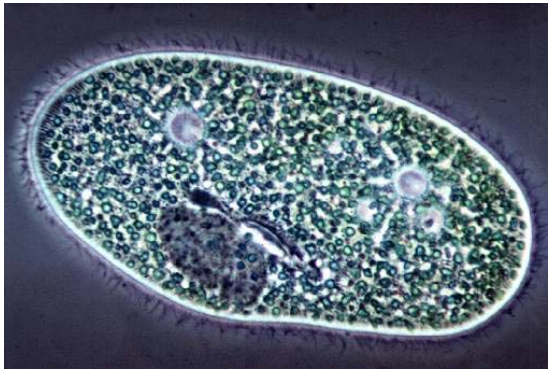
CS440/ECE 448 Lecture 4: Search Intro

Slides by Svetlana Lazebnik, 9/2016

Modified by Mark Hasegawa-Johnson, 9/2017

Types of agents

Reflex agent



- Consider how the world IS
- Choose action based on current percept
- Do not consider the future consequences of actions

Goal-directed agent

- Consider how the world WOULD BE
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Must formulate a goal

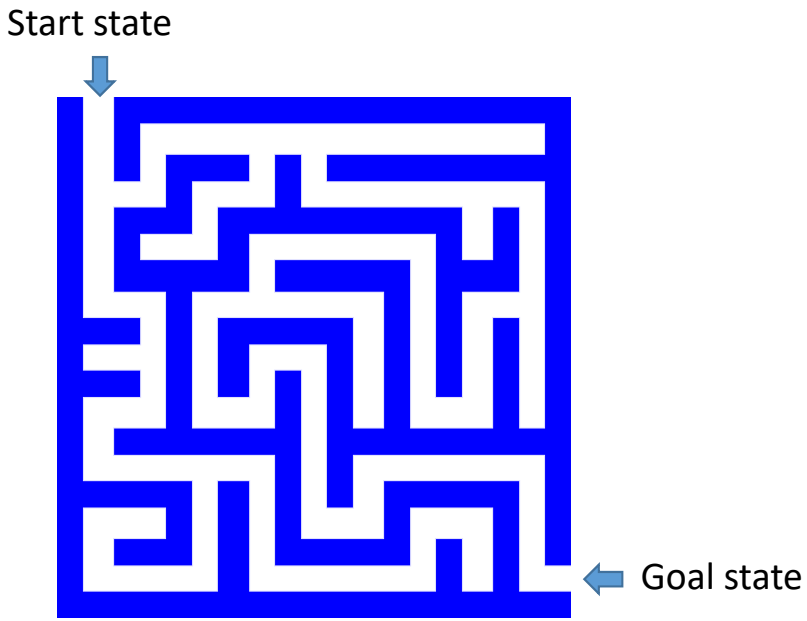
Source: D. Klein, P. Abbeel

Outline of today's lecture

1. How to turn ANY problem into a SEARCH problem:
    1. Initial state, goal state, transition model
    2. Actions, path cost
2. General algorithm for solving search problems
    1. First data structure: a frontier list
    2. Second data structure: a search tree
    3. Third data structure: a "visited states" list

Search

- We will consider the problem of designing **goal-based agents** in **fully observable**, **deterministic**, **discrete**, **static**, **known** environments



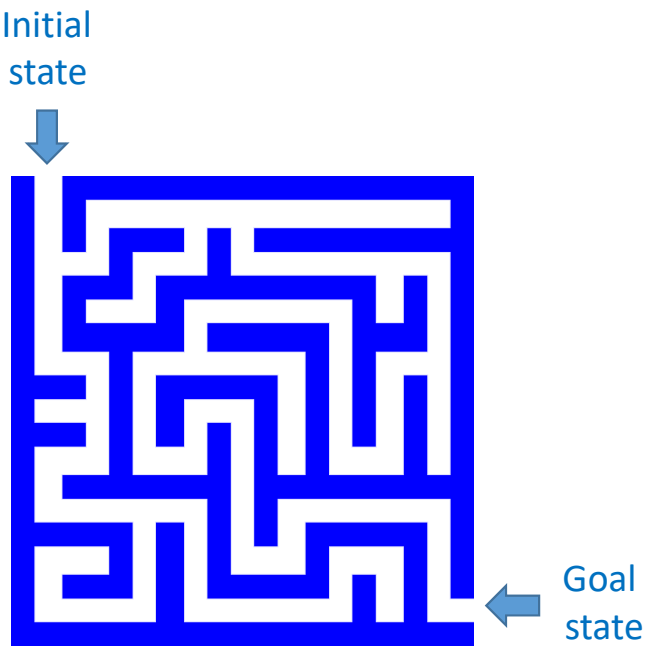
Search

We will consider the problem of designing **goal-based agents** in **fully observable**, **deterministic**, **discrete**, **known** environments

- The agent must find a *sequence of actions* that reaches the goal
- The **performance measure** is defined by (a) reaching the goal and (b) how "expensive" the path to the goal is
  - The **agent doesn't know** the performance measure. This is a goal-directed agent, not a utility-directed agent
  - The **programmer (you) DOES know** the performance measure. So you design a goal-seeking strategy that minimizes cost.
- We are focused on the process of finding the solution; while executing the solution, we assume that the agent can safely ignore its percepts (**static environment, open-loop system**)

Search problem components

- **Initial state**
- **Actions**
- **Transition model**
  - What state results from performing a given action in a given state?
- **Goal state**
- **Path cost**
  - Assume that it is a sum of nonnegative *step costs*



- The **optimal solution** is the sequence of actions that gives the *lowest* path cost for reaching the goal

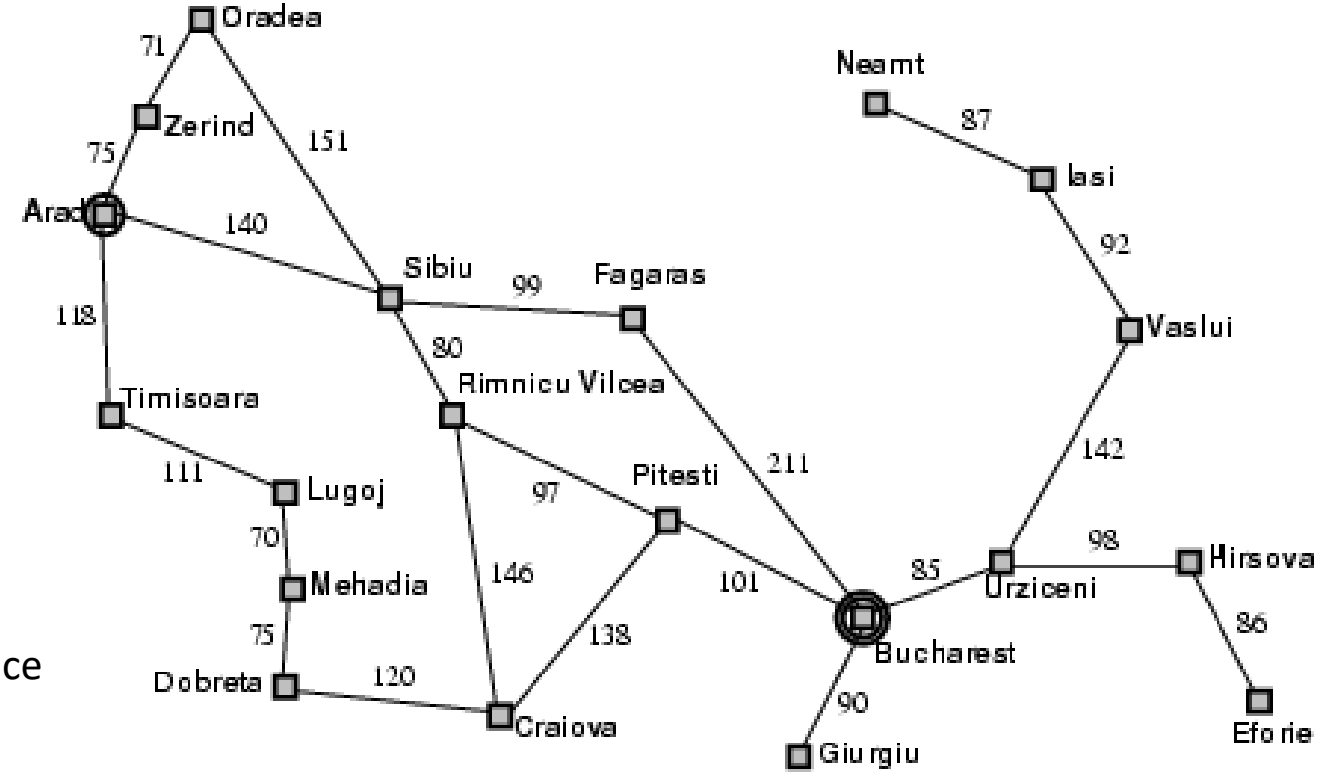
Knowledge Representation: State

- State = description of the world
  - Must have enough detail to decide whether or not you're currently in the **initial state**
  - Must have enough detail to decide whether or not you've reached the **goal state**
  - Often but not always: "defining the state" and "defining the transition model" are the same thing

