ECE 445

SENIOR DESIGN LABORATORY

FINAL REPORT

Low-Latency Analog Differential Equation Solver

Team #42

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May 15, 2026

Abstract

This project presents a low-latency analog differential equation solver for real-time demonstration of dynamic system behavior. Instead of using a digital computer to solve ordinary differential equations step by step, this design uses analog circuits to perform the main mathematical operations directly [1]. The system is implemented as a PCB-based hardware platform. The analog computing section is the core of the design, constructed using operational amplifiers, adjustable potentiometers, etc. These circuit blocks physically perform mathematical operations such as weighted summation, scaling, integration and feedback

The final system includes an STM32 control unit, an OLED display, a USB-C powered supply circuit and a PCB-based analog computing section. Users can select different input waveforms and adjust basic parameters, while measuring the output waveform with an oscilloscope. This project demonstrates that analog hardware can serve as a direct and rapid way to visualize simple dynamic systems.

Contents

1 Introduction

1.1 Background

Ordinary differential equations (ODEs) are widely used to describe dynamic systems, such as mechanical vibration systems, circuits and control systems. Therefore, ODE serves as an important bridge between mathematical modeling and physical engineering systems.

Traditionally, ODEs have been solved by digital computers through numerical methods such as the Euler method and the Runge-Kutta method. Although these methods are widely used, in real-time hardware applications, repeated computations may increase power consumption and introduce computational latency.

In contrast, analog circuits can implement differential equations directly through continuous-time signal processing. By representing system variables as voltage signals and using operational amplifier circuits, such as integrators and scaling circuits, the mathematical relationships in ODEs can be implemented directly at the circuit level [2].

This project developed a hardware-based differential equation solver for calculating the system response in continuous time. The objective is to reduce computational latency and achieve continuous-time observation of system dynamics.

1.2 Proposal Overview

Our project proposes a PCB-based analog ODE solver. This system does not solve differential equations step by step using digital computers, but performs calculations continuously through circuits.System variables are represented by voltage signals. Operational amplifier circuits are used to build integrators, summing amplifiers, scaling circuits, and feedback loops.

By choosing the circuit mode and adjusting the parameters, the system can solve different types of ODEs. The output signal can be directly observed on the oscilloscope, making this system very useful for studying dynamic systems, control systems and circuit responses.

1.3 Component Units

This system consists of four main component units: the STM32-based control and waveform generation unit, the analog computing unit, and the waveform observation unit. The STM32 unit manages waveform selection, frequency adjustment, user input and display information. The analog computing unit uses operational amplifiers and passive components to achieve summation, scaling, integration and feedback. The waveform observation unit uses an OLED display to show status information, and an oscilloscope performs real-time output measurements.

2 Design Details

2.1 Top-level Design

The system is designed as a low-latency analog differential equation solver with digital control, analog computation, and real-time waveform observation. At the system level, the project is divided into four major parts: the power supply unit, the STM32-based control and waveform generation unit, the analog computing unit, and the waveform display and observation unit.

The STM32 control unit is responsible for user interaction, mode selection, parameter control, OLED display, and waveform generation. The generated input signal is sent to the analog computing circuit, where operational amplifier stages implement the mathematical operations required for solving ordinary differential equations. The analog computing circuit produces an output voltage corresponding to the solution $x(t)$. This output can be observed using an oscilloscope, while the OLED display provides local system information such as operation mode and parameter settings.

The system follows a closed-loop analog computing structure. The input signal $u(t)$ is combined with feedback signals from the system state variables. The weighted summation result is then processed by integrator stages to generate the output state. The output is fed back into the circuit so that the governing differential equation is satisfied continuously in real time.

2.1.1 System Block Diagram

Figure 1 shows the system-level block diagram of the analog differential equation solver. The USB-C power input provides the initial 5 V supply. The power supply unit generates the voltage rails required by both the digital control circuit and the analog computation circuit. The STM32 unit generates the input waveform and controls the user interface. The analog computing unit solves the target differential equation through summing, scaling, integration, and feedback. The output waveform is finally observed using an oscilloscope and display-related interface.

2.1.2 Signal Flow Description

The signal flow begins with the waveform generation module. The STM32 generates or controls the input excitation signal $u(t)$, which can be configured as a step, square, sinusoidal, or other test waveform. This signal is routed to the analog computing unit as the external input of the differential equation.

Inside the analog computing unit, the input signal is combined with feedback signals from the circuit output. For a first-order ODE, the feedback signal is the state variable $x(t)$. For a second-order ODE, the feedback signals include both $x(t)$ and $\dot{x}(t)$. These signals are scaled by resistor networks and summed by an operational amplifier stage.

For a second-order system, the implemented equation can be written as

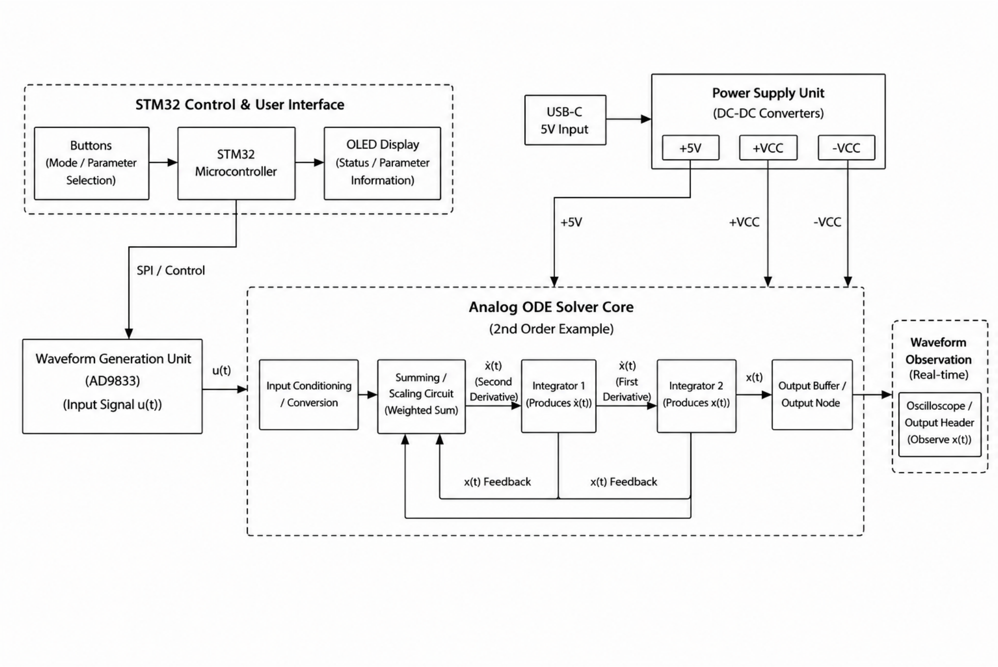


Figure 1: System-level block diagram of the analog differential equation solver.

$$\ddot{x} + a\dot{x} + bx = cu(t). \tag{1}$$

The circuit realizes the rearranged form

$$\ddot{x} = -a\dot{x} - bx + cu(t). \tag{2}$$

Therefore, the summing stage generates a voltage corresponding to $\ddot{x}(t)$. This signal is passed through the first integrator to obtain $\dot{x}(t)$, and then through the second integrator to obtain $x(t)$. The output $x(t)$ is routed to the oscilloscope for observation and is also fed back into the summing stage.

