UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

CS 440/ECE 448 Artificial Intelligence
Spring 2020

SAMPLE EXAM 2

Actual Exam will be held on Compass, Monday, March 30, 2020

- This is will be an OPEN BOOK exam. You will be allowed to use textbook, notes, and calculator.
- You will NOT be allowed to use internet search, or to consult with any other human being, while taking the exam.
- You will need to install Proctorio. See instructons on the course web page or on piazza.
- There will be 40 points on the exam: 8 problems, with 1 or (usually) 2 parts each.

Name: ____

netid: ____

Problem 1 (5 points)

Consider three binary events, A, B, and C, with probabilities given by $P(A) = 0.7$, $P(B) = 0.4$, and $P(C) = 0.3$.

(a) What's the smallest possible $P(B \wedge C)$?

Solution: B and C could be mutually exclusive, so

$$\min P(B \wedge C) = 0$$

(b) If A and B are independent, what's $P((\neg A) \wedge B)$?

Solution:

$$P(\neg A \wedge B) = P(\neg A)P(B) = (0.3)(0.4) = 0.12$$

Problem 2 (5 points)

Consider three binary events, A, B, and C, with probabilities given by $P(A) = 0.7$, $P(B) = 0.4$, and $P(C) = 0.3$.

(a) If B and C are mutually exclusive, what's $P\left((\neg B) \wedge (\neg C)\right)$?

Solution: If B and C are mutually exclusive, then $P(B \wedge C) = 0$, so

$$\begin{aligned} P(B \wedge \neg C) &= P(B) \\ P(\neg B \wedge C) &= P(C) \\ P(\neg B \wedge \neg C) &= 1 - P(B) - P(C) = 0.3 \end{aligned}$$

(b) What's the largest possible $P\left(\neg(B \vee C)\right)$?

Solution: $\neg(B \vee C)$ is $(\neg B \wedge \neg C)$:

$$\max P\left(\neg B \wedge \neg C)\right) = \min\left(P(\neg B), P(\neg C)\right) = 0.6$$

Problem 3 (5 points)

You're trying to determine whether a particular newspaper article is of class $Y = 0$ or $Y = 1$. The prior probability of class $Y = 1$ is $P(Y = 1) = 0.4$. The newspaper is written in a language that only has four words, so that the i^{th} word in the article must be $W_i \in \{0, 1, 2, 3\}$, with probabilities given by:

$$\begin{array}{ll} P(W_i = 0|Y = 0) = 0.1 & P(W_i = 0|Y = 1) = 0.3 \\ P(W_i = 1|Y = 0) = 0.3 & P(W_i = 1|Y = 1) = 0.2 \\ P(W_i = 2|Y = 0) = 0.3 & P(W_i = 2|Y = 1) = 0.4 \end{array}$$

The article is only three words long; it contains the words

$$A = (W_1 = 0, W_2 = 0, W_3 = 0)$$

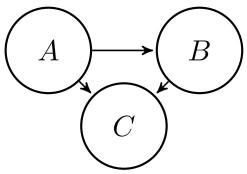
What is $P(Y = 1, A)$?

Solution:

$$P(Y = 1, A) = P(Y = 1)P(W_1 = 0|Y = 1)P(W_2 = 0|Y = 1)P(W_3 = 0|Y = 1) = (0.4)(0.3)(0.3)(0.3)$$

Problem 4 (5 points)

Consider the following Bayes network (all variables are binary):



$P(A) = 0.4$

A	$P(B\|A)$
False	0.1
True	0.2

A, B	$P(C\|A, B)$
False,False	0.9
False,True	0.3
True,False	0.7
True,True	0.5

(a) What is $P(C)$? Write your answer in numerical form, but you don't need to simplify.

Solution:

$$\begin{aligned} P(C) &= P(\neg A, \neg B, C) + P(\neg A, B, C) + P(A, \neg B, C) + P(A, B, C) \\ &= (0.6)(0.9)(0.9) + (0.6)(0.1)(0.3) + (0.4)(0.8)(0.7) + (0.4)(0.2)(0.5) \end{aligned}$$

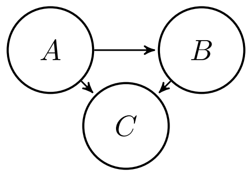
(b) What is $P(A|B = True, C = True)$? Write your answer in numerical form, but you don't need to simplify.

Solution:

$$\begin{aligned} P(A|B, C) &= \frac{P(A, B, C)}{P(A, B, C) + P(\neg A, B, C)} \\ &= \frac{(0.4)(0.2)(0.5)}{(0.4)(0.2)(0.5) + (0.6)(0.1)(0.3)} \end{aligned}$$

Problem 5 (5 points)

Consider the following Bayes network (all variables are binary):



You've been asked to re-estimate the parameters of the network based on the following observations:

Observation	A	B	C
1	True	True	False
2	False	True	True
3	False	True	False
4	False	False	True

(a) Given the data in the table, what are the maximum likelihood estimates of the model parameters? If there is a model parameter that cannot be estimated from these data, mark it "UNKNOWN."

