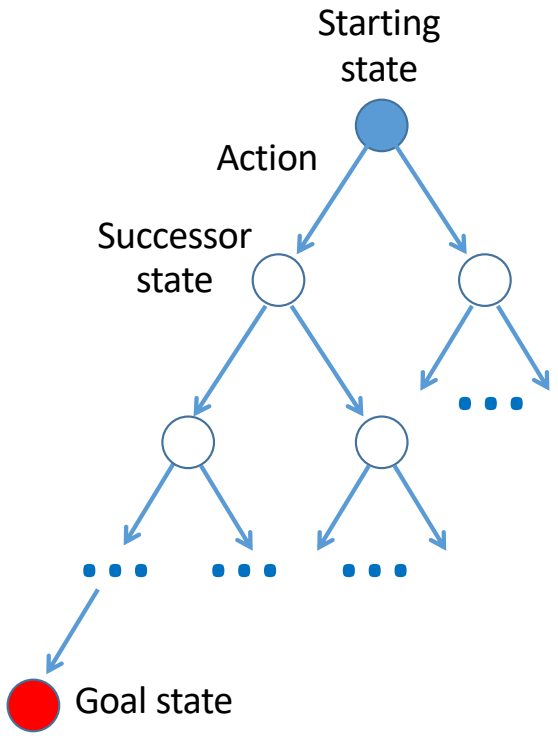


Tree Search: Basic idea

1. SEARCH for an optimal solution
   - Maintain a **frontier** of unexpanded states
   - At each step, pick a state from the frontier to **expand:**
     - Check to see whether or not this state is the goal state. If so, DONE!
     - If not, then list all of the states you can reach from this state, add them to the frontier, and add them to the tree
2. BACK-TRACE: go back up the tree; list, in reverse order, all of the actions you need to perform in order to reach the goal state.
3. ACT: the agent reads off the sequence of necessary actions, in order, and does them.

Nodes vs. States

- State = description of the world
  - Must have enough detail to decide whether or not you're currently in the initial state
  - Must have enough detail to decide whether or not you've reached the goal state
  - Often but not always: "defining the state" and "defining the transition model" are the same thing
- Node = a point in the search tree
  - Knows the ID of its STATE
  - Knows the ID of its PARENT NODE
  - Knows the COST of the path



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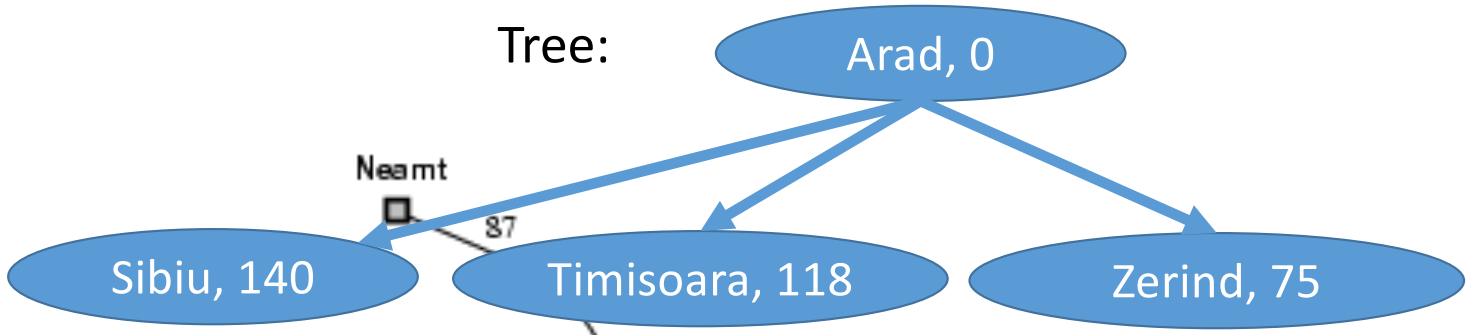
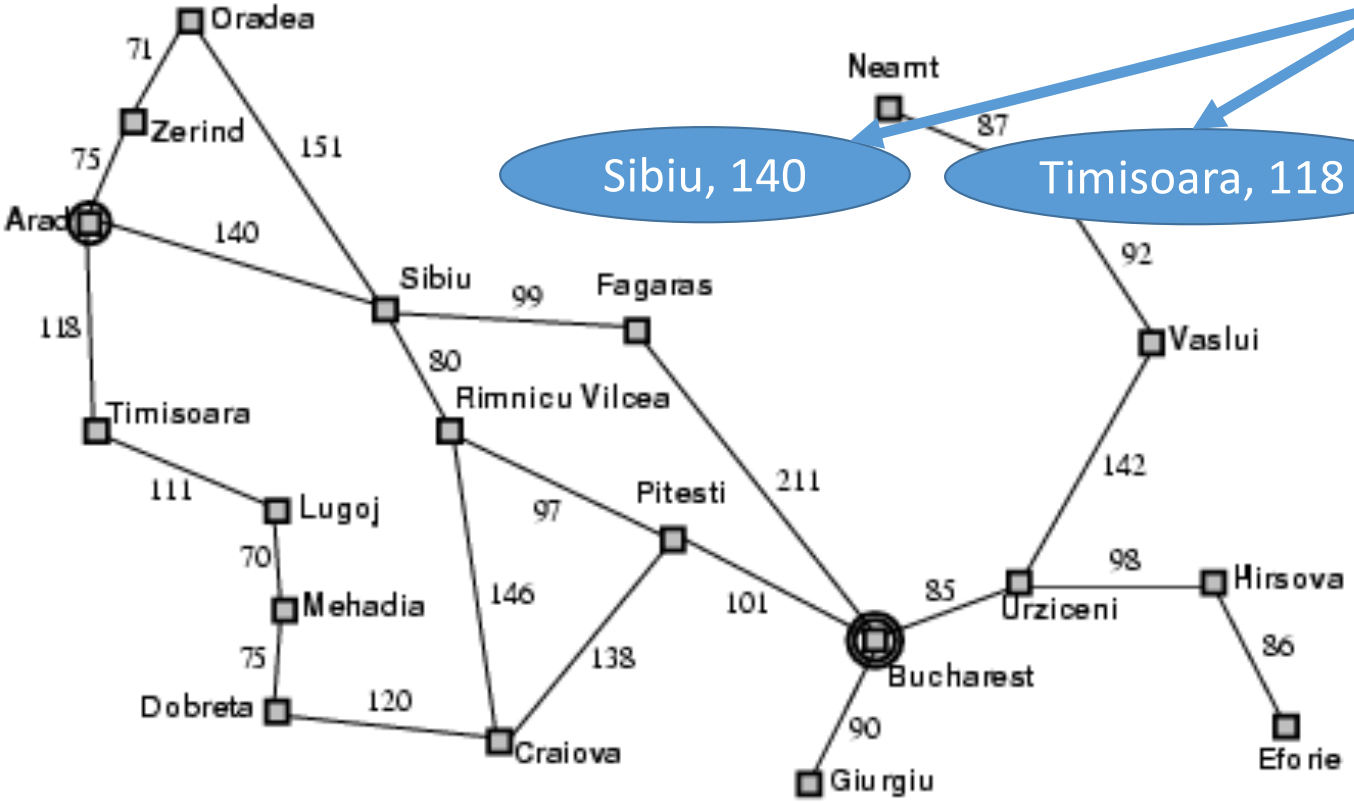
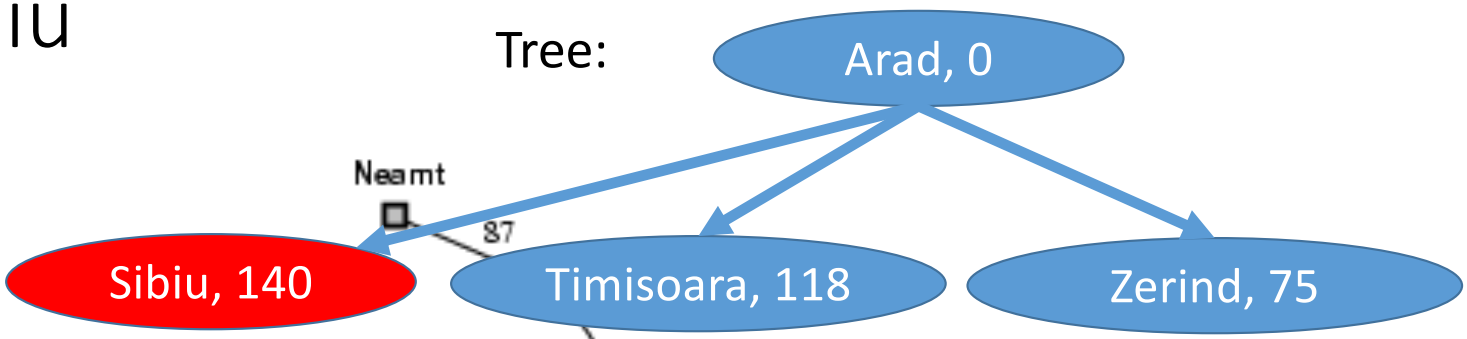