2.1.3 Overall Hardware Architecture

The overall hardware architecture consists of the digital subsystem, analog subsystem, power subsystem, and observation subsystem.

The digital subsystem is built around the STM32 microcontroller. It provides waveform generation, button input reading, mode selection, and OLED display control. The user can select the operation mode and adjust parameters through the interface. The OLED module displays the current mode and related system information.

The analog subsystem is built using LM358 operational amplifiers and passive components. The op-amp stages implement weighted summation, scaling, integration, and feedback. Resistors determine the equation coefficients, while capacitors define the integrator time constants. The analog subsystem is the part that physically solves the differential equation.

The power subsystem receives power from the USB-C input. The digital circuit uses the 5 V rail, while the analog circuit requires positive and negative supply rails so that the op-amps can process signals around the analog ground reference. The power conversion circuit therefore provides the required supply rails for stable operation.

The observation subsystem includes the OLED display and the oscilloscope output. In the current implementation, the OLED display is used as a local status and waveform preview interface. It can show the selected waveform type, input/output mode, frequency setting, amplitude-related setting, and a simplified preview of the input or output waveform. This allows the user to quickly confirm the current operating configuration.

However, because the OLED has limited resolution and refresh rate, it is not used as the main device for quantitative waveform verification. The oscilloscope output is used to observe the actual real-time analog solution waveform $x(t)$. This separation allows the OLED to provide convenient local feedback, while the oscilloscope provides accurate measurement of waveform amplitude, frequency, phase, and transient response.

2.2 Computing Unit

Fig. 2 shows the schematic of the analog computing circuit used for continuous-time ODE solving. The circuit consists of weighted summing stages, cascaded integrators, adjustable coefficient networks, and feedback paths implemented using LM358 operational amplifiers.

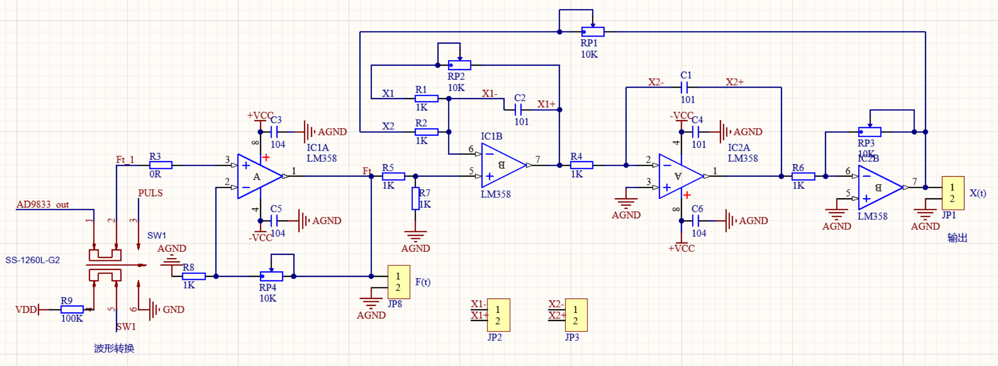


Figure 2: Schematic of the analog computing circuit used for continuous-time ODE solving.

The computing unit is the core analog computing section of the system. Its main purpose is to physically implement the mathematical operations required for solving ordinary differential equations using analog circuits. In this project, the computing unit consists of weighted summing circuits, cascaded integrator circuits, feedback paths, and adjustable coefficient networks implemented using operational amplifiers, resistors, capacitors, and potentiometers.

The analog computing circuit receives the excitation waveform generated by the waveform generation module and continuously processes the signal in real time. Unlike digital numerical methods, the computation is directly represented by continuous analog voltages inside the circuit. Different system states are represented by voltage amplitudes, while the mathematical coefficients of the differential equation are implemented through resistor ratios and adjustable potentiometers.

2.2.1 Operational Amplifier Selection

The LM358 dual operational amplifier was selected as the main analog computing component of the system[3]. Two LM358 chips are used on the PCB, providing four independent operational amplifier channels for implementing the weighted summing stages, integrator stages, and output buffering circuits.

The LM358 was chosen because it provides stable low-frequency analog signal processing performance while maintaining low power consumption and simple peripheral circuitry. Since the analog computer mainly operates on slowly varying continuous-time voltage signals, the bandwidth and slew rate of the LM358 are sufficient for this application. In addition, the dual-op-amp package helps reduce PCB area and simplifies circuit integration.

Another important advantage of the LM358 is its compatibility with dual power supply operation. In this design, the analog computing circuit uses positive and negative voltage rails so that the output signal can swing above and below the analog ground reference. This is necessary because the ODE solution may contain both positive and negative values during the computation process.

The operational amplifiers are placed near the center of the PCB analog section to shorten feedback routing paths and reduce noise coupling. Decoupling capacitors are also placed close to the power pins of the LM358 chips to improve power stability and suppress high-frequency interference.

2.2.2 Weighted Summing Circuit

The weighted summing circuit is responsible for combining the excitation input signal and the feedback state variables from the analog computing loop. This stage performs the linear combination operation required by the ordinary differential equation.

The input waveform generated by the waveform generation module is first routed into the summing circuit. At the same time, the feedback signals representing the system states are also connected back into the summing node through resistor networks and adjustable potentiometers. The operational amplifier then produces a weighted sum of these input signals.

The mathematical relationship of the summing stage can be expressed as:

$$V_{\text{sum}} = k_1 F(t) + k_2 x(t) + k_3 \dot{x}(t)$$

where $F(t)$ is the external excitation signal, $x(t)$ is the output state variable, and $\dot{x}(t)$ is the intermediate state generated by the integrator stages. The coefficients k_1, k_2, and k_3 are determined by the resistor ratios and potentiometer settings.