Example: Romania

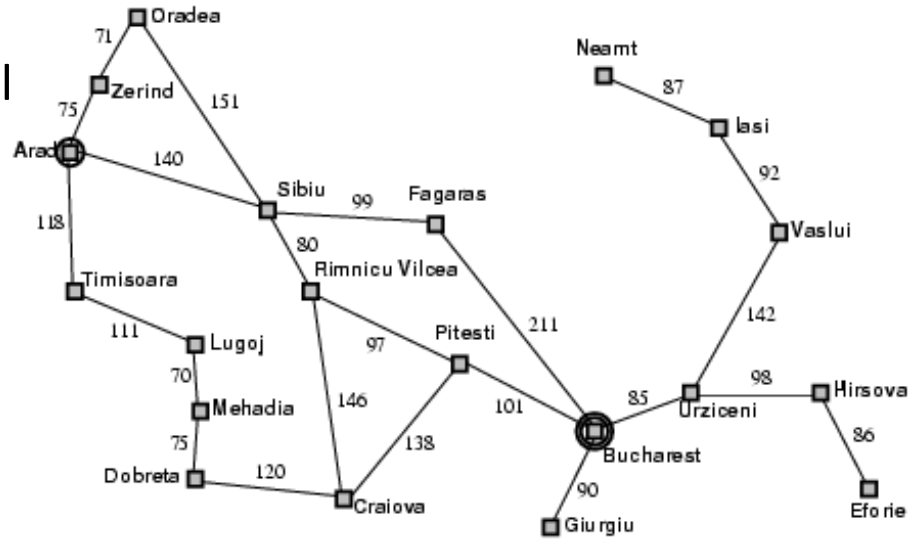
- On vacation in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

- **Initial state**
  - Arad
- **Actions**
  - Go from one city to another
- **Transition model**
  - If you go from city A to city B, you end up in city B
- **Goal state**
  - Bucharest
- **Path cost**
  - Sum of edge costs (total distance traveled)

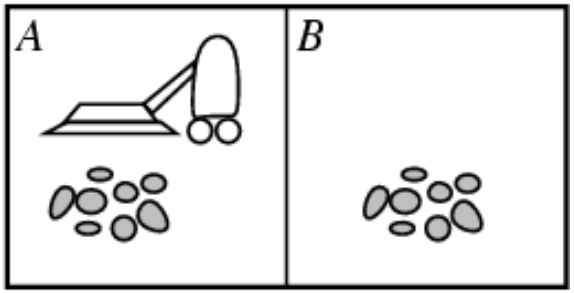


State space

- The initial state, actions, and transition model define the **state space** of the problem
  - The set of all states reachable from initial state by any sequence of actions
  - Can be represented as a **directed graph** where the nodes are states and links between nodes are actions
- What is the state space for the Romania problem?

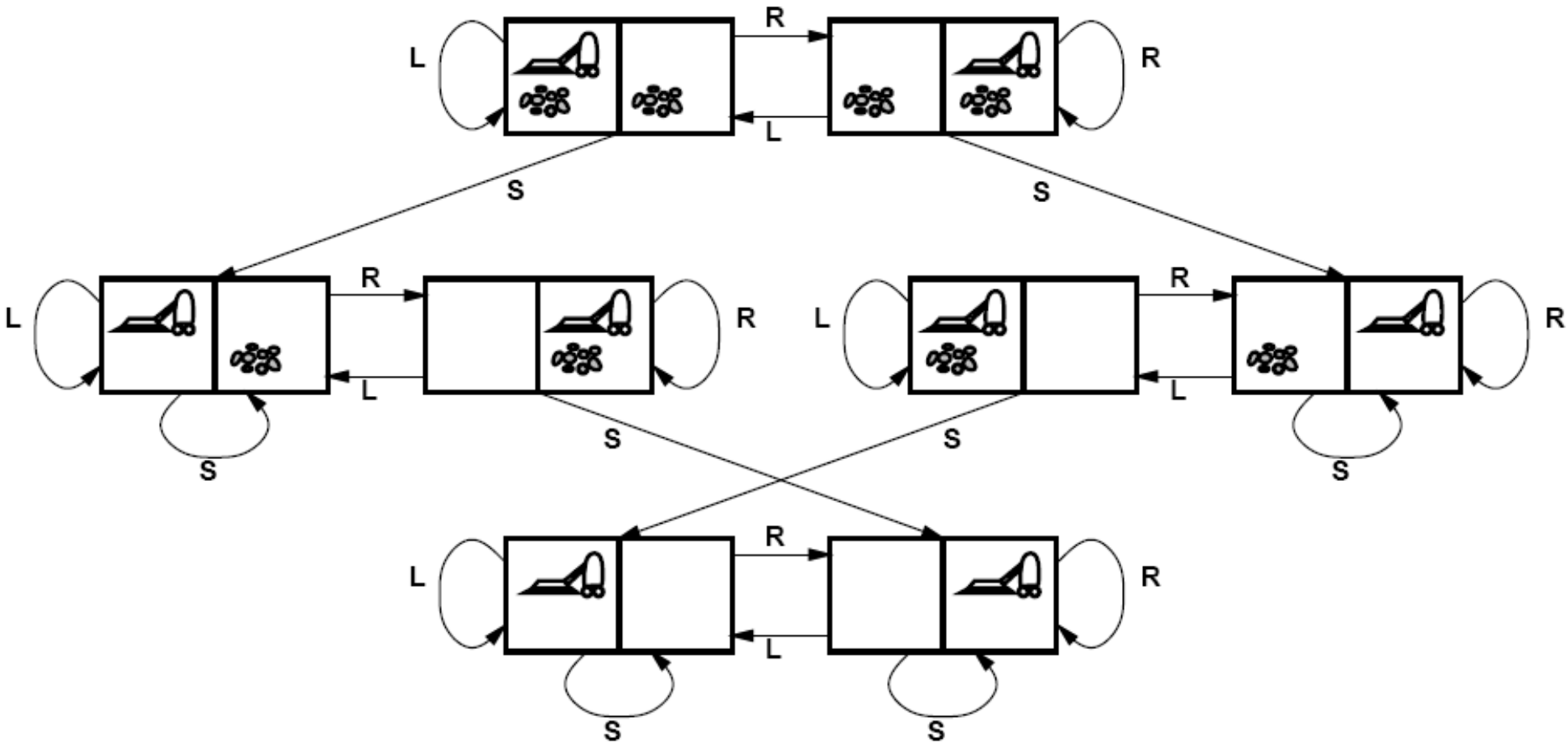


Example: Vacuum world



- **States**
  - Agent location and dirt location
  - How many possible states?
  - What if there are *n* possible locations?
    - The size of the state space grows exponentially with the "size" of the world!
- **Actions**
  - Left, right, suck
- **Transition model**

Vacuum world state space graph

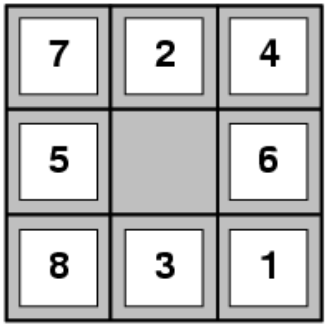


Complexity of the State Space

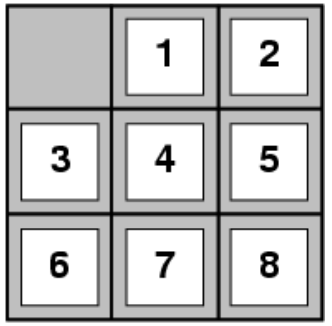
- Many “video game” style problems can be subdivided:
  - There are M different things your character needs to pick up: 2^M different world states
  - There are N locations you can be in while carrying any subset of those M objects: total number of world states = $O\{2^M N\}$
- Why a maze is nice: you don’t need to pick anything up
  - Only N different world states to consider

Example: The 8-puzzle

- **States**
  - Locations of tiles
    - 8-puzzle: 181,440 states (9!/2)
    - 15-puzzle: ~10 trillion states
    - 24-puzzle: ~10^{25} states
- **Actions**
  - Move blank left, right, up, down
- **Path cost**
  - 1 per move