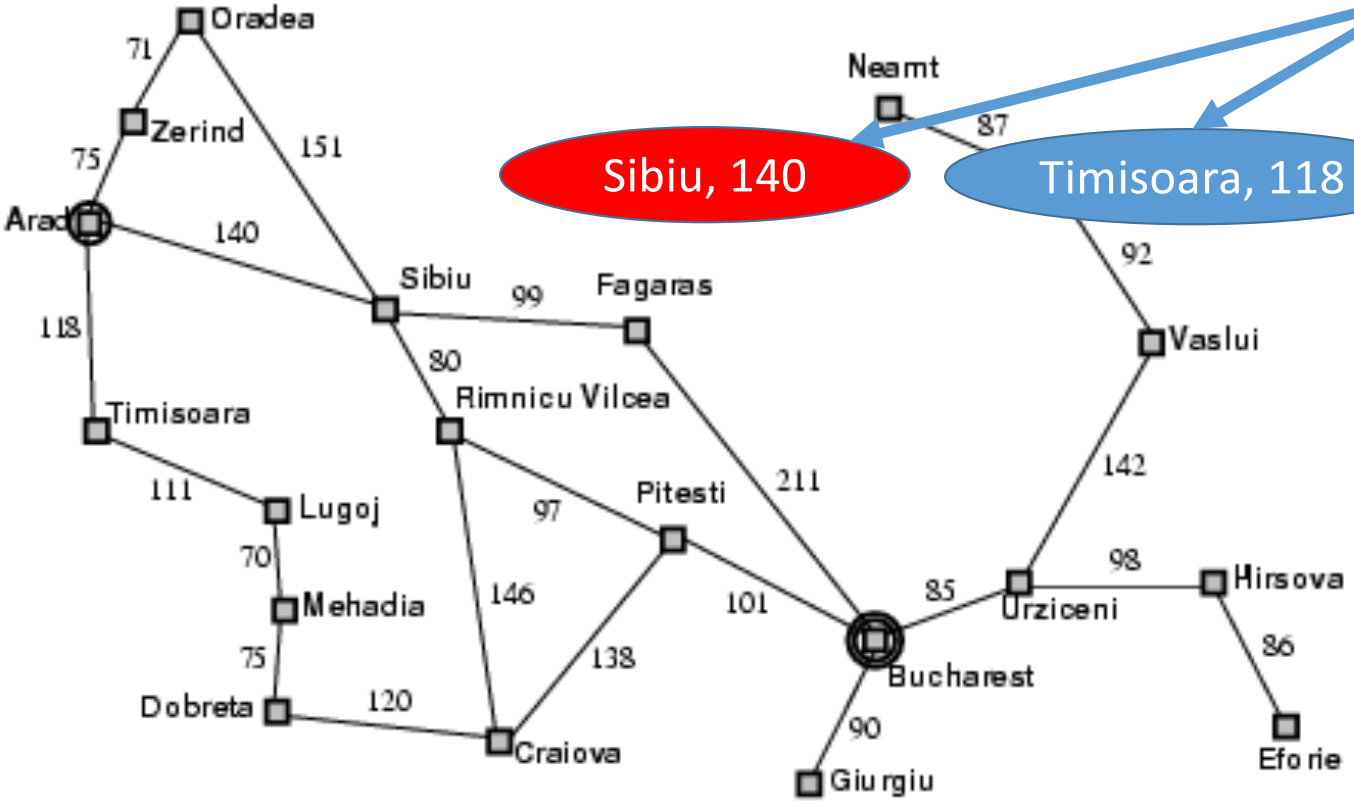


Search step 2
Expand Sibiu

Frontier: { Sibiu, Zerind, Timisoara }

Tree:

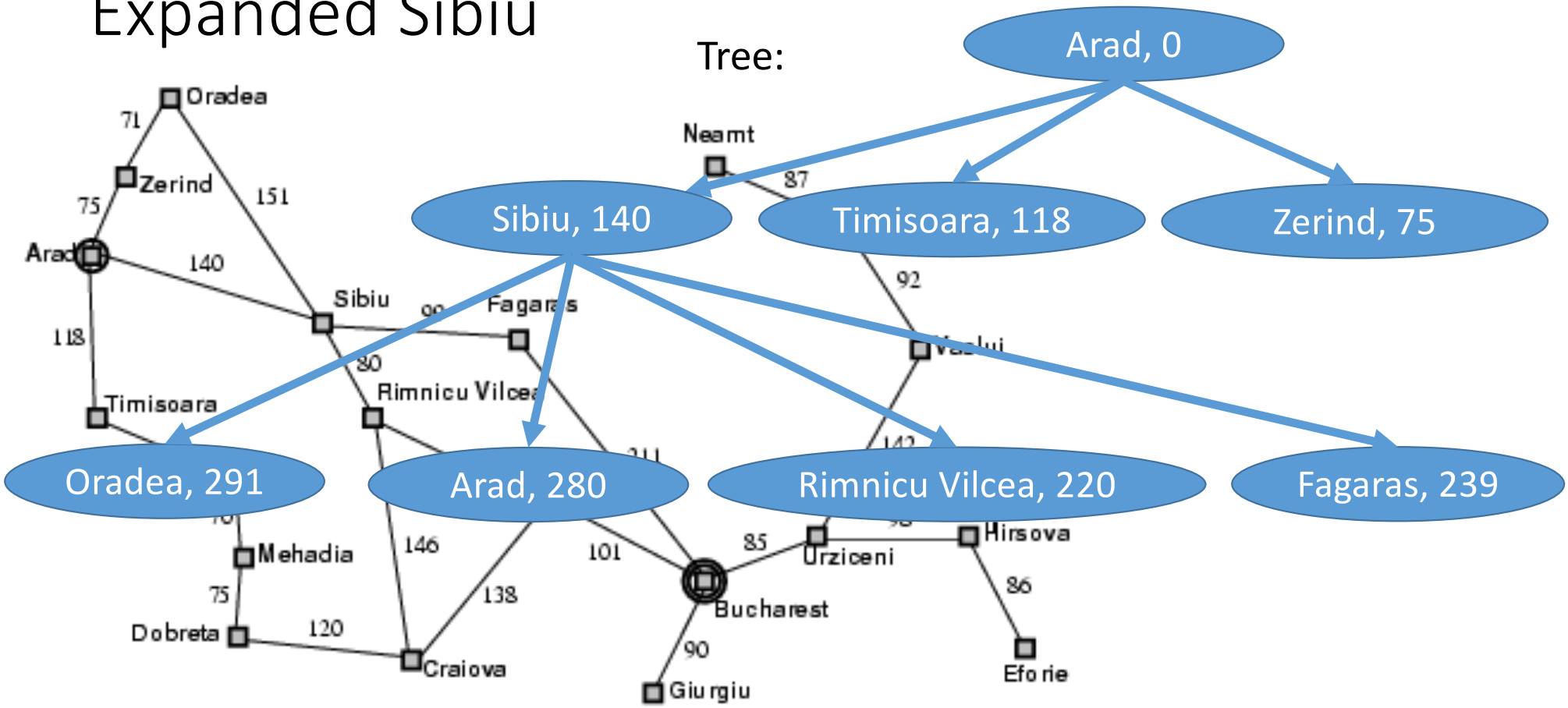




Search step 2
Expanded Sibiu

Frontier: { Zerind, Timisoara, Oradea, Arad, Rimnicu Vilcea, Fagaras }

Tree:



Tree Search: Computational Complexity

Without an EXPLORED set

- b = “branching factor” = max # states you can reach from any given state
- d = “depth” = # layers in the tree (# moves that you have made)
- Without an explored set: complexity = O{b^d}

Solution: keep track of the states you have explored

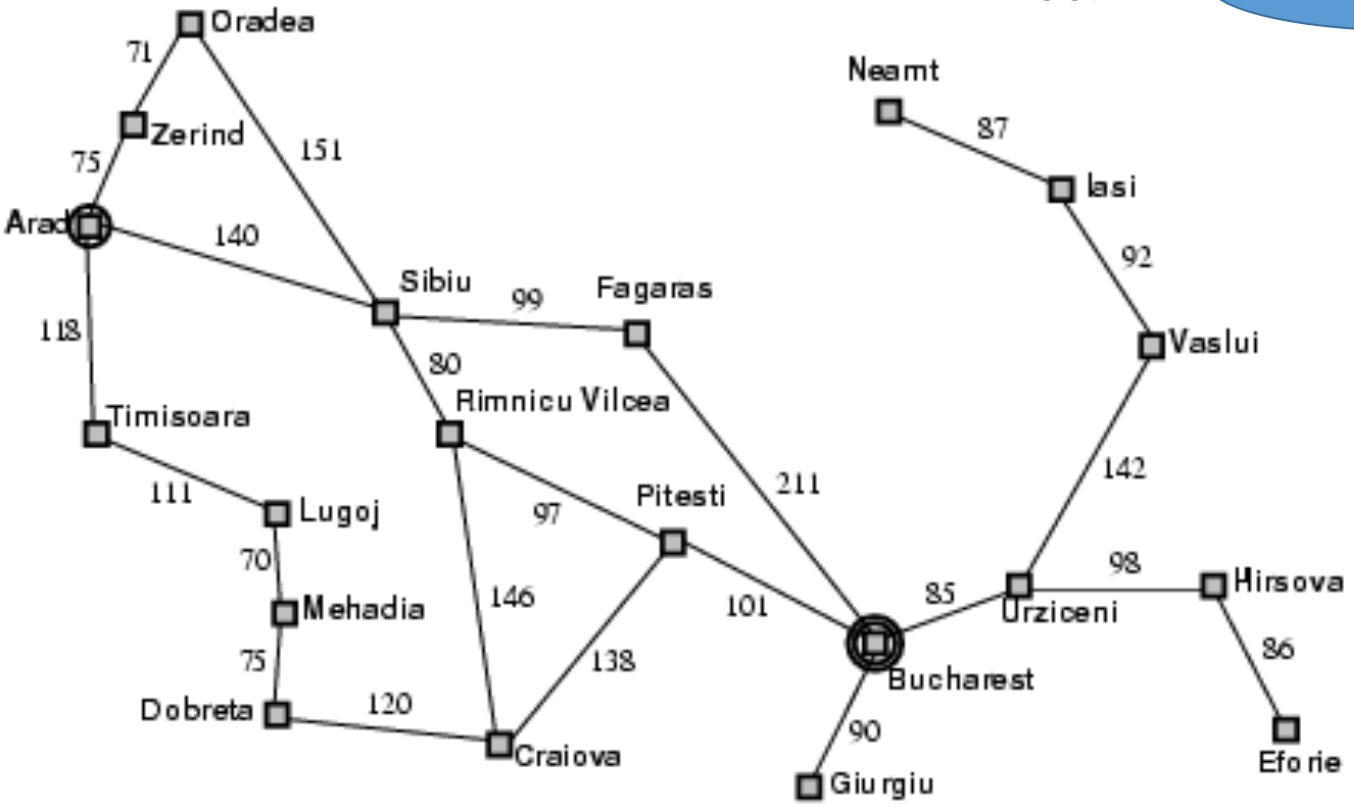
- When you expand a state, you get the list of its possible child states
- ONLY IF a child state is not already explored, put it on the frontier, and put it on the explored set.
- Result: complexity = min(O{b^d}, O{# possible world states})

Search step 0

Frontier: { Arad }

Explored: { Arad }

Tree: Arad, 0



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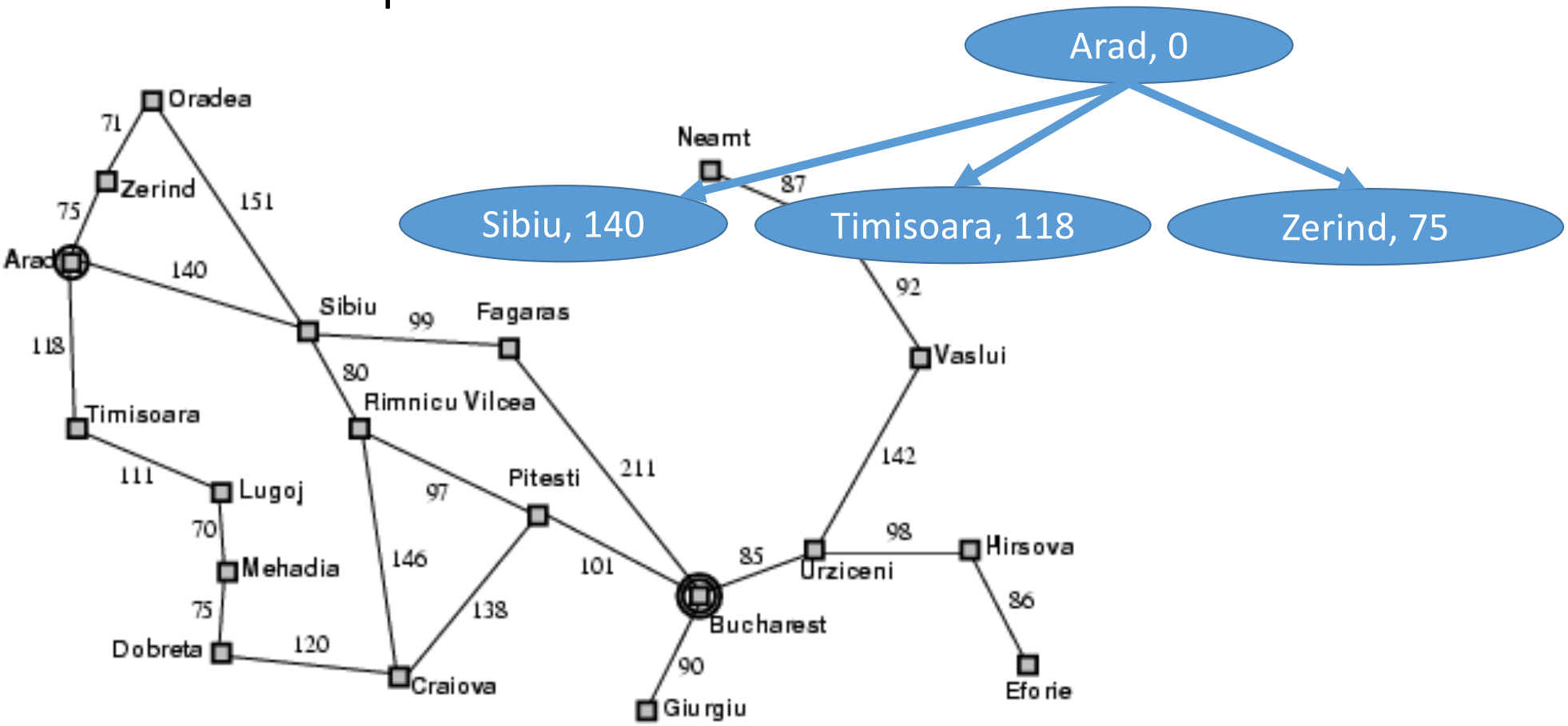
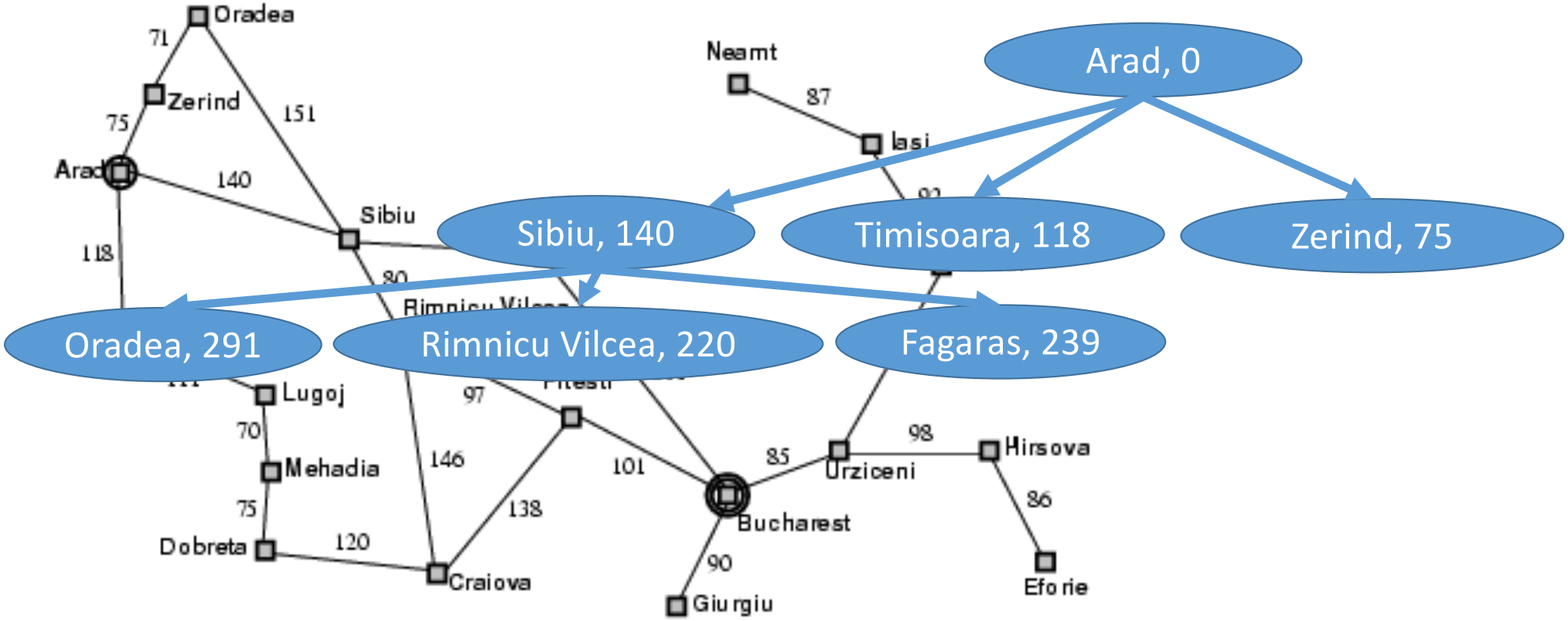