The weighted summing operation is implemented using resistor networks connected to the inverting input of the operational amplifier. By carefully selecting resistor values, the circuit can represent different mathematical coefficients of the target differential equation. This method allows the analog circuit to directly represent mathematical relationships using physical electrical parameters.

2.2.3 Integrator Circuit

The integrator circuit is the most important part of the analog computer because it performs continuous-time integration of the voltage signals. In this project, the integrator stages are implemented using operational amplifiers with resistor-capacitor feedback networks.

For an ideal inverting integrator, the output voltage is related to the input voltage by:

$$V_{\text{out}}(t) = -\frac{1}{RC} \int V_{\text{in}}(t)\, dt$$

where R is the input resistance and C is the feedback capacitance.

The feedback capacitor stores electrical charge, allowing the operational amplifier output to continuously represent the time integral of the input signal. This property makes the integrator suitable for solving differential equations using analog voltages.

In the PCB design, two cascaded integrator stages are used. The first integrator generates an intermediate state variable, while the second integrator generates the final output state $x(t)$. The output signal is then fed back into the weighted summing circuit, forming a closed-loop analog computation structure.

The resistor and capacitor components associated with the integrators are placed very close to the operational amplifiers on the PCB. This layout strategy reduces parasitic noise and improves integrator stability. Since integrator circuits are highly sensitive to offset voltages and noise accumulation, minimizing routing length is important for maintaining stable analog computation.

2.2.4 Analog Feedback Structure

Fig. 3 shows the signal flow of the closed-loop analog computing structure used for continuous-time ODE solving. The weighted summing stage combines the external excitation input and feedback signals from the state variables. The cascaded integrators then generate the corresponding system states in continuous time.

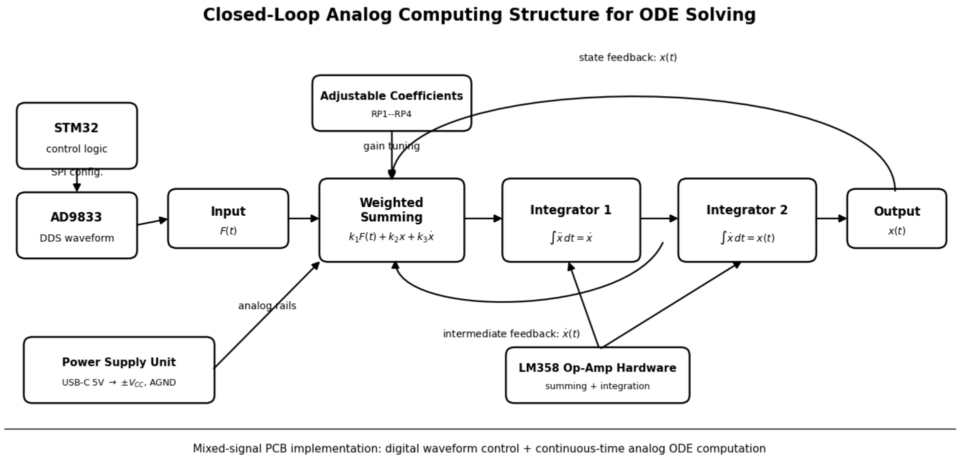


Figure 3: Block diagram of the closed-loop analog computing structure for continuous-time ODE solving.

The analog computing circuit adopts a closed-loop feedback structure to continuously solve the differential equation in real time. After the weighted summing stage generates the combined signal, the cascaded integrators process the signal and generate the system state variables. These output states are then routed back into the summing circuit through feedback paths.

This feedback structure allows the analog hardware to physically reproduce the dynamic behavior of the mathematical system. Instead of calculating discrete numerical values step by step, the analog circuit continuously evolves according to the differential equation represented by the hardware connections.

The closed-loop structure also allows the system to respond immediately to changes in the excitation waveform or coefficient settings. As a result, the analog computer can demonstrate real-time system behavior such as oscillation, damping, and transient response directly on the output waveform.

The signal flow of the analog computing structure can be summarized as:

$$F(t) \rightarrow \text{Weighted Summing} \rightarrow \text{Integrator 1} \rightarrow \text{Integrator 2} \rightarrow x(t)$$

with the output state signals continuously fed back into the weighted summing stage.

2.2.5 Adjustable Coefficient Network

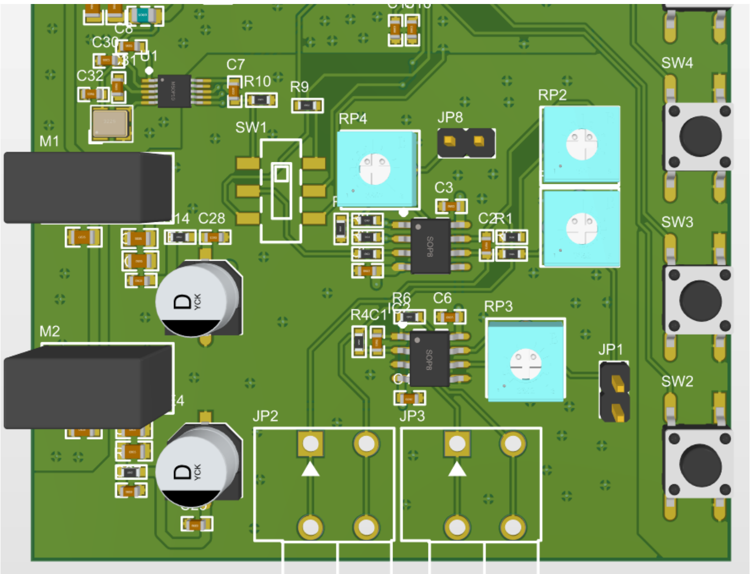


Figure 4: PCB layout of the analog computing section, including LM358 operational amplifiers, RC integrator networks, and adjustable potentiometers.

As shown in Fig. 4, the analog computing section is concentrated near the lower-center region of the PCB to shorten feedback routing paths and reduce noise coupling.

The circuit includes four adjustable potentiometers, RP1, RP2, RP3, and RP4, which are used to tune the coefficients of the analog differential equation. These potentiometers provide adjustable resistance values in the feedback and input paths of the analog computing circuit.

By rotating the potentiometers, the gain of different signal paths can be modified, allowing the analog computer to represent different system parameters without redesigning the circuit. This flexibility allows the hardware to simulate multiple differential equation configurations using the same PCB platform.

The potentiometers are placed near the analog computing section on the PCB to simplify routing and improve user accessibility. This placement allows users to tune the system parameters while observing the waveform output in real time using an oscilloscope.

2.3 Control Unit

The control unit is the digital control part of this system. It is built based on an STM32 microcontroller, which is responsible for managing the waveform generation module and providing a simple interface for users to control the input signals of the analog ODE circuit.