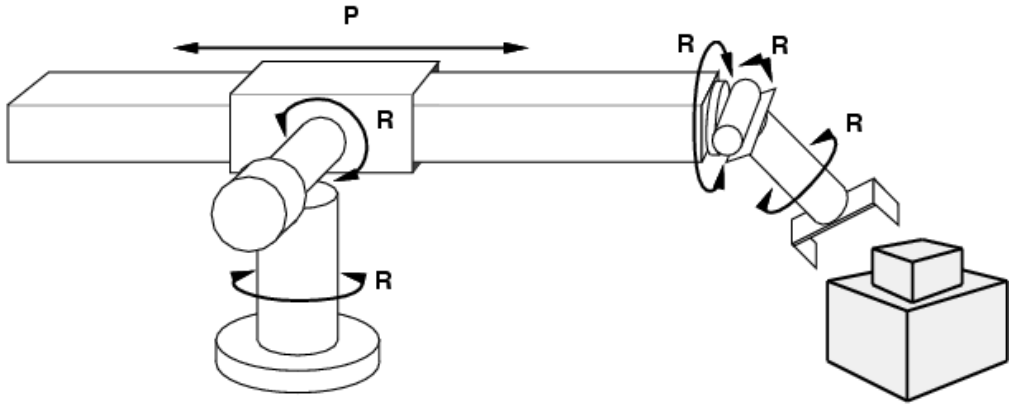
Start State



Goal State

- Finding the optimal solution of n-Puzzle is NP-hard

Example: Robot motion planning



- **States**
  - Real-valued joint parameters (angles, displacements)
- **Actions**
  - Continuous motions of robot joints
- **Goal state**
  - Configuration in which object is grasped
- **Path cost**
  - Time to execute, smoothness of path, etc.

Traveling Salesman Problem

- Goal: visit every city in the United States
- Path cost: total miles traveled
- Initial state: Champaign, IL
- Action: travel from one city to another
- Transition model: when you visit a city, mark it as “visited.”

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Search

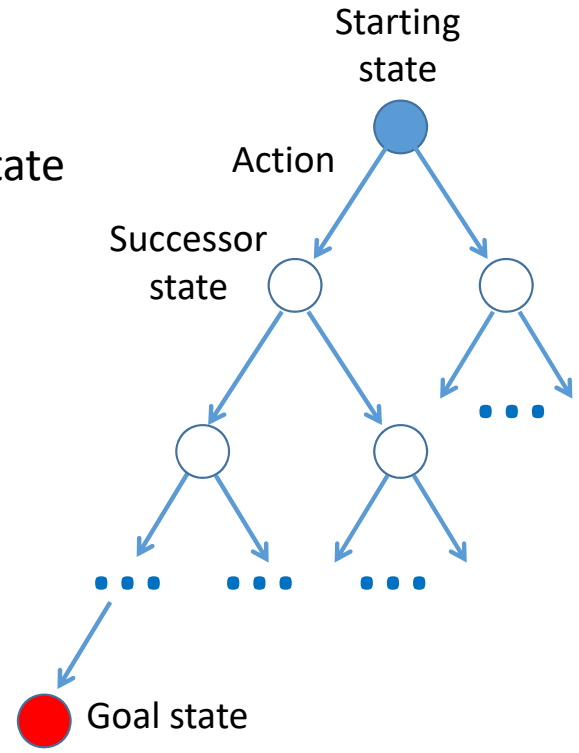
- Given:
  - **Initial state**
  - **Actions**
  - **Transition model**
  - **Goal state**
  - **Path cost**
- How do we find the optimal solution?
  - How about building the state space and then using Dijkstra's shortest path algorithm?
    - Complexity of Dijkstra's is $O(E + V \log V)$, where E is the number of transitions, V is the number of states
    - Usually, V = O{2^{# variables you need to keep track of}}
    - Therefore the shortest-path algorithm is exponential in the # variables

Tree Search: Basic idea

- Let's begin at the start state and **expand** it by making a list of all possible successor states
- Maintain a **frontier** or a list of unexpanded states
- At each step, pick a state from the frontier to expand (EXPAND = list all of the other states that can be reached from this state)
- Keep going until you reach a goal state
- BACK-TRACE: go back up the tree; list, in reverse order, all of the actions you need to perform in order to reach the goal state.
- ACT: the agent reads off the sequence of necessary actions, in order, and does them.

Search tree

- "What if" tree of sequences of actions and outcomes
- The root node corresponds to the starting state
- The children of a node correspond to the **successor states** of that node's state
- A path through the tree corresponds to a sequence of actions
  - A solution is a path ending in the goal state
- Nodes vs. states
  - A state is a representation of the world, while a node is a data structure that is part of the search tree
    - Node has to keep pointer to parent, path cost, possibly other info



Knowledge Representation: States and Nodes

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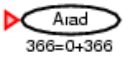
  - Often but not always: "defining the state" and "defining the transition model" are the same thing
- Node = a point in the search tree
  - Private data: ID of the state reached by this node
  - Private data: the ID of the parent node, and of all child nodes

Search: Computational complexity

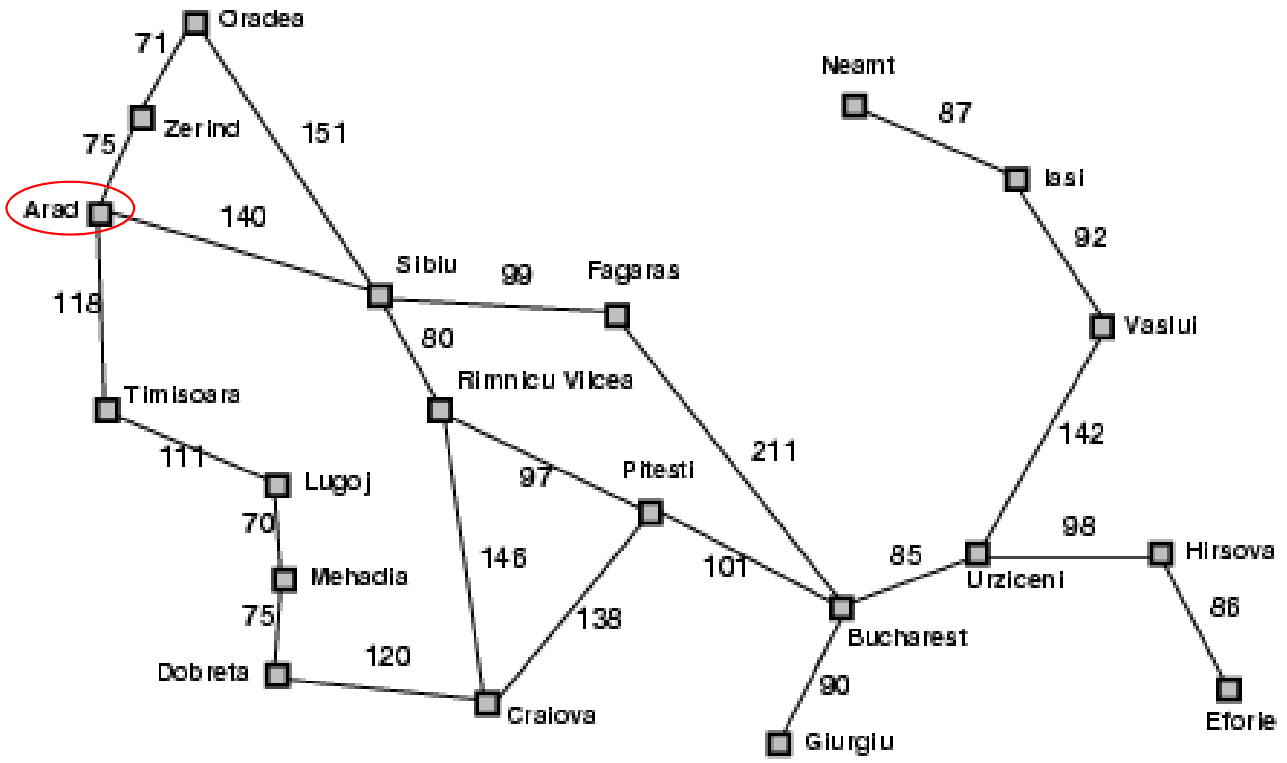
- In the typical case, your search algorithm will need to expand about half of all of the world-states
- The number of world-states is exponential in the number of sub-tasks you need to perform, $\mathrm{V} = O\{2^M\}$
- Therefore, in the typical case, search is exponential complexity
- Most of what we discuss, in the next three lectures, will be methods for limiting the mantissa and/or limiting the exponent.