Search step 2

Frontier: { Zerind, Timisoara, Oradea, Rimnicu Vilcea, Fagaras }

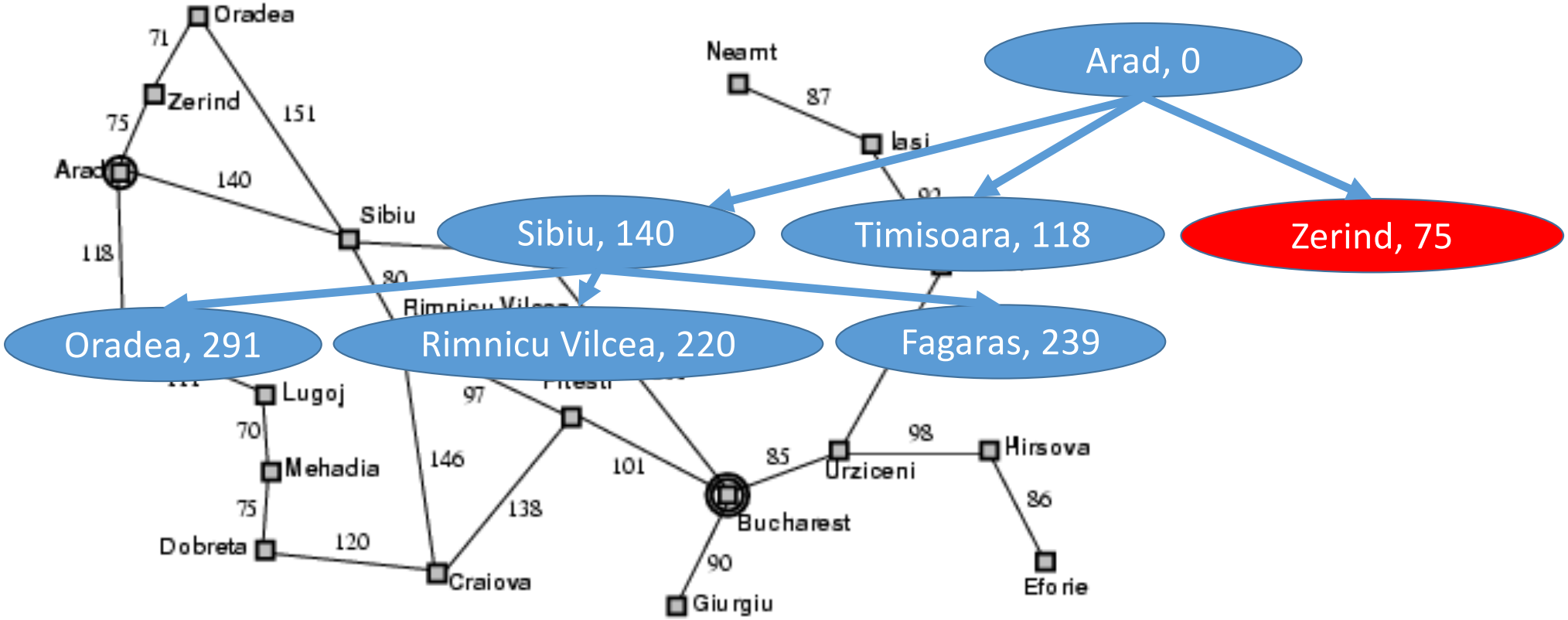
Explored: { Arad, Sibiu, Zerind, Timisoara, Oradea, Rimnicu Vilcea, Fagaras }