In this project, the STM32 does not directly perform the mathematical solution process. The differential equation is still solved by the analog circuit based on operational amplifiers. Instead, the STM32 is responsible for generating and controlling the input waveforms and sending them to the analog circuit. Users can select waveform types, adjust frequencies, and monitor the current settings through the OLED display screen.

The control unit is mainly composed of five parts: the STM32 microcontroller, SPI communication with the AD9833 waveform generator, the switching system, the parameter regulator, and the frequency and waveform control logic. These parts work together to make the system easier to operate and enable testing of different input conditions without manually changing the circuit components.

2.3.1 STM32 Microcontroller

The STM32 microcontroller plays the role of the main digital controller for the waveform generation subsystem. Its responsibility is to receive user input, process control decisions, and send configuration commands to the waveform generator.

Throughout the system, the STM32 acts as a bridge between the user and the analog ODE circuit. The user interacts with the system through buttons, and the STM32 converts these button operations into waveform settings. For example, when the user selects a sine wave or changes the frequency, the STM32 updates the waveform generator accordingly. The STM32 also controls the OLED display. It continuously updates the displayed information, such as the selected waveform type, output frequency, and pulse state. This makes the system more user-friendly as the user can clearly see the current working state of the waveform generator.

Therefore, the STM32 is not only a controller but also the coordination center of the digital part in the system. By allowing the input of waveforms in a digital manner for modification, it enhances the flexibility of the analog computing system.

2.3.2 SPI Communication with AD9833

The waveform generation module is controlled through communication between the STM32 and the AD9833 waveform generator. The AD9833 is a direct digital synthesis device capable of generating various periodic waveforms, including sine waves, triangular waves and square waves**stm32f334-datasheet**, [4].

The STM32 sends digital control instructions to the AD9833 via the serial communication interface. These instructions contain relevant information about the required waveform type and output frequency. After receiving the instruction, the AD9833 will generate the corresponding analog waveform. The main advantage of using AD9833 lies in its ability to generate stable and adjustable waveforms without the need for STM32 to manually calculate and output each waveform sample. STM32 only needs to send control parameters, while AD9833 will internally complete the waveform synthesis.

This communication structure makes the waveform generation process both efficient and reliable. At the same time, it also reduces the computational burden of STM32, enabling the microcontroller to focus on user input, display updates, and system control, among other aspects.

2.3.3 Switching System

This switching system is operated through four buttons. These buttons enable users to directly control the waveform generator.

The first button is used to switch the waveform type. The system supports three common waveforms: sine wave, triangle wave, and square wave. By pressing this button, users can switch to different waveform modes and test the response of the analog ODE circuit to different input signals. The second and third buttons are used to adjust the output frequency. One button raises the frequency, while the other lowers it. Thus, users can gradually adjust the input signal and observe the performance of the analog circuit under different frequency conditions. The fourth button is used to control the pulse output signal. This pulse signal can be used as additional digital control or trigger signal in the system. It provides greater flexibility for testing or activating certain parts of the analog circuit.

This switching system makes project demonstrations much simpler, as all major control operations can be directly performed on the hardware. Users do not need to reprogram the STM32 or connect the computer each time they change the waveform settings.

2.3.4 Parameter Tuner

The main responsibility of the parameter tuner is to adjust the waveform frequency. In this system, frequency is an important parameter because the response of the analog differential equation circuit depends on the speed of the input signal.

The frequency can be increased or decreased by pressing the corresponding buttons. Each time a button is pressed, the frequency changes by a fixed step size. This makes the adjustment process simple, repeatable, and easy to observe. After the user changes the frequency, the STM32 will update the waveform generator according to the new settings. The OLED display will also refresh the frequency value, so the user can immediately confirm the current output frequency.

This parameter tuner function is crucial for testing analog differential equation circuits. For example, low-frequency signals can be used to observe the slow and stable operation state of the circuit, while high-frequency signals can be used to test the dynamic response and bandwidth limitations of the operational amplifier circuit.

2.3.5 Frequency and Waveform Control

The frequency and waveform control module determines the type of the generated signal and the speed at which the signal changes over time. The STM32 sends the selected waveform mode and frequency settings to the AD9833 waveform generator.

For sine wave output, the waveform generator generates a smooth periodic signal. This type of signal can be used to test the steady-state and frequency response of analog differential equation circuits. For triangle wave output, the voltage increase and decrease are approximately linear. This waveform can be used to observe the response of analog circuits to gradually changing input signals. For square wave output, the signal switches rapidly between high and low levels. This waveform is suitable for testing transient response, rise time, delay, overshoot, and stability performance.

Frequency control determines the frequency of waveform repetition. By changing the frequency, users can set different test conditions for the same analog circuit. This makes the waveform generator a flexible input source for real-time differential equation solving systems.

2.4 Power Supply Unit

Fig. 5 shows the power supply circuit used in the system. The power supply unit generates regulated analog and digital voltage rails for the analog computing circuit and embedded control system. The circuit includes a USB-C power input interface, isolated DC-DC conversion modules, filtering networks, and decoupling capacitors for stable mixed-signal operation.

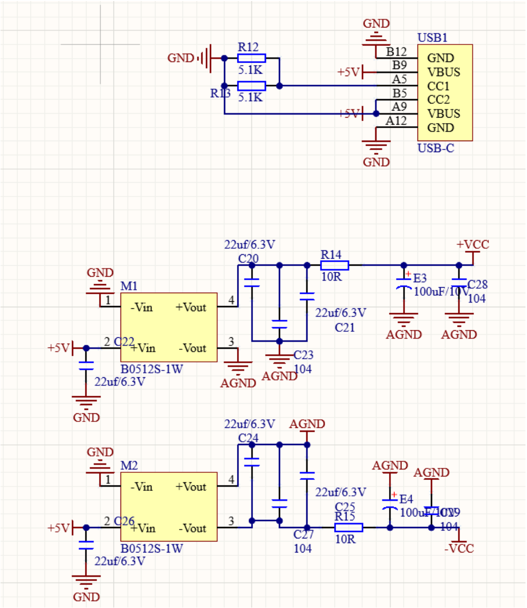


Figure 5: Power supply circuit used to generate regulated analog and digital voltage rails.

2.4.1 USB-C Input Power

The system is powered through a USB-C connector that provides a 5 V DC input supply. Pull-down resistors are connected to the CC pins to ensure proper USB-C power negotiation and stable voltage delivery. The 5 V rail serves as the primary power source for both the digital control circuits and the analog computing section.