Tree Search Algorithm Outline

- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - Choose a frontier node according to **search strategy** and take it off the frontier
  - If the node contains the **goal state**, return solution
  - Else **expand** the node and add its children to the frontier



Tree search example



Start: Arad
Goal: Bucharest

Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
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Hirsova	151
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Tree search example

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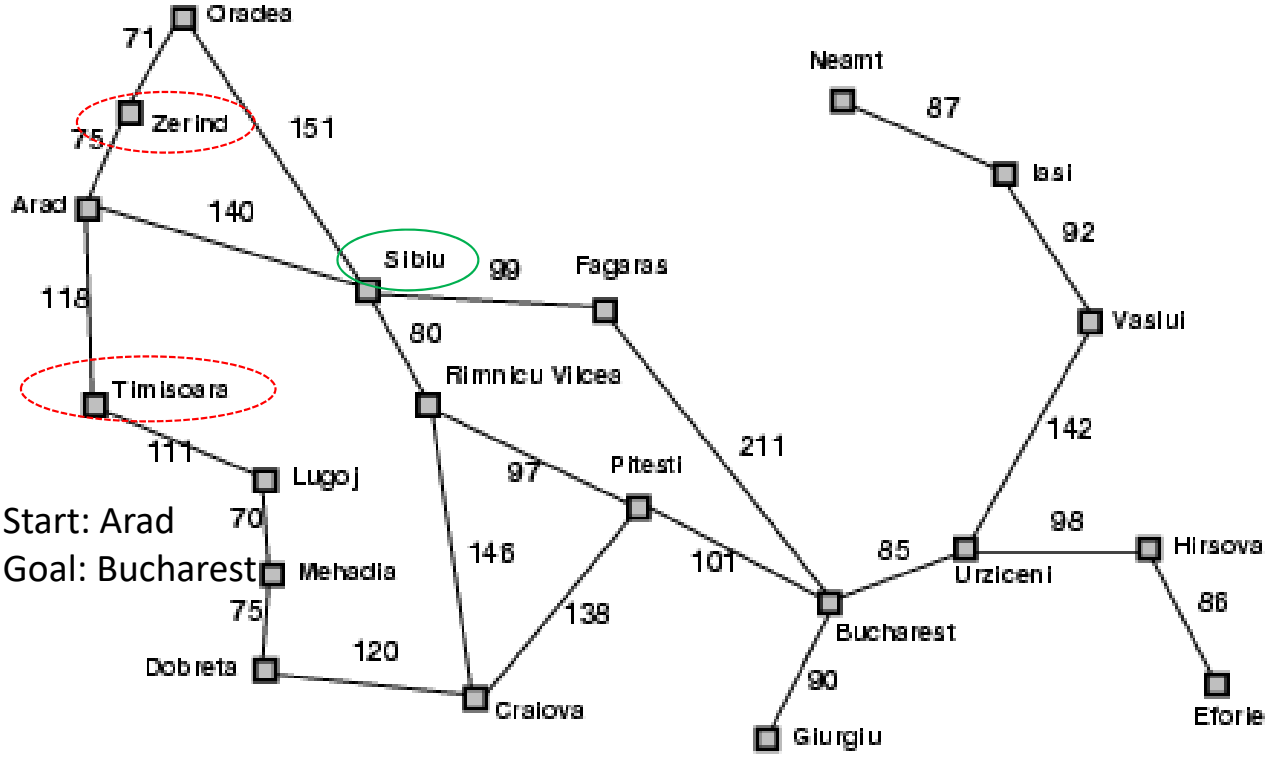

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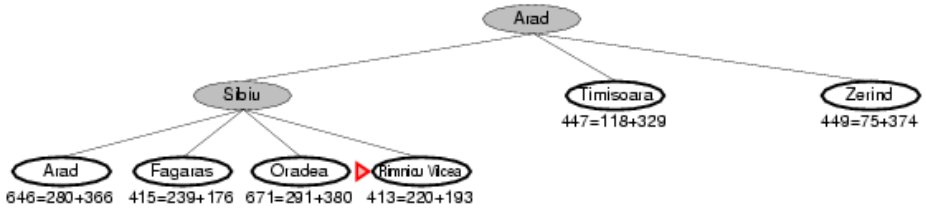

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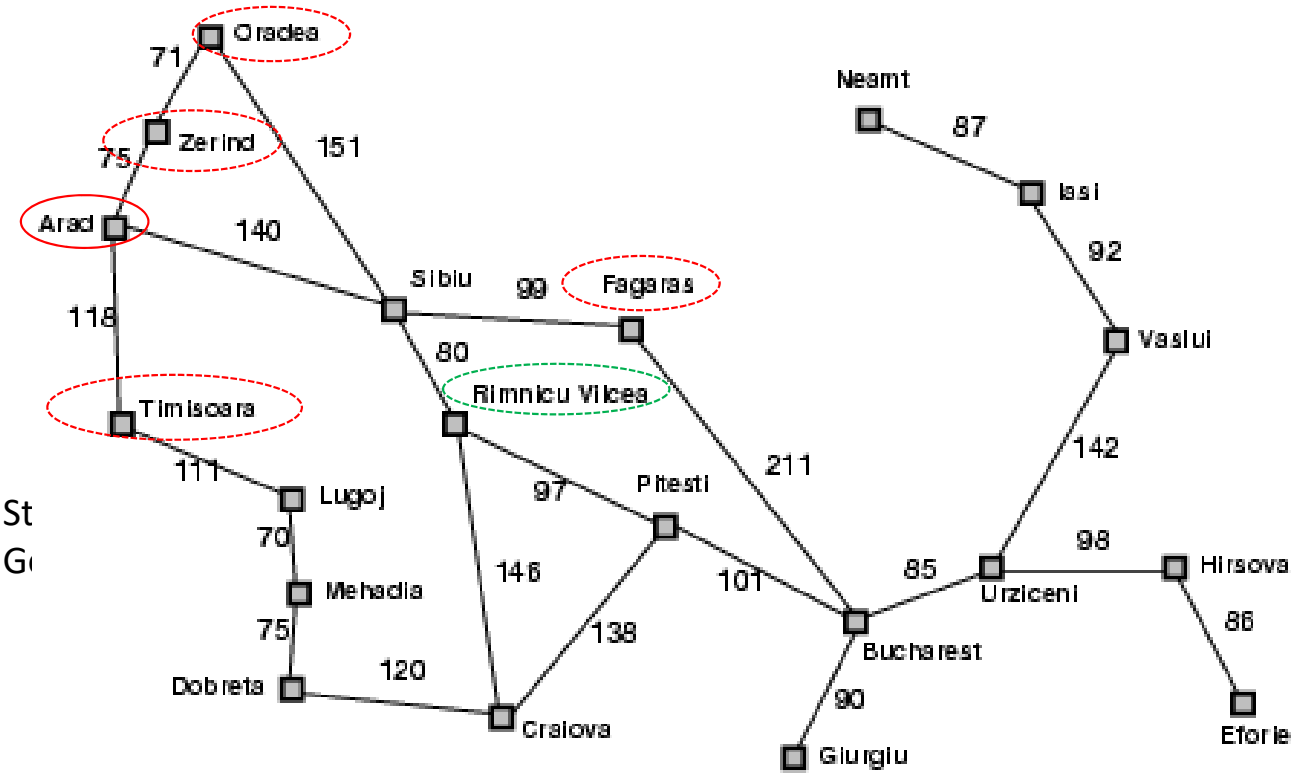

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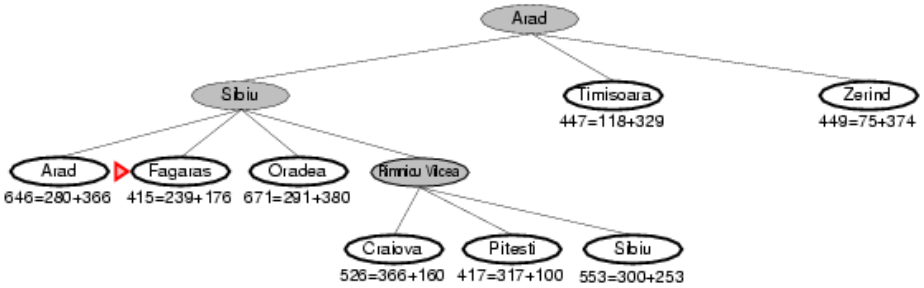