Search step 3: expand Zerind

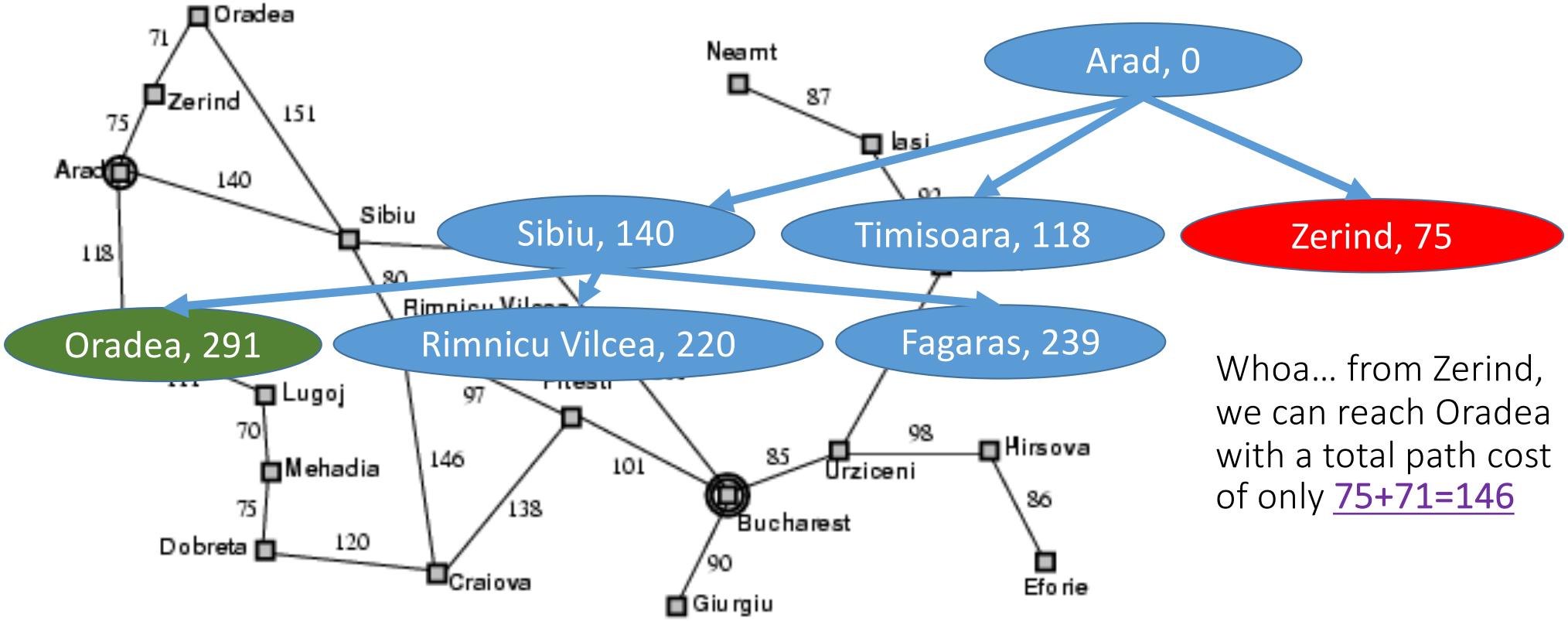
Frontier: { **Zerind**, Timisoara, Oradea, Rimnicu Vilcea, Fagaras }

Explored: { Arad, Sibiu, Zerind, Timisoara, Oradea, Rimnicu Vilcea, Fagaras }



Frontier: { **Zerind**, Timisoara, Oradea, Rimnicu Vilcea, Fagaras }

Explored: { Arad, Sibiu, Zerind, Timisoara, Oradea, Rimnicu Vilcea, Fagaras }



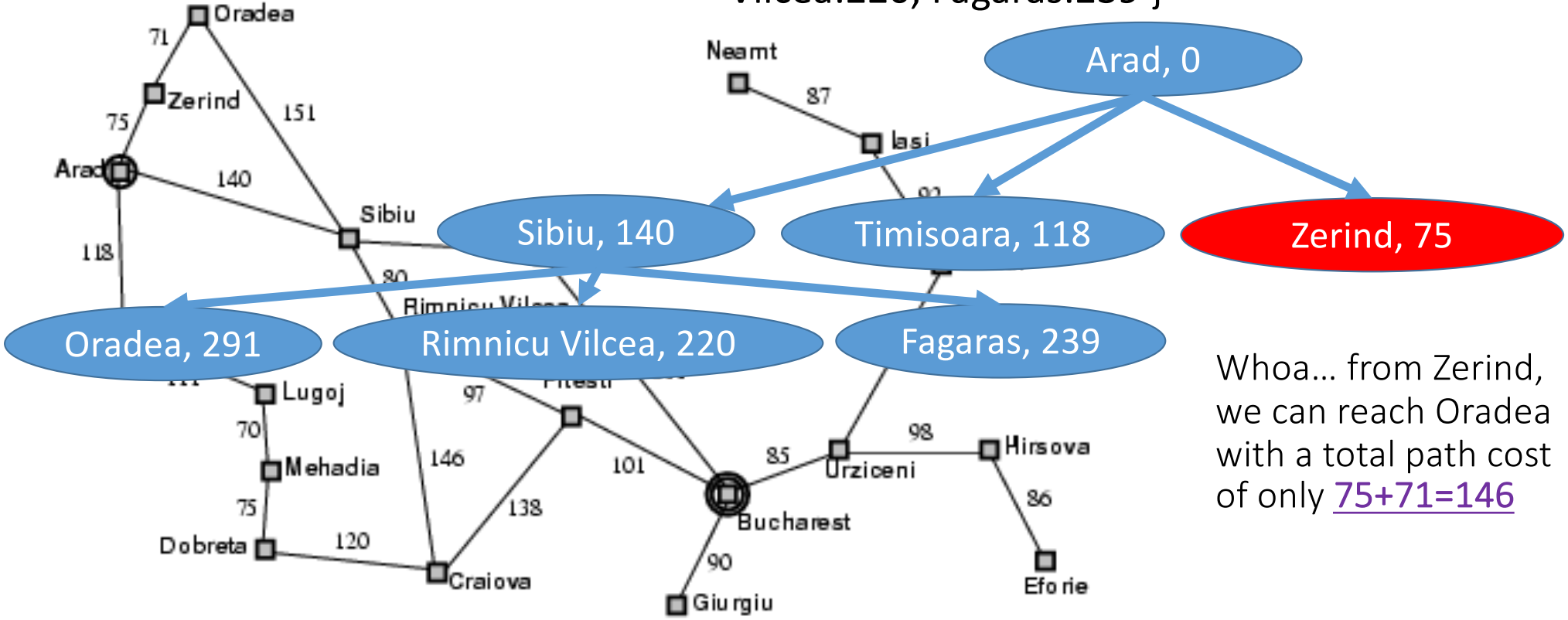
Whoa… from Zerind, we can reach Oradea with a total path cost of only 75+71=146

Third data structure: Explored Dictionary

- Explored = dictionary mapping from state ID to path cost
- If we find a new path to the same state, with HIGHER COST, then we ignore it
- If we find a new path to the same state, with LOWER COST, then we expand the new path

Frontier: { **Zerind:75**, Timisoara:118, Oradea:291, Rimnicu Vilcea:220, Fagaras:239 }

Explored: { Arad:0, Sibiu:140, Zerind:75, Timisoara:118, Oradea:291, Rimnicu Vilcea:220, Fagaras:239 }