Solution:

$$P(A) = 1/4$$
$$P(B|\neg A) = 2/3$$
$$P(B|A) = 1/1$$
$$P(C|\neg A, \neg B) = 1/1$$
$$P(C|\neg A, B) = 1/2$$
$$P(C|A, \neg B) = UNKNOWN$$
$$P(C|A, B) = 0/1$$

(b) Use the table of data, but this time, estimate the data using Laplace smoothing, with a smoothing parameter of $k = 1$.

Solution:

$$
\begin{aligned}
P(A) &= 2/6 \\
P(B|\neg A) &= 3/5 \\
P(B|A) &= 2/3 \\
P(C|\neg A, \neg B) &= 2/3 \\
P(C|\neg A, B) &= 2/4 \\
P(C|A, \neg B) &= 1/2 \\
P(C|A, B) &= 1/3
\end{aligned}
$$

Problem 6 (5 points)

Consider the following probabilistic context-free grammar:

S→	NP VP	$P = 1.0$
NP→	N	$P = 0.9$
NP→	J N	$P = 0.1$
VP→	V	$P = 0.3$
VP→	V NP	$P = 0.7$
J→	beautiful	$P = 0.4$
J→	complicated	$P = 0.6$
N→	birds	$P = 0.8$
N→	flowers	$P = 0.2$
V→	enjoy	$P = 0.5$
V→	grow	$P = 0.5$

Consider the sentence

"Complicated flowers enjoy."

What is the probability that this sentence would be generated by the grammar shown above? (Ignore capitalization and punctuation.)

Solution: The rules that fire are

S→	NP VP	$P = 1.0$
NP→	J N	$P = 0.1$
J→	complicated	$P = 0.6$
N→	flowers	$P = 0.2$
VP→	V	$P = 0.3$
V→	enjoy	$P = 0.5$

The product of these probabilities is

$$P = (1.0)(0.1)(0.6)(0.2)(0.3)(0.5)$$

Problem 7 (5 points)

A particular hidden Markov model (HMM) has state variable X_t, and observation variables E_t, where t denotes time. Suppose that this HMM has two states, $X_t \in \{0, 1\}$, and three possible observations, $E_t \in \{0, 1, 2\}$. The initial state probability is $P(X_1 = 1) = 0.3$. The transition and observation probability matrices are

X_{t-1}	$P(X_t = 1 \mid X_{t-1})$	X_t	$P(E_t = 0 \mid X_t)$	$P(E_t = 1 \mid X_t)$
0	0.6	0	0.4	0.1
1	0.4	1	0.1	0.6

Suppose that, in a particular test of the HMM, the observation sequence is

$$\{E_1, E_2\} = \{2, 1\}$$

(a) What is the total probability $P(E_1 = 2, E_2 = 1)$?

Solution:

$$\begin{aligned} P(E_1 = 2, E_2 = 1) &= \sum_{X_1, X_2} P(X_1) P(E_1 = 2 \mid X_1) P(X_2 \mid X_1) P(E_2 = 1 \mid X_2) \\ &= (0.7)(0.5)(0.4)(0.1) + (0.7)(0.5)(0.6)(0.6) + (0.3)(0.3)(0.6)(0.1) + (0.3)(0.3)(0.4)(0.6) \end{aligned}$$

(b) If it is observed that $X_2 = 1$, what is the most likely value of X_1? In other words, what is $\arg\max_{X_1} P(X_1, E_1 = 2, X_2 = 1, E_2 = 1)$?

$$\begin{aligned} \arg\max_{X_1} P(X_1, E_1 = 2, X_2 = 1, E_2 = 1) &= \arg\max_{X_1} P(X_1) P(E_1 = 2 \mid X_1) P(X_2 = 1 \mid X_1) \\ &= \arg\max_{0,1} \left((0.7)(0.5)(0.6), (0.3)(0.3)(0.4)\right) \\ &= 0 \end{aligned}$$

Problem 8 (5 points)

Your camera is 6.3m above the ground, i.e., the ground is at $y = -6.3$. There are a pair of railroad tracks in front of you, at positions in the (x, z) plane given by

$$2.3x - 4.5z = 1.2$$
$$2.3x - 4.5z = 2.6$$

The focal length of your camera is $f = 0.05$. Find the coordinates (x', y') of the vanishing point.

Solution: The pinhole camera equations are

$$\frac{x'}{x} = \frac{y'}{y} = \frac{-0.05}{z}$$

Combining this with the equations in the problem statement gives

$$-\frac{2.3x'z}{0.05} - 4.5z = 1.2$$
$$-\frac{2.3x'z}{0.05} - 4.5z = 2.6$$
$$-\frac{y'z}{0.05} = -6.3$$

Dividing through by z, and taking the limit as $z \to \infty$, we get

$$-\frac{2.3x'}{0.05} - 4.5 = 0$$
$$-\frac{y'}{0.05} = 0$$

Or $(x', y') = (-(4.5)(0.05)/(2.3), 0)$