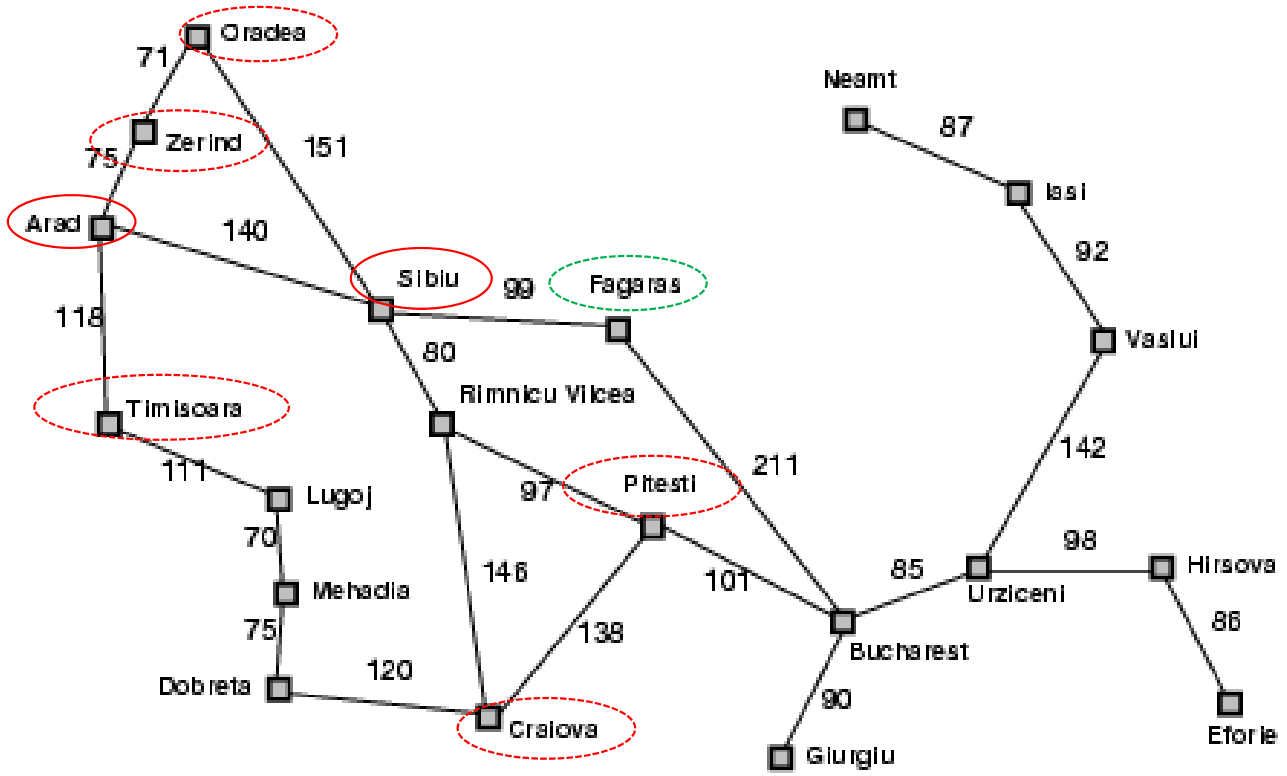

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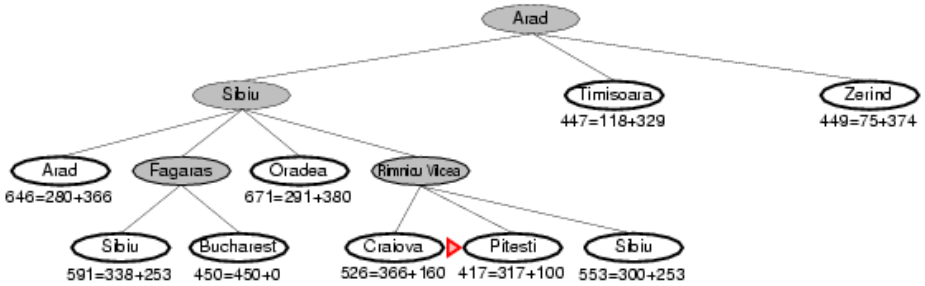

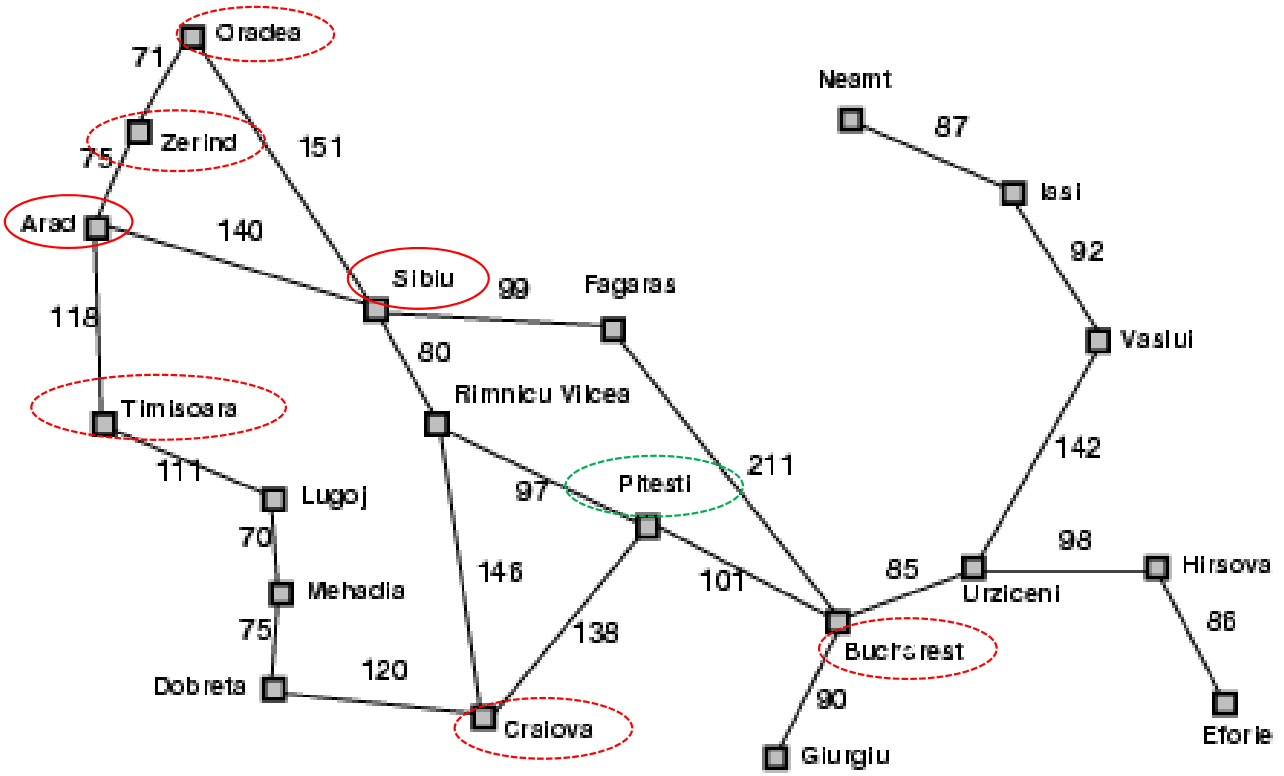

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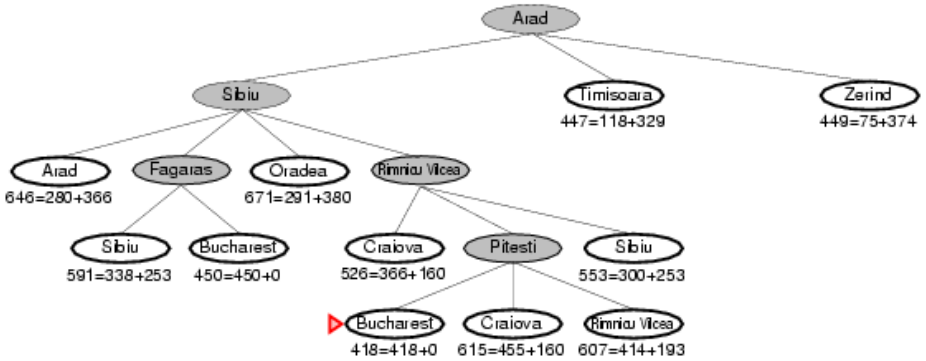