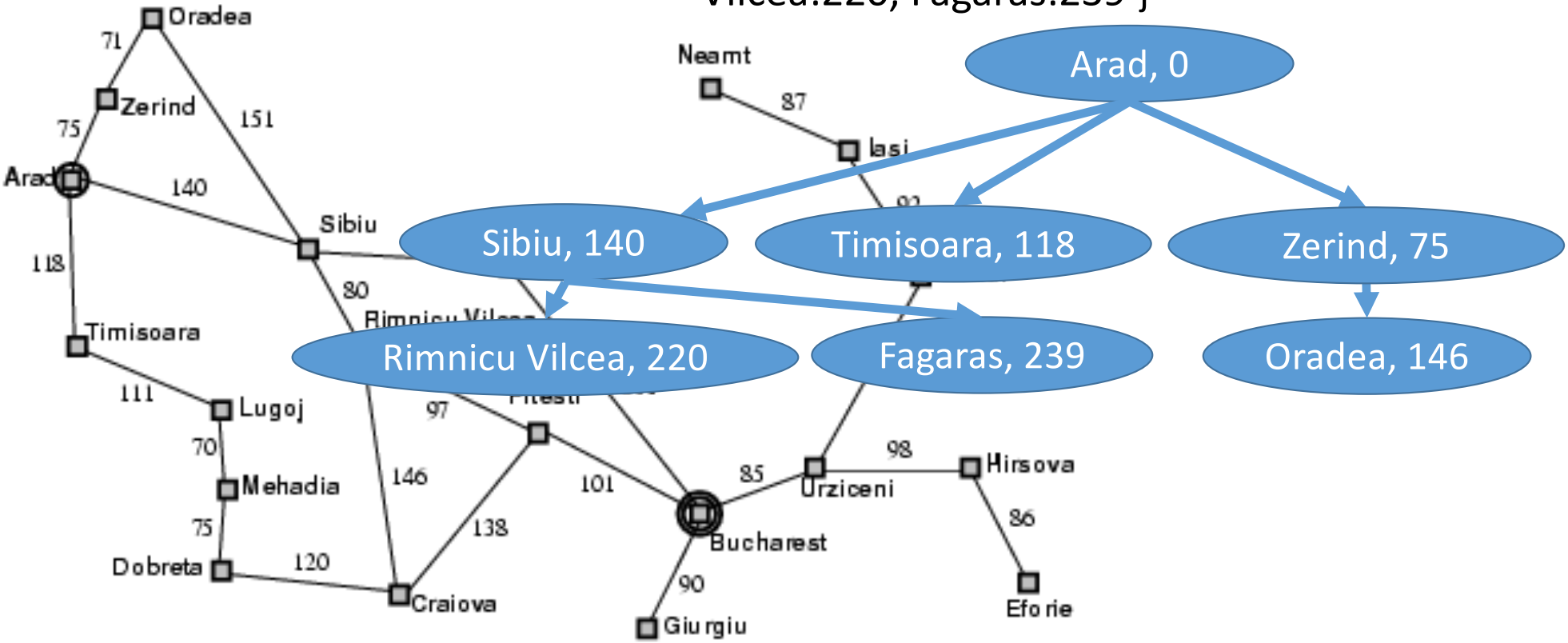


Whoa… from Zerind, we can reach Oradea with a total path cost of only 75+71=146

Search step 3: expanded Zerind

Frontier: { Timisoara:118, Oradea:**146**, Rimnicu Vilcea:220, Fagaras:239 }

Explored: { Arad:0, Sibiu:140, Zerind:75, Timisoara:118, Oradea:**146**, Rimnicu Vilcea:220, Fagaras:239 }



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In which order should you pick nodes from the frontier?

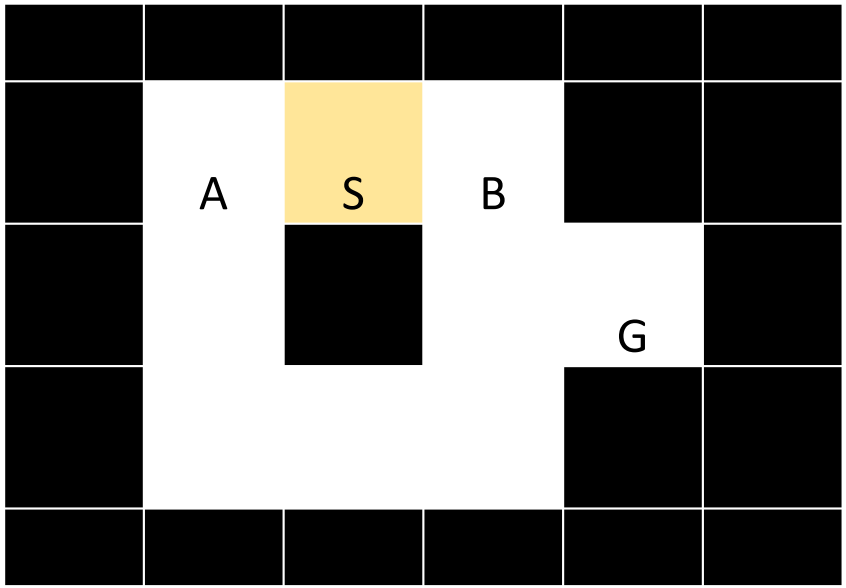
- FIFO (first-in, first-out):
  - the next node you expand will always be the one **least** recently added to the frontier.
  - corresponds to a breadth-first search (BFS), i.e., the same order that Dijkstra's algorithm would use
- LIFO (last-in, first-out):
  - the next node you expand will always be the one **most** recently added to the frontier.
  - corresponds to a depth-first search (DFS)

BFS: Frontier = FIFO

Example:

explored = { S }

frontier = { A, B }

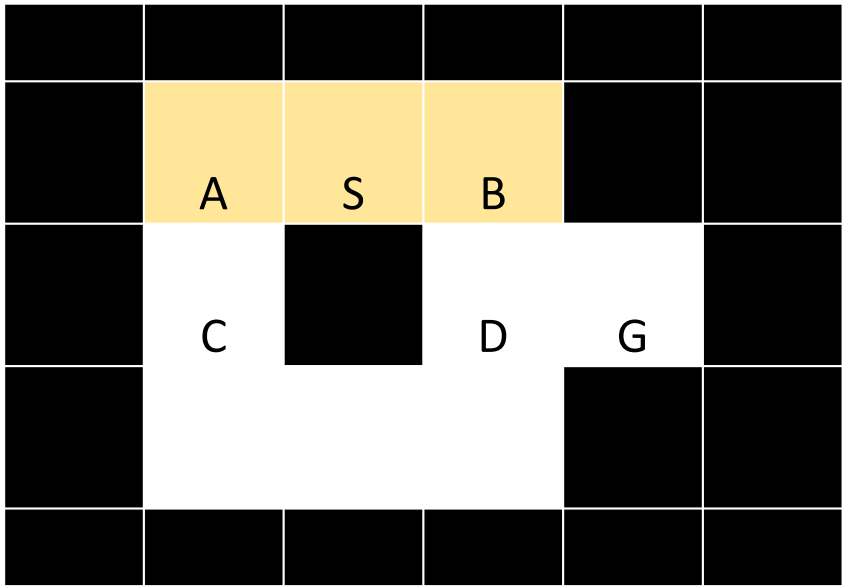


BFS: Frontier = FIFO

Example:

explored = { S, A, B }

frontier = { C, E }

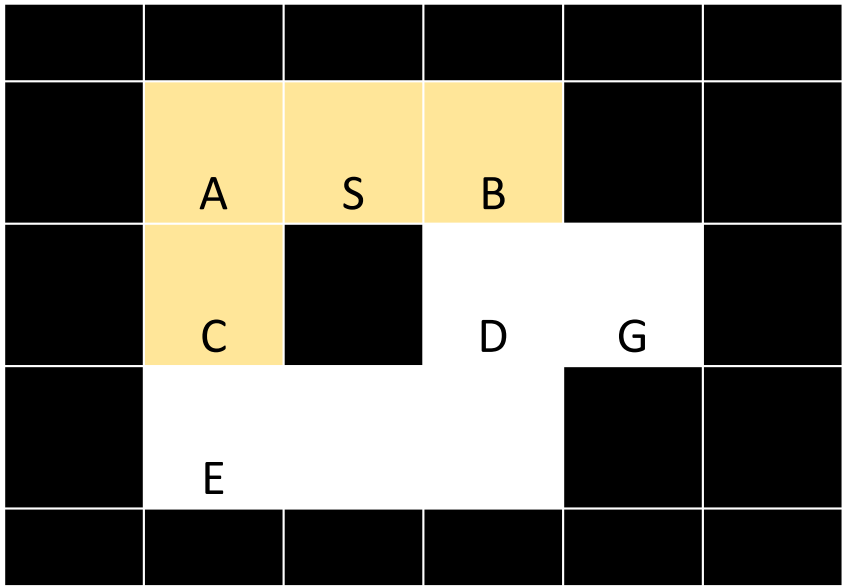


BFS: Frontier = FIFO

Example:

explored = { S, A, B, C }

frontier = { D, E }

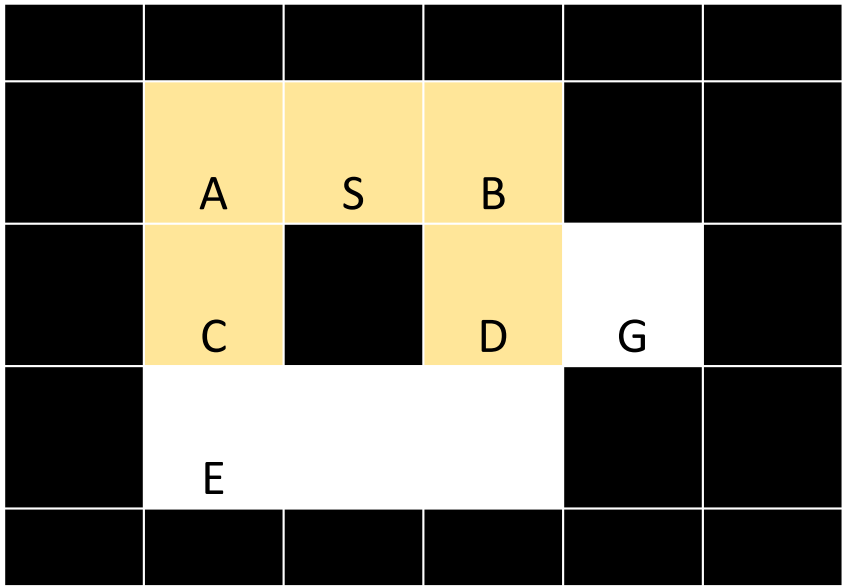


BFS: Frontier = FIFO

Example:

explored = { S, A, B, C, D }

frontier = { E, G }

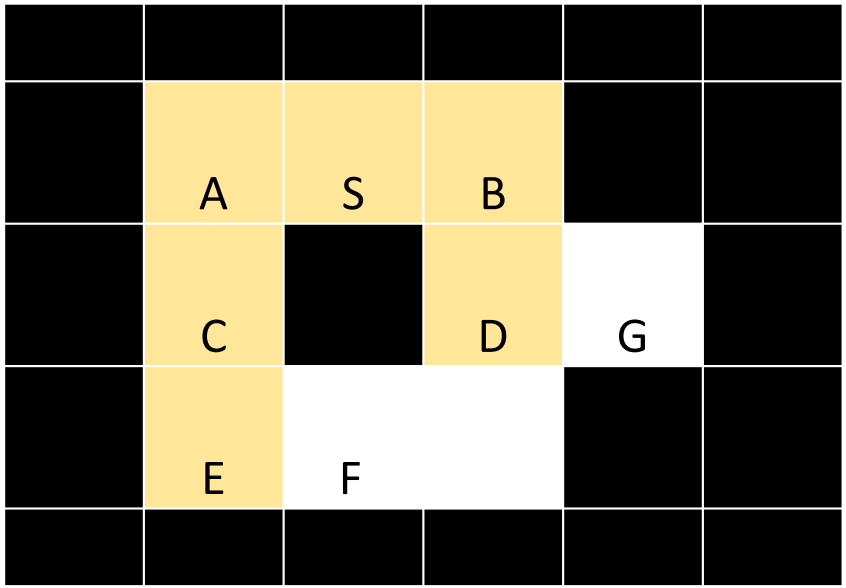


BFS: Frontier = FIFO

Example:

explored = { S, A, B, C, D, E }

frontier = { G, F }



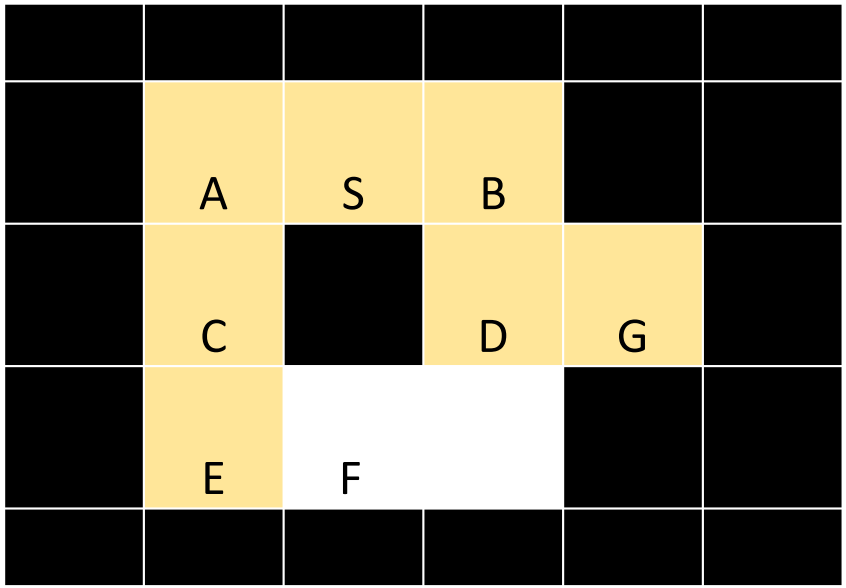
BFS: Frontier = FIFO

Example:

explored = { S, A, B, C, D, E, G }

frontier = { F }

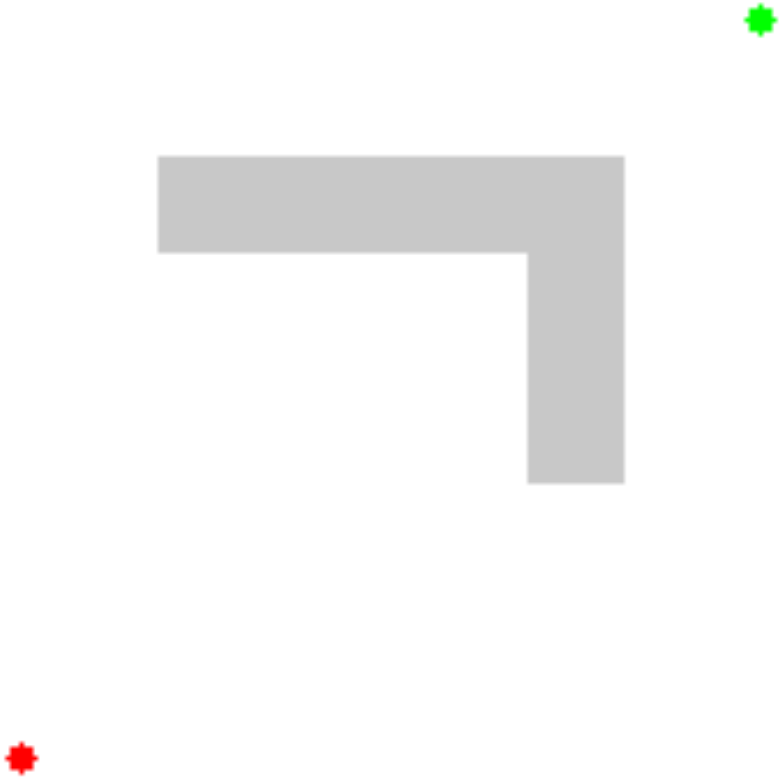
Goal reached!



BFS: How to do it

- Notice that BFS searches in exactly the same order as Dijkstra's algorithm.

- BFS is the normal way you would implement Dijkstra's algorithm for a possibly-infinite search space.



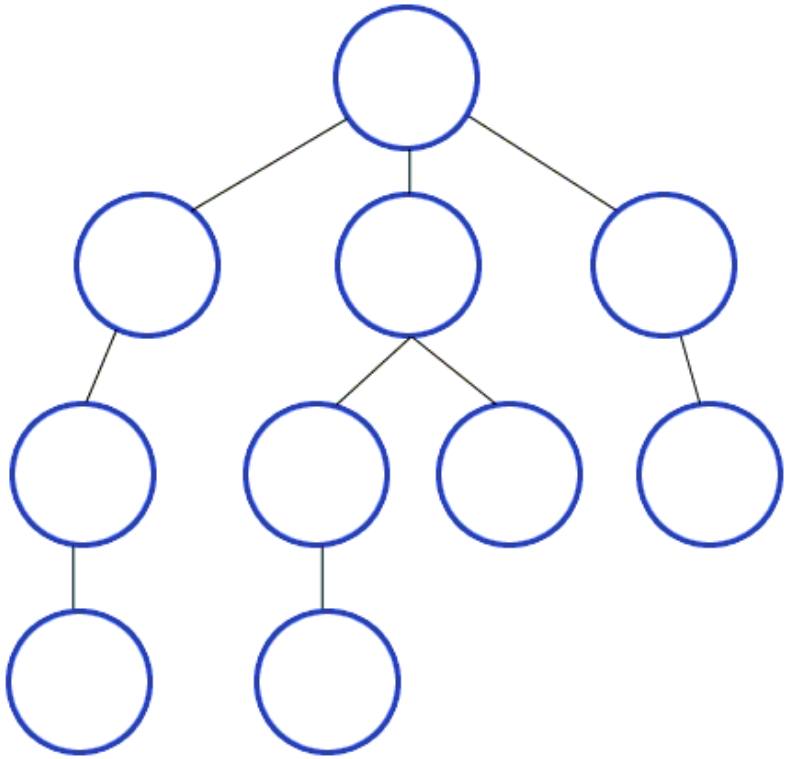
Dijkstra's progress, CC-BY 3.0, Subh83, 2011
https://commons.wikimedia.org/wiki/File:Dijkstras_progress_animation.gif

Depth-first search (DFS)

Exactly like BFS, but instead of expanding the frontier in FIFO order, expand it in LIFO order (last in, first out).