2.4.2 Positive and Negative Supply Generation

Continuous-time analog computation requires both positive and negative supply voltages for proper operational amplifier operation. To generate the dual supply rails, the system uses B0512S-1W isolated DC-DC converter modules.

The generated voltage rails are denoted as $+V_{CC}$ and $-V_{CC}$ and are used to power the LM358 operational amplifiers in the analog computing circuit. The dual-supply configuration allows the analog signals to swing around ground potential and enables stable implementation of integrator circuits and feedback networks.

2.4.3 Power Filtering and Decoupling

Filtering capacitors and decoupling capacitors are placed near the power conversion modules and integrated circuits to reduce voltage ripple and suppress high-frequency noise. Electrolytic capacitors are used for bulk energy storage, while ceramic capacitors are used for high-frequency filtering and local decoupling.

Additional RC filtering stages are included at the analog supply outputs to further improve voltage stability and reduce noise coupling into the analog computing section. These filtering components help maintain stable analog computation performance and improve signal integrity.

2.4.4 Analog and Digital Ground Separation

The PCB design separates analog ground (AGND) and digital ground (GND) to reduce switching noise coupling from the digital control circuits into the analog computing circuits. Since the analog integrator and feedback networks are highly sensitive to voltage fluctuations and high-frequency interference, careful grounding and power routing are necessary to maintain stable continuous-time computation.

The analog supply rails and grounding paths are concentrated near the analog computing section to shorten return current paths and reduce unwanted electromagnetic interference.

2.5 User I/O Interface

The user I/O interface connects the user to the STM32 control system. It includes a waveform generator, buttons, an OLED display screen, and a section that connects to the output of the analog ODE circuit. The purpose of this interface is to make the system easy to operate. Users do not need to use external instruments to manually change the waveform, but can directly control the type and frequency of the waveform through this board. The user I/O interface can also provide feedback information. The OLED display screen will show the current waveform type, frequency, and pulse output status. This helps users verify the system settings before applying the signal to the analog circuit.

Overall, the user I/O interface integrates the STM32 and the waveform generator into an independent control module. This makes the analog computing system more interactive, adjustable, and more suitable for demonstrations.

2.5.1 Waveform Generator

The waveform generator provides input signals to the analog differential equation circuits. In this project, the waveform generator can output sine waves, triangular waves, and square waves. These waves are used as test input signals for the differential equation solver based on operational amplifiers.

Sine waves are helpful for analyzing the characteristics of periodic systems. Since sine signals are widely used in circuit analysis and frequency response tests, this waveform is one of the most important input types. Triangular waves provide a linearly changing input signal. It can be used to test whether the analog circuit can smoothly follow the gradual process and whether there is distortion during the rising or falling slope. Square waves change suddenly between two voltage levels. This characteristic helps observe transient phenomena, such as response delay, overshoot, undershoot, and settling time, etc.

The waveform generator is controlled by STM32. When the user changes the waveform type or frequency, STM32 updates the waveform generator, causing the output signal to change accordingly. Subsequently, the generated waveform is sent to the analog ODE circuit as an excitation input.

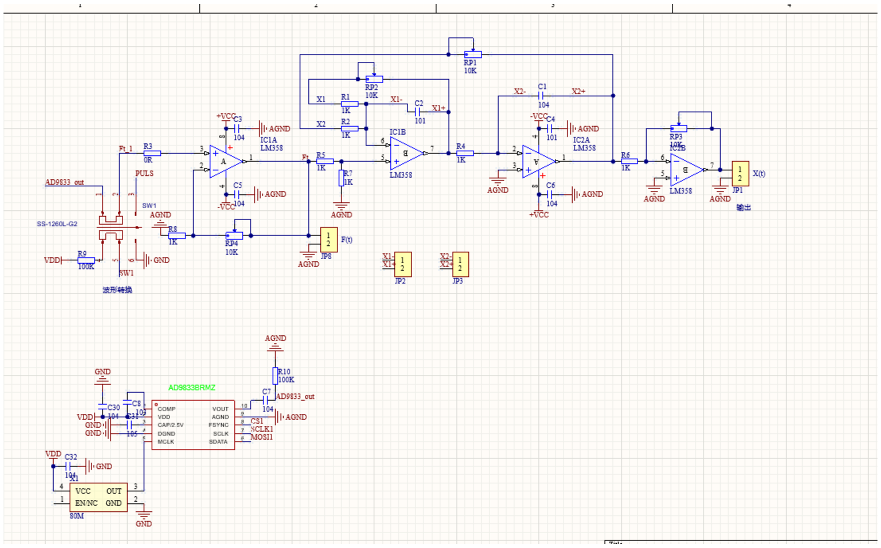


Figure 6: Schematic of the waveform generator and analog ODE signal processing stage.

2.5.2 Push Buttons

The push buttons provide the main user input for the STM32 control system. In this project, four buttons, K1, K2, K3, and K4, are used to control the waveform generator directly. This allows the user to change the waveform settings without connecting the system to a computer or modifying the code.

Each button has a specific function. K1 is used to switch the type of output waveform. The available waveform options include sine wave, triangle wave and square wave. K2 and K3 are used to adjust the output frequency. K2 gradually increases the frequency, while K3 gradually decreases the frequency. K4 is used to switch the pulse output between high level and low level.

Button	**Function**	**Description**
K1	Waveform Selection	Switches between sine, triangle, and square wave outputs.
K2	Frequency Increase	Increases the output frequency by a fixed step.
K3	Frequency Decrease	Decreases the output frequency by a fixed step.
K4	Pulse Control	Toggles the pulse output between high and low states.

Table 1: Functions of the Push Buttons

This push-button interface makes the system operation simple and practical. During the testing process, users can quickly change the waveform type or frequency and observe the response of the simulated ODE circuit. This design enhances the usability of the system and supports real-time experimental demonstrations.

2.6 Waveform Display Device

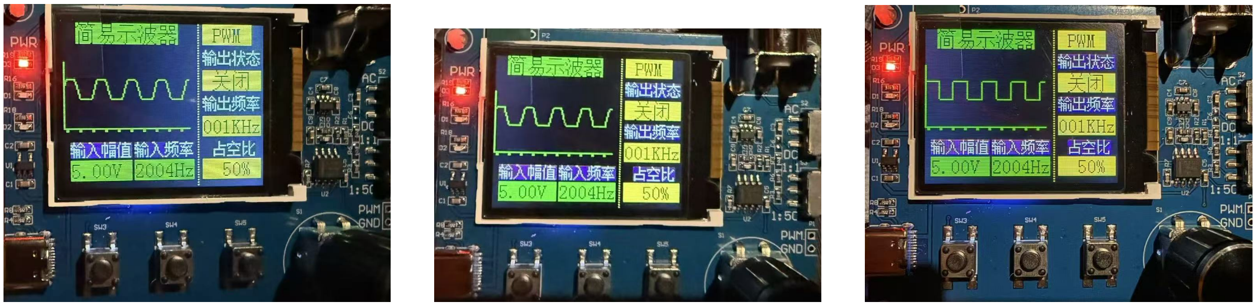
The waveform display subsystem provides visual feedback for both system operation and waveform generation. In the final implementation, the OLED module is used as a local status and waveform preview display, while the oscilloscope is used as the primary device for accurate real-time waveform observation.