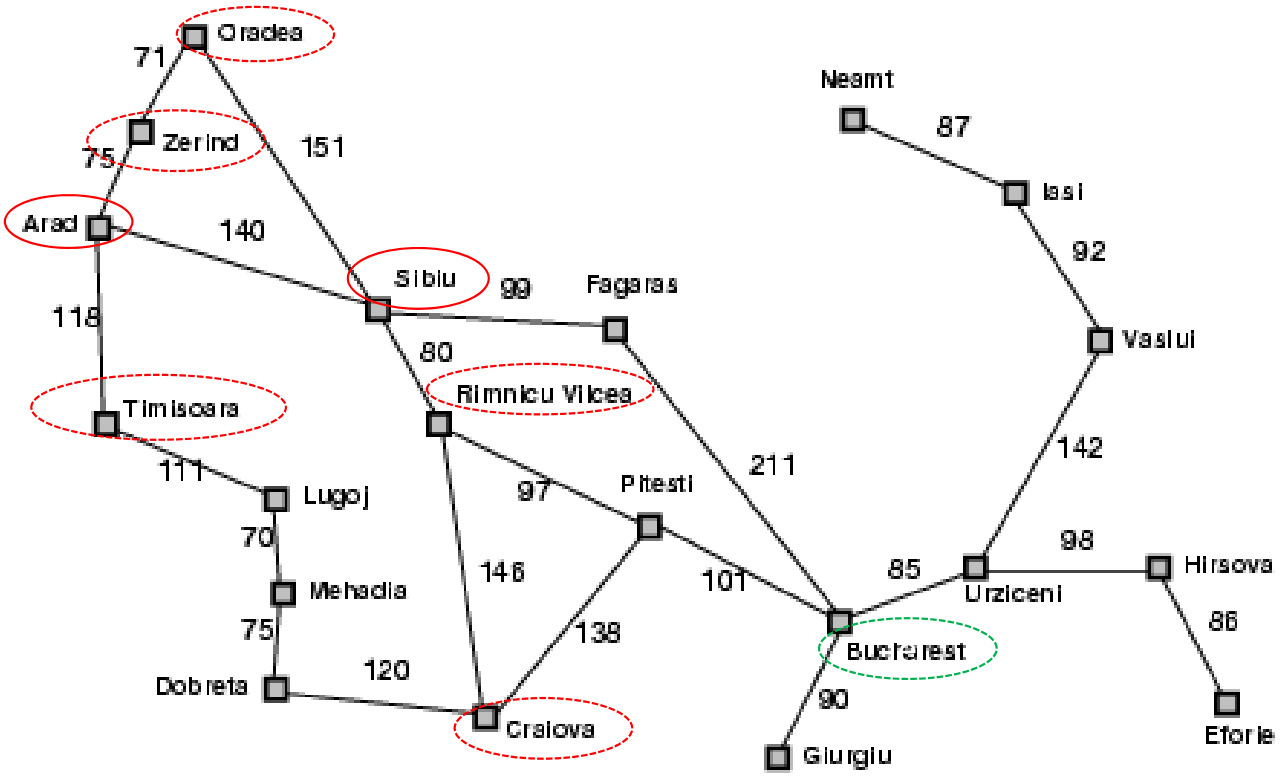

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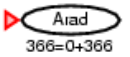
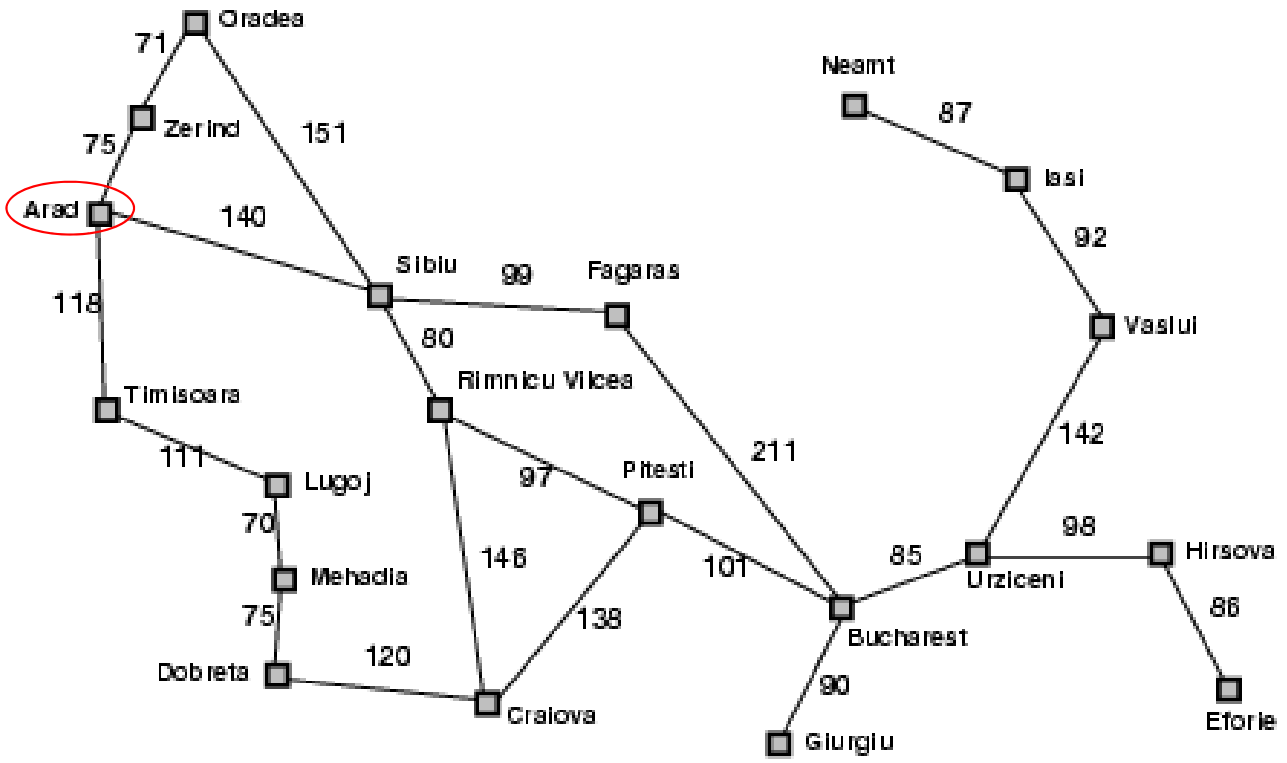
Handling repeated states

- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - Choose a frontier node according to **search strategy** and take it off the frontier
  - If the node contains the **goal state**, return solution
  - Else **expand** the node and add its children to the frontier

- To handle repeated states:
  - Every time you expand a node, add that state to the **explored set**; do not put explored states on the frontier again
  - Every time you add a node to the frontier, check whether it already exists in the frontier with a higher path cost, and if yes, replace that node with the new one



Tree search w/o repeats



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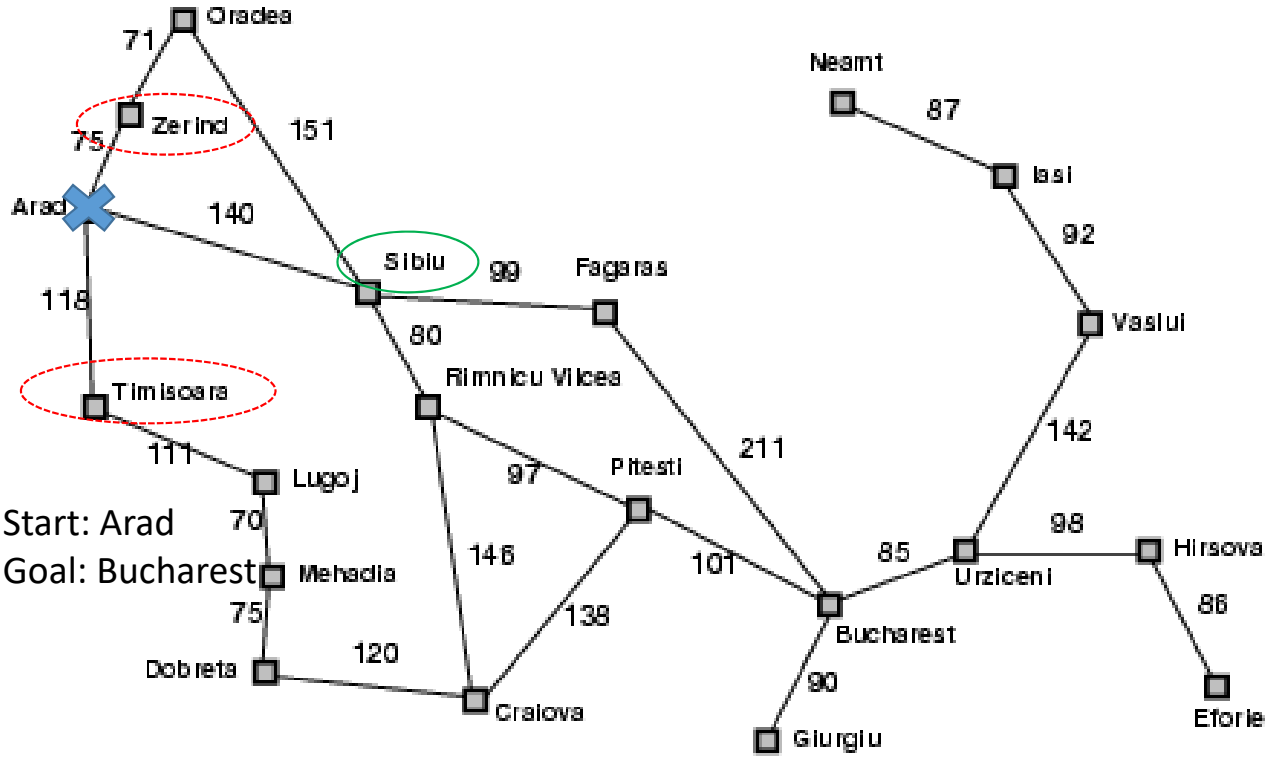
Tree search w/o repeats