Result: most recently discovered path is pursued, all the way to the end.

Very efficient if there are many solutions, very inefficient if there are few solutions.



Depth-first-search. CC-BY-SA 3.0, Mre, 2009
https://commons.wikimedia.org/wiki/File:Depth-First-Search.gif

Analysis of search strategies

- Strategies are evaluated along the following criteria:
  - **Completeness:** does it always find a solution if one exists?
  - **Optimality:** does it always find a least-cost solution?
  - **Time complexity:** number of nodes generated
  - **Space complexity:** maximum number of nodes in memory
- Time and space complexity are measured in terms of
  - ***b****:* maximum branching factor of the search tree
  - ***d****:* depth of the optimal solution
  - ***m***: maximum length of any path in the state space (may be infinite)

Properties of breadth-first search

- **Complete?**

  Yes (if branching factor b is finite).

  Even w/o explored-set checking, it still works!

- **Optimal?**

  Yes

- **Time?**

  Number of nodes in a b-ary tree of depth d: $O(b^d)$

  (d is the depth of the optimal solution)

- **Space?**

  $O(b^d)$

Properties of depth-first search

- **Complete? (always finds a solution if one exists?)**
  - Fails in infinite-depth spaces
  - Fails if there are loops (unless you keep an "Explored Set")
- **Optimal? (always finds an optimal solution?)**
  - No – returns the first solution it finds
- **Time? (how long does it take, in terms of b, d, m?)**
  - $O(b^m)$ (remember BFS was $O(b^d)$)
  - Terrible if m is much larger than d
- **Space? (how much storage space?)**
  - $O(bm)$, i.e., linear space!
  - The frontier doesn't need to keep track of failed paths, only the currently active path

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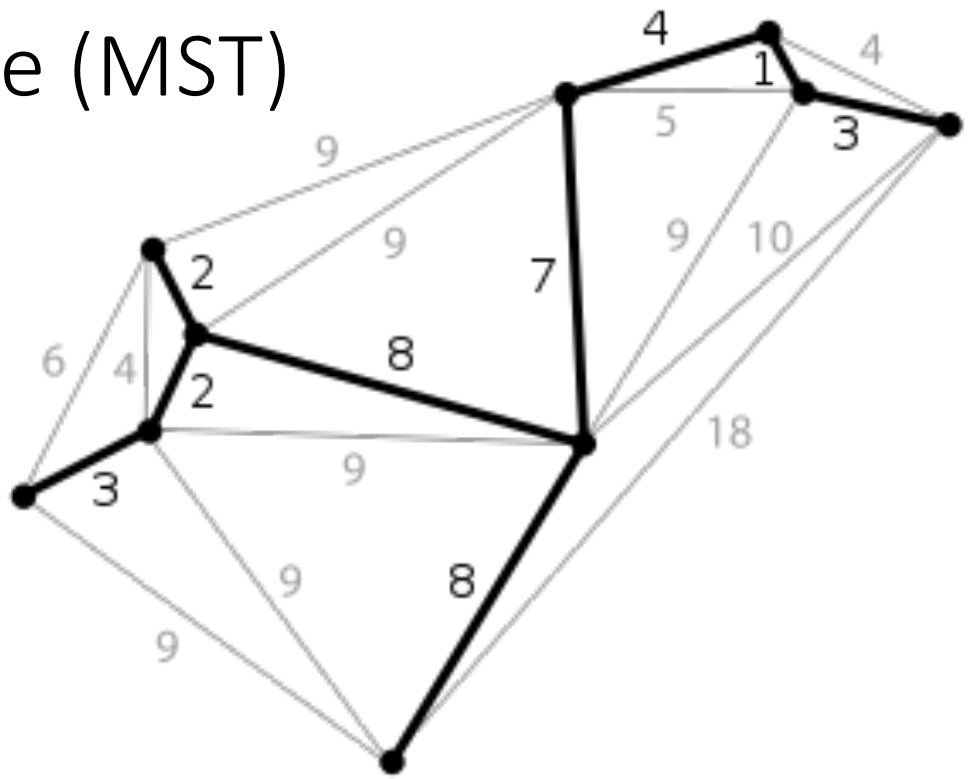
Minimum Spanning Tree (MST)

Digression: something totally different…

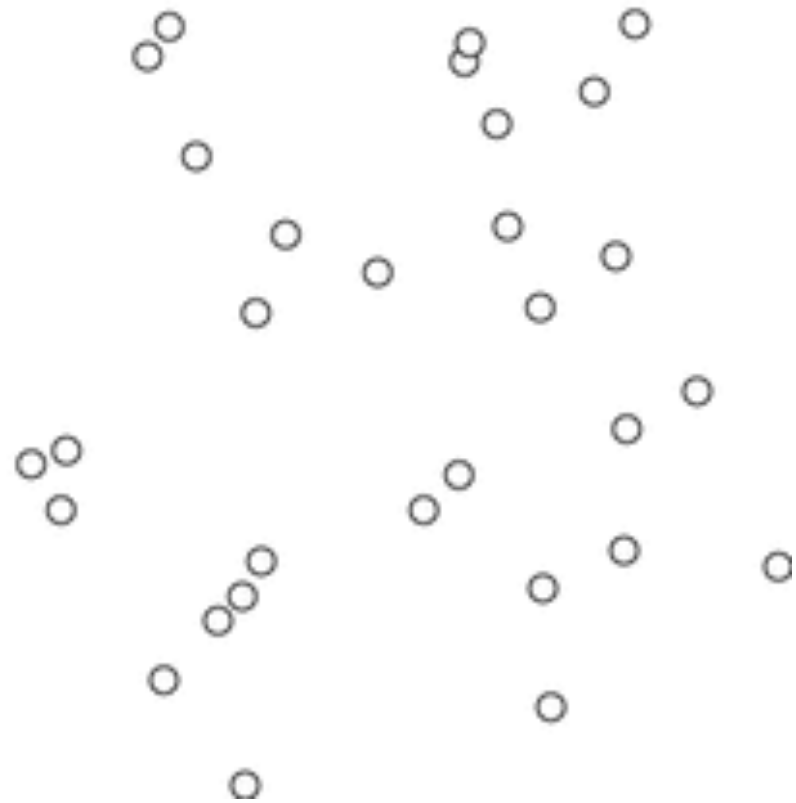
An MST is a **tree** (no cycles) that **spans** a graph (touches every vertex) with the **minimum** possible total distance of the selected edges.



In the example at right, total cost of the MST is the cost of all of its selected eges, i.e., 3+2+2+8+8+7+4+1+3=38

Kruskal's Algorithm

1. Sort the edges in increasing order of cost
2. Start state: every vertex is an independent tree, disconnected from every other
3. While there is more than one tree:
   1. Expand the next-least-cost edge
   2. If it connects two different trees, merge them into the same tree
   3. If not, discard it



Kruskal's algorithm. CC-BY-SA 4.0, Shiyu Ji, 2016
https://commons.wikimedia.org/wiki/File:KruskalDemo.gif

Outline of today's lecture

1. Initial state, goal state, transition model
2. General algorithm for solving search problems
    1. First data structure: a frontier queue
    2. Second data structure: a search tree
    3. Third data structure: a "visited states" dict
3. Breadth-first search (BFS) and Depth-first search (DFS)
4. Minimum spanning tree (MST)