The OLED display shows the selected waveform type, input/output status, frequency, amplitude-related settings, and a simplified waveform preview. This allows the user to check the current configuration directly from the device. However, because the OLED has limited resolution, limited refresh rate, and a small display area, it is not ideal for precise waveform measurement. Therefore, the OLED is mainly used for user-interface feedback rather than final quantitative verification.

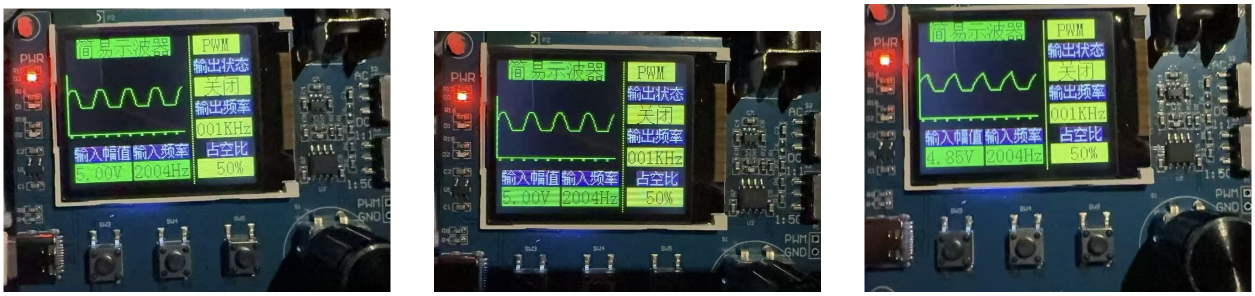
The actual analog output waveform $x(t)$ is observed using an oscilloscope. The oscilloscope provides higher temporal resolution and more accurate amplitude and frequency measurement. Therefore, the oscilloscope is used to verify the real-time response of the analog differential equation solver.

2.6.1 OLED Status and Waveform Preview

The OLED display is connected to the control and waveform generation subsystem. It shows the waveform mode selected by the user and provides a preview of the generated waveform. Figure 7 shows the OLED display results for sine, triangle, and square wave modes. The first row shows the input waveform display, and the second row shows the output waveform display.



(a) Input sine wave (b) Input triangle wave (c) Input square wave



(d) Output sine wave (e) Output triangle wave (f) Output square wave

Figure 7: OLED display preview for sine, triangle, and square waveform modes. The first row shows input waveform display, and the second row shows output waveform display.

Although the OLED display successfully shows the selected waveform type and provides a basic waveform preview, the displayed waveform is mainly qualitative. The OLED image is useful for confirming that the control interface and waveform selection are working, but it is not used as the main source for measuring amplitude, frequency, phase shift, or transient response.

2.6.2 Oscilloscope-based Real-time Waveform Observation

The oscilloscope is used as the main waveform observation device. The output node of the analog computing circuit is connected to the oscilloscope so that the solved voltage signal $x(t)$ can be observed directly in real time.

Compared with the OLED display, the oscilloscope provides more accurate waveform measurement. It can be used to measure amplitude, frequency, phase shift, rise time, overshoot, damping behavior, and settling time. These values are important for comparing the measured circuit response with the theoretical first-order and second-order ODE models.

For the first-order ODE mode, the oscilloscope is used to verify the exponential rise or decay response and to measure the time constant. For the second-order ODE mode, the oscilloscope is used to observe the oscillation frequency, damping envelope, and settling time. Therefore, the OLED display supports user interaction, while the oscilloscope provides the final real-time waveform verification.

2.7 Analog Computer Hardware Integration

Fig. 8 shows the complete PCB implementation of the analog computing system. The PCB integrates the analog computing section, STM32 control unit, waveform generation circuit, power supply modules, and user interface components into a compact mixed-signal hardware platform.

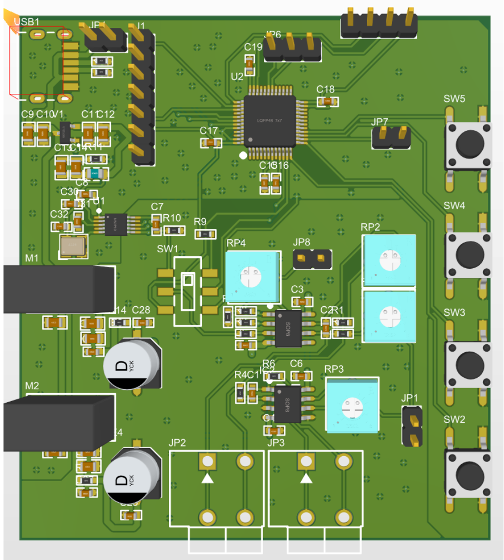


Figure 8: Complete PCB layout of the analog computing system.

2.7.1 Analog Computer Circuit Integration

The analog computer system integrates multiple functional modules on a two-layer PCB platform. The PCB includes the analog computing section, embedded control unit, waveform generation circuit, power supply modules, user interface buttons, and output headers.

The analog computing section is concentrated near the lower-center region of the PCB to shorten feedback routing paths and reduce noise coupling. The operational amplifiers, resistor networks, capacitors, and adjustable potentiometers are placed close to each other to improve analog signal integrity and minimize parasitic effects.

The STM32 microcontroller and digital control circuits are placed near the upper region of the PCB to separate the digital switching activity from the sensitive analog computing circuits. The waveform generation circuit based on the AD9833 DDS module is located near the control section to simplify SPI communication routing.

The power supply modules are placed on the left side of the PCB to isolate the DC-DC conversion stages from the analog feedback network. Separate analog ground (AGND) and digital ground (GND) regions are used to reduce noise interference between mixed-signal subsystems.

The PCB also includes multiple headers and test interfaces for waveform observation, debugging, and parameter adjustment. The final integrated hardware platform supports continuous-time analog computation and real-time waveform observation using external oscilloscopes.