Explored:
Arad



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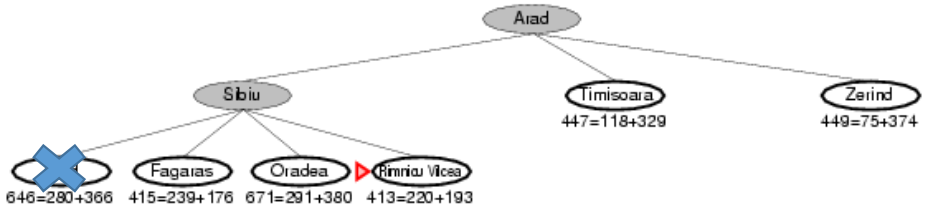
Tree search example



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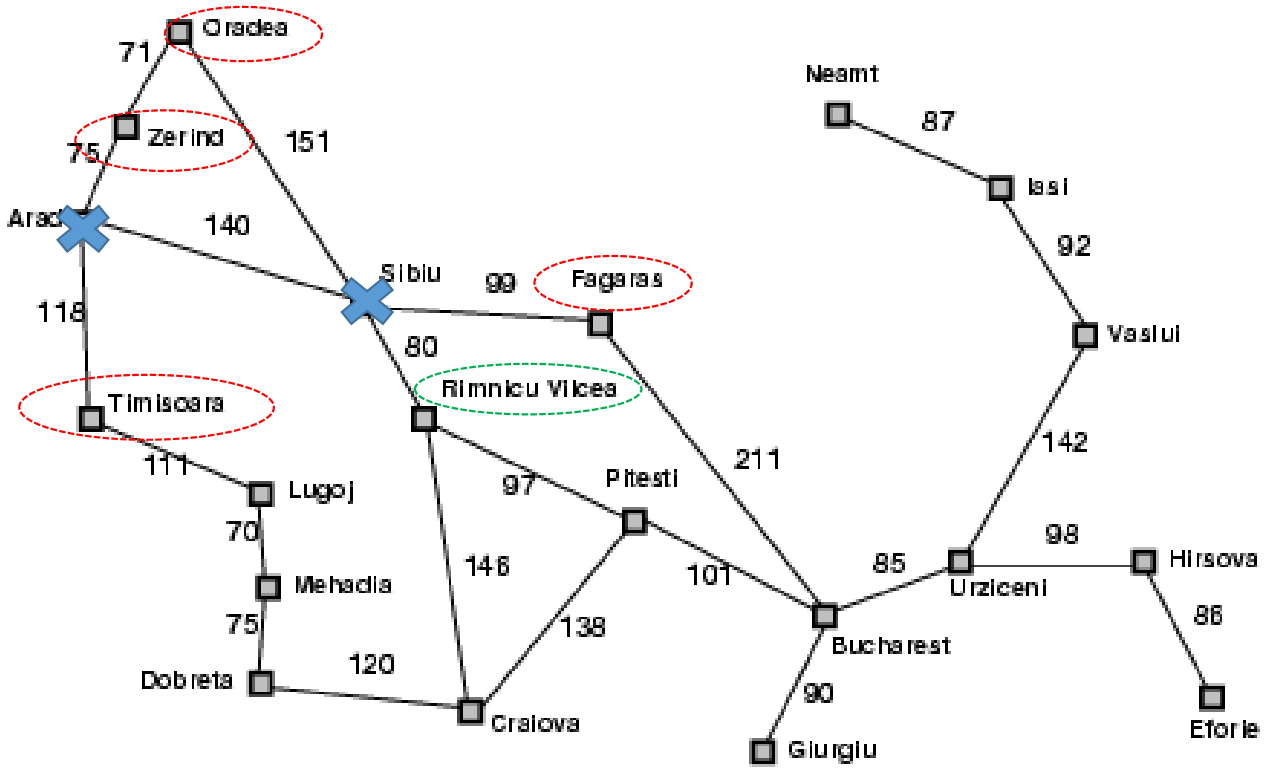

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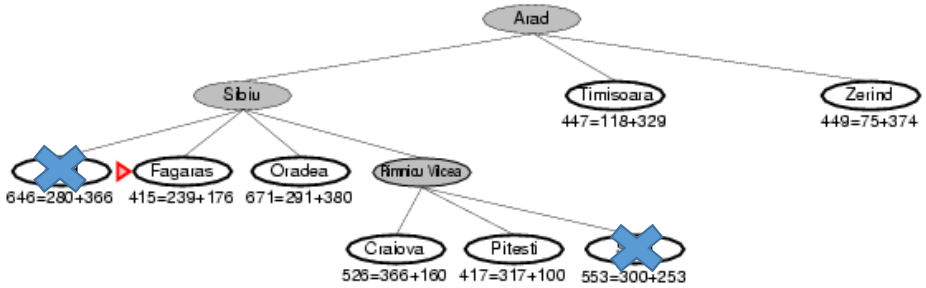

Explored:
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Rimnicu Vilces

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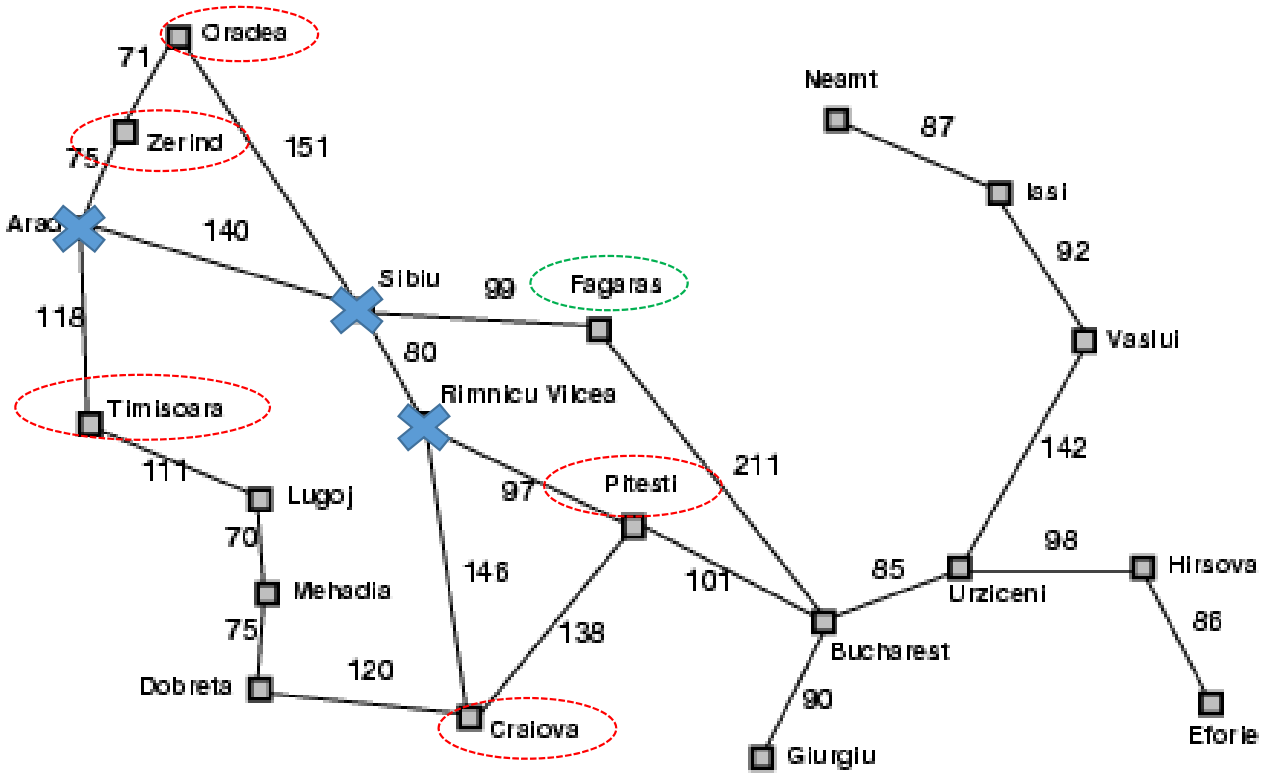