2.7.2 PCB Functional Partitioning

To improve system stability and reduce interference between mixed-signal subsystems, the PCB is divided into several functional regions, including the analog computing section, digital control section, waveform generation section, and power supply section.

The analog computing section occupies the lower-center region of the PCB and contains the LM358 operational amplifiers, resistor-capacitor integrator networks, and adjustable potentiometers. This region is designed to minimize feedback routing length and reduce parasitic coupling in the continuous-time analog computation paths.

The digital control section is located near the upper region of the PCB and includes the STM32 microcontroller, SPI communication routing, and digital interface circuitry. Separating the digital subsystem from the analog subsystem helps reduce switching noise and electromagnetic interference.

The waveform generation circuit based on the AD9833 DDS module is placed near the digital control section to simplify SPI signal routing and reduce communication trace complexity. The power supply modules are concentrated near the left side of the PCB to isolate DC-DC conversion noise from the analog feedback network.

2.7.3 PCB Layout Design

The analog computer system is implemented on a two-layer PCB structure consisting of top and bottom signal routing layers. The PCB layout is designed to balance routing simplicity, signal integrity, and compact hardware integration.

Special attention is given to the routing of analog feedback paths and integrator networks. The analog traces are kept as short as possible to reduce parasitic resistance, capacitance, and external noise coupling. Sensitive analog routing is separated from high-frequency digital control traces to maintain stable analog computation performance.

The placement of the operational amplifiers and RC integrator networks is optimized to reduce loop area and improve feedback stability. Decoupling capacitors are placed close to integrated circuits and power conversion modules to suppress high-frequency noise and voltage ripple.

Ground routing is carefully managed using separate analog ground (AGND) and digital ground (GND) regions. The analog ground network is concentrated near the analog computing section to improve return current stability and reduce interference from the digital subsystem.

The PCB also includes multiple external headers for waveform observation, debugging, parameter tuning, and signal output connection to external oscilloscopes.

2.7.4 Circuit Functionality and Mode Switching

The analog computer system supports multiple operating modes and waveform configurations through a combination of digital control and analog parameter adjustment.

The STM32 control unit configures the AD9833 waveform generator to produce different excitation waveforms, including sine waves, square waves, and triangular waves. The generated excitation signal is then fed into the analog computing section as the system input signal.

The analog computing circuit processes the excitation signal using weighted summing stages, integrator networks, and closed-loop feedback paths to generate the corresponding ODE solution waveform. Adjustable potentiometers are used to modify coefficient weights and feedback gains, allowing the system dynamics to be tuned in real time.

Push-button switches are used for waveform selection, parameter switching, and user interaction. The final output waveform can be observed through external oscilloscope interfaces connected to the PCB output headers.

The combination of digital waveform generation and continuous-time analog computation allows the system to demonstrate real-time physical implementation of ordinary differential equation solving.

2.7.5 PCB Design Rule Verification

PCB design rule verification (DRC) is performed using Altium Designer to check electrical connectivity, routing constraints, and manufacturing compatibility.

The verification process includes clearance checking, routing validation, solder mask inspection, and silk layer overlap analysis. The PCB design does not contain short-circuit violations or unrouted nets, indicating that the electrical connections are correctly completed.

Most reported DRC items are related to silk layer overlap and solder mask spacing, which are primarily associated with component labeling density and compact PCB placement rather than functional electrical errors.

3 Tolerance Analysis

The performance of the analog differential equation solver is affected by the tolerance of physical circuit components. Since the system represents mathematical coefficients using resistor ratios and represents integration using resistor-capacitor networks, component variations can cause the actual circuit response to deviate from the ideal theoretical response.

The main tolerance sources include resistor tolerance, capacitor tolerance, potentiometer adjustment error, op-amp offset voltage, finite op-amp bandwidth, and power supply variation. Among these factors, resistor and capacitor tolerances have the most direct effect on the implemented ODE coefficients and the integration time constants.

For an op-amp integrator, the ideal relationship is

$$V_{\mathrm{out}}(t) = -\frac{1}{RC}\int V_{\mathrm{in}}(t)\,dt. \tag{3}$$

Therefore, the integration behavior depends on the time constant

$$\tau = RC. \tag{4}$$

The relative error of the time constant can be estimated as

$$\frac{\Delta\tau}{\tau} = \sqrt{\left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta C}{C}\right)^2}. \tag{5}$$

If 1% resistors and 5% capacitors are used, the expected relative error is

$$\frac{\Delta\tau}{\tau} = \sqrt{(0.01)^2 + (0.05)^2} \approx 0.051, \tag{6}$$

which is approximately 5.1%. This means that the actual integration rate may be slightly different from the nominal design value. As a result, the measured waveform may show differences in rise time, decay rate, phase shift, or oscillation behavior.

In the summing and scaling circuit, the ODE coefficients are determined by resistor ratios. For an inverting summing amplifier, the output can be expressed as

$$V_{\mathrm{out}} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right). \tag{7}$$

Thus, resistor tolerance changes the effective coefficient values in the implemented differential equation. Potentiometers are used in the circuit to provide adjustable feedback gains, which helps compensate for component mismatch during testing.

3.1 First-order ODE

For the first-order mode, the target equation can be written as

$$\dot{x} + ax = bu(t). \tag{8}$$

The circuit implements the rearranged form

$$\dot{x} = -ax + bu(t). \tag{9}$$

In this equation, a determines the feedback strength and response speed, while b determines the input gain. In the physical circuit, these coefficients are implemented using resistor ratios in the summing amplifier. Therefore, resistor tolerance causes the actual coefficient values to deviate from the designed values.

If the coefficient a is larger than the nominal value, the first-order response becomes faster. For a step-like input, the output reaches its steady-state value more quickly. If a is smaller than the nominal value, the response becomes slower. Therefore, tolerance in the feedback resistor path mainly affects the rise time or decay time of the output waveform.

The capacitor and resistor values in the integrator also affect the first-order response. Since the integration rate depends on the RC time constant, variation in either component changes the effective time scale of the circuit. This can cause the measured waveform to have a slightly different exponential slope compared with the ideal model.

For a stable first-order system, the expected step response has an exponential form:

$$x(t) = x_\infty \left(1 - e^{-at}\right), \tag{10}$$

where x_∞ is the steady-state value. Component tolerance does not change the general first-order behavior, but it can change how quickly the response approaches steady state and the final output amplitude.

In practice, the first-order tolerance effect is observed by comparing the measured waveform with the expected exponential trend. A small mismatch in time constant, amplitude, or phase is expected due to resistor and capacitor tolerances, op-amp offset, and measurement noise. The response is considered acceptable if the waveform remains stable and follows the expected first-order behavior.

3.2 Second-order ODE

For the second-order mode, the target equation can be written as

$$\ddot{x} + a\dot{x} + bx = cu(t). \tag{11}$$

The circuit implements the rearranged form

$$\ddot{x} = -a\dot{x} - bx + cu(t). \tag{12}$$

In this equation, a corresponds to the damping-related term, b corresponds to the state feedback term, and c corresponds to the input gain. These coefficients are implemented using resistor ratios in the summing and feedback network. Therefore, resistor tolerance can change the effective damping, oscillation behavior, and output amplitude of the second-order response.

The second-order circuit also uses two cascaded integrator stages. Since each integrator depends on an RC time constant, tolerance in the resistors and capacitors affects the time scale of both integration stages. Compared with the first-order case, the second-order mode is more sensitive to tolerance because errors from multiple stages can accumulate.

If the effective damping coefficient is higher than expected, the output waveform may decay faster and show less oscillation. If the damping coefficient is lower than expected, the output may show stronger oscillation or a longer settling time. Similarly, changes in the effective state feedback coefficient can shift the oscillation frequency or change the shape of the transient response.

In the ideal case, different parameter selections can produce underdamped, critically damped, or overdamped responses. In the actual circuit, component tolerance may cause the measured waveform to deviate from the ideal damping condition. For example, a response designed to be close to critically damped may appear slightly underdamped or overdamped due to resistor and capacitor mismatch.

The second-order tolerance effect is observed by checking the waveform shape, oscillation trend, damping behavior, and stability. Small deviations in amplitude, frequency, and settling behavior are expected. The circuit is considered acceptable if the output remains stable and the measured waveform is qualitatively consistent with the expected second-order ODE response.

4 Costs

4.1 Parts

The main cost of this project comes from the PCB and components. The approximate parts cost of this project is shown in Table 2.

Table 2: Parts Costs

Part	Quantity	Retail Unit Cost (RMB)	Actual Cost (RMB)	Notes
PCB fabrication	2	100.00	200.00	Main PCB
Capacitors (all types)	34	0.26	8.70	all capacitors
LM358 dual op-amps	2	1.20	2.40	IC1, IC2
OLED display	1	15.00	15.00	J1
Connectors	9	0.34	3.05	HDR-1X2/1X3/1X4
10uH SMD inductor	1	1.20	1.20	L1
B0512S-1W DC-DC modules	2	18.00	36.00	M1, M2
Fixed resistors	15	0.02	0.30	fixed resistors
10K potentiometers	4	1.50	6.00	RP1–RP4
Switches (all types)	5	0.36	1.80	switches
waveform generator	1	65.00	65.00	AD9833BRMZ
MCU	1	35.00	35.00	STM32F334C8T6
LDO regulator	1	0.60	0.60	XC6206P332MR
80M crystal oscillator	1	2.00	2.00	X1
spare parts	1	40.00	40.00	Spare
Total			**417.05**	

4.2 Labors

During the project, our team worked on circuit design, PCB layout, soldering, debugging, testing, and report preparation. Each of the four team members spent about 9 hours per week over a 14-week period. Therefore, the total labor time is estimated as 504 hours. With an estimated labor rate of 40 RMB per hour, the total labor cost is 20,160 RMB.

5 Conclusion

5.1 Accomplishments

This project designed and built an analog ODE solver based on amplifier circuits. The system uses voltage signals to represent variables in a second-order differential equation. Using integrators, amplifiers, and feedback loops to form the computing structure. The input and solution waveform can be clearly shown on the screen.

The circuit is able to generate continuous-time output signals and can be directly observed on an oscilloscope. This project connected the theory of differential equations with real implementation of analog hardware. Overall, the project shows that analog circuits can provide a low-latency way to visualize simple dynamic systems.

5.2 Ethical Considerations

This project is mainly used for demonstration and engineering design. Since the system is driven by circuits, safety is an important consideration. During the testing process, we need to ensure that the power connection is correct. The circuit should operate within the rated voltage and current limits of all components. Improper wiring or over-voltage can damage the circuit or cause safety hazards.

Another ethical consideration is the accuracy of the output results. Since this simulation solver can be used to demonstrate system behavior, we should clearly explain its limitations. Component tolerances and limited operational amplifier bandwidth can cause errors between the measured waveform and the theoretical solution. Therefore, without further verification, this system should not be used as a final decision-making tool.

5.3 Future Work

There are several possible improvements for future work. Firstly, the accuracy of the circuit can be enhanced by using higher-precision resistors and operational amplifiers. Secondly, it can optimize the PCB layout, reduce noise and improve signal stability. In addition, more ODE modes can be added to enable the system to solve a wider range of equations. A better user interface can also be developed to make parameter adjustment easier.

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Appendix A Appendix

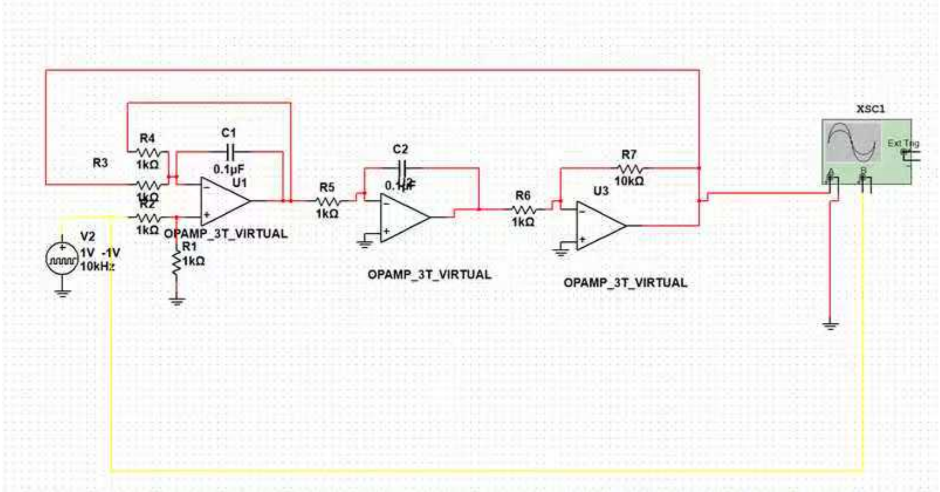

Figure 9: Initial principle design schematic of the analog ODE solver circuit.

Fig. 9 shows our initial principle design of the analog ODE solver. At the beginning of the project, we used this schematic to check the basic idea of the circuit. The op-amp stages were arranged to perform summing, integration, feedback, and output observation. This helped us understand how the differential equation could be represented by real circuit blocks before moving to PCB design.