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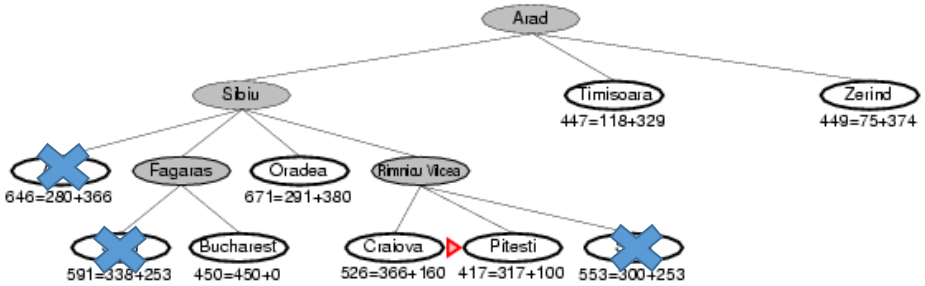
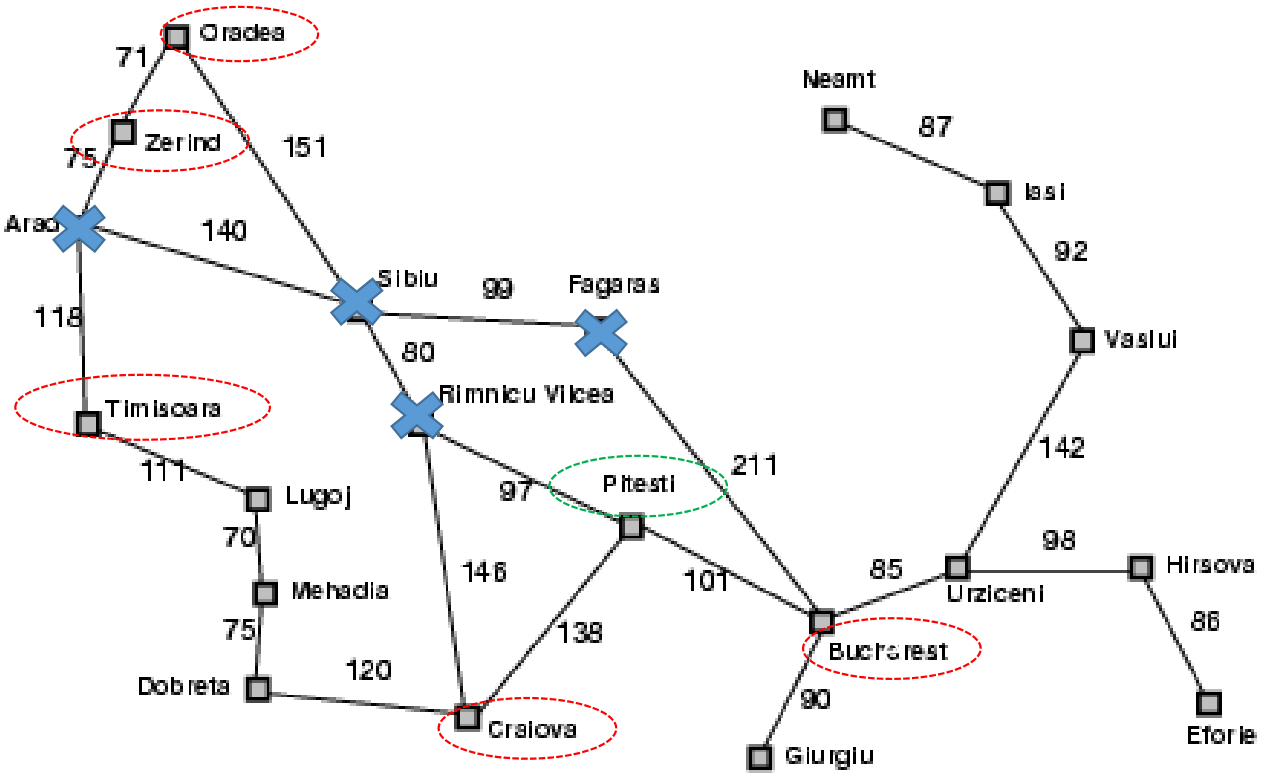


Explored:
Arad
Sibiu
Rimnicu Vilces
Fagaras

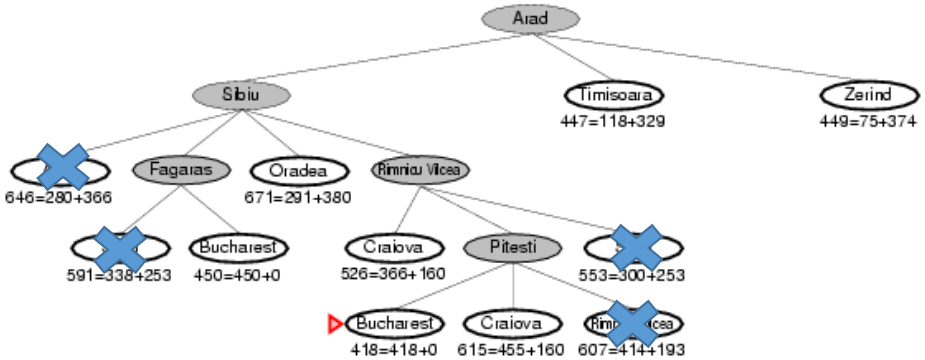
Start: Arad
Goal: Bucharest



Straight–line distance to Bucharest

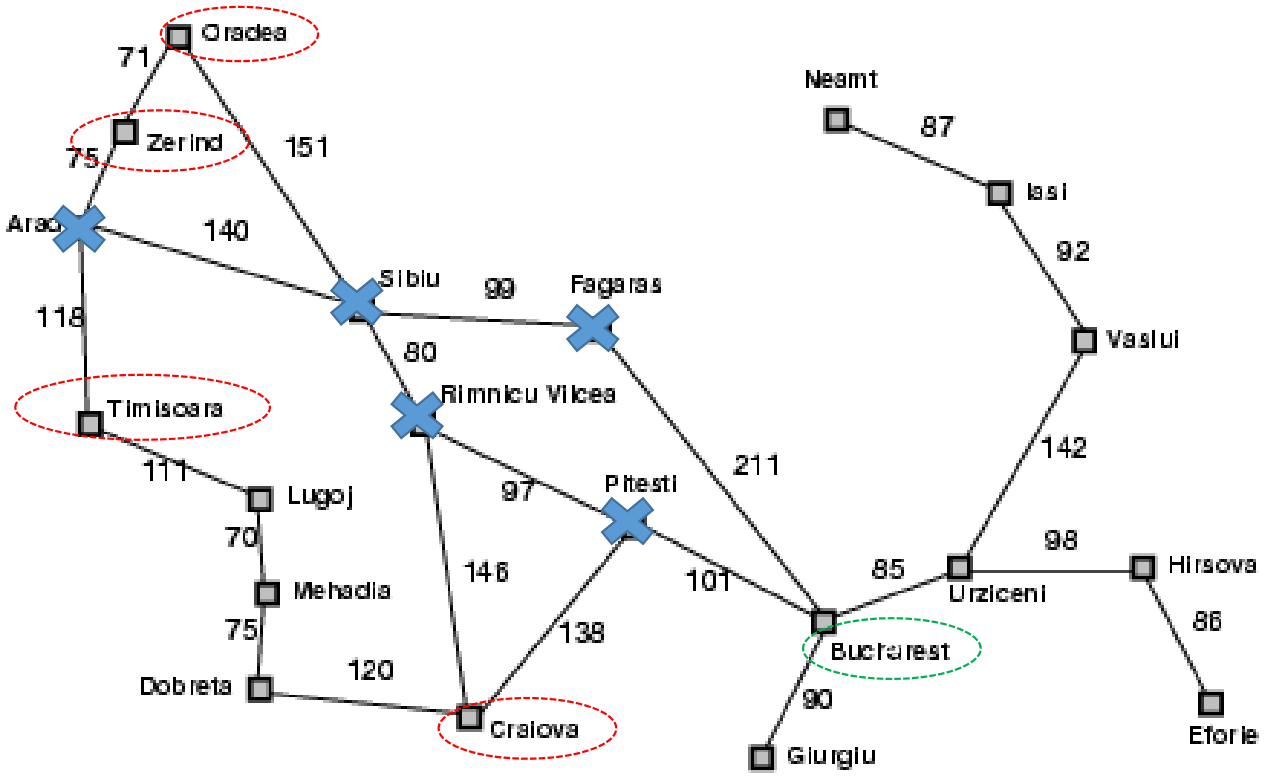
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Tree search example



Explored:
Arad
Sibiu
Rinnicu Vilces
Fagaras
Pitesti

Start: Arad
Goal: Bucharest



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Our first computational savings: avoid repeated states

- Complexity if you allow repeated states: mantissa = number of states you can transition to from any source state, exponent = depth of the tree
- Complexity if you don't allow repeated states: never larger than the total number of states in the world
- For a maze search: # states = # positions you can reach, therefore avoiding repeated states might be all the computational savings you need
- For a task with multiple sub-tasks, e.g., search a maze while cleaning dirt: # states is exponential in the # sub-tasks, therefore we still need better algorithms. That's the topic for